INDIAN AGRICULTURAL
RESEARCH INSTITUTE, NEW DELHI.

I. A. R. I. 6.
MGIPC—S1—51 AR/57—3-4-58—5,000.

29058/36

PROCEEDINGS

OF THE

ROYAL SOCIETY OF EDINBURGH

PROCEEDINGS

OF

THE ROYAL SOCIETY OF EDINBURGH

VOL. LIII.

29058/06

1932-1933

29058
IARI

EDINBURGH
PRINTED BY NEILL AND COMPANY, LIMITED
MCMXXXIV

CONTENTS

PAGE

1 On the Development of the Olfactory Organ in *Protopterus* By Margaret H Fullarton, B Sc, Assistant in the Zoology Department, University of Glasgow. *Communicated by* Professor J Graham Kerr, F R S (With Six Text-figures) Issued separately January 16, 1933, . . . 1

2 A Matrix Notation for Mendelian Populations By Professor Lancelot Hogben, M A, D Sc (From the Department of Social Biology in the University of London) Issued separately January 16, 1933, . 7

3 Supernumerary Pectoral Fins in *Raia circularis* Loudon By W F Harper, Ph.D., M B., Department of Anatomy, University College (University of St Andrews), Dundee *Communicated by* Professor A D Peacock (With Two Text-figures) Issued separately January 16, 1933, . . 26

4 On Properties of Null Geodesics, and their Application to the Theory of Radiation By W O. Kermack, M A, D.Sc, W. H M'Crea, M A, Ph D, and Professor E T. Whittaker, F R S Issued separately January 24, 1933, 31

5 Expansion and Contraction of Coal caused respectively by the Sorption and Discharge of Gas By Professor Henry Briggs, D Sc, Ph.D, and R P Sinha, B Sc., Assoc H-W Coll (With Two Text-figures) Issued separately January 30, 1933, ... 48

6. On the Graduation of Data by the Orthogonal Polynomials of Least Squares By A. C. Aitken, M A, D Sc Issued separately February 3, 1933, . . 54

7. On the Measurement of Spatial Distance in a Curved Space-time. By H S. Ruse, M.A, D Sc. Issued separately March 30, 1933, . . 79

*8 A Case of Non-disjunction in the Fowl By Professor F A E Crew, M D., D Sc., Institute of Animal Genetics, University of Edinburgh. (With Five Plates.) Issued separately March 30, 1933. 89

9 The Intellectual Resemblance of Twins By Louis Herrman, M.A, Research Assistant in the Department of Social Biology, and Lancelot Hogben, M.A., D.Sc, Research Professor of Social Biology in the University of London Issued separately April 12, 1933, ... 105

*10. Spermatogenesis in *Drosophila obscura* Fallen. I. The Cytological Basis of Suppression of Crossing-over. By P. Ch. Koller, Ph.D, and Thelma Townson, M.A., Institute of Animal Genetics, University of Edinburgh. *Communicated by* Professor F A E. Crew, M.D., D.Sc. (With Three Plates.) Issued separately April 22, 1933, ... 130

PAGE

11 On the Pituitary in *Lepidosiren* and its Development. By T. Kerr, M.A., Assistant in the Zoology Department, Queen's University of Belfast. *Communicated by* Professor J Graham Kerr, F.R.S. (With Five Text-figures.) Issued separately June 13, 1933. 147

12. Matrices and Continued Fractions By Professor H W. Turnbull, F.R.S. Issued separately June 23, 1933. 151

13 The Formation of Rock Joints and the Cleat of Coal By Emeritus Professor P F Kendall, D Sc, F.R.S., and Professor Henry Briggs, D.Sc, Ph.D. (With a Plate.) Issued separately June 28, 1933. 164

14 Studies in Genetic Psychology The Intellectual Resemblance of Collateral Relatives By J L. Gray, M A, and Pearl Moshinsky, B.Sc. (From the Department of Social Biology, University of London) *Communicated by* Professor Lancelot Hogben, M.A, D.Sc Issued separately July 3, 1933. . 188

15 Matrices and Continued Fractions, II. By Professor H W. Turnbull, F R.S. Issued separately July 13, 1933. 208

*16 Studies on the Reproductive System in the Guinea-Pig: Variations in the Œstrous Cycle in the Virgin Animal, after Parturition, and during Pregnancy. By Thomas Nicol, M.B, Ch B, F.R C.S.E., Senior Lecturer in Anatomy, University of Glasgow Issued separately August 4, 1933.. . 220

17 The Effect of Consanguineous Parentage upon Metrical Characters of the Offspring. By Professor Lancelot Hogben, M A, D Sc (From the Department of Social Biology in the University of London) Issued separately September 1, 1933. 239

18 The Fæcal Pellets of *Hippa asiatica*. By Hilary B. Moore, Ph D *Communicated by* Professor J Graham Kerr, F.R.S. Issued separately September 1, 1933. 252

19. The Diffusion Coefficients of Bromine-Argon, Bromine-Methane, Bromine-Hydrogen Chloride, Bromine-Nitrous Oxide. By John E Mackenzie, D.Sc., and Harry W Melville, Ph.D Issued separately September 1, 1933. . 255

20 On the Theory of Statistical Regression. By M. S Bartlett, B.A, B.Sc. (Queens' College, Cambridge) *Communicated by* J. Wishart, M A., D.Sc. Issued separately September 29, 1933. 260

*21 Geology and Petrology of the Dolerites of Spitsbergen. By G. W. Tyrrell, D Sc, F G S (University of Glasgow), and K. S. Sandford, M.A., Ph.D. (University of Oxford) (With a Plate and Nine Text-figures.) Issued separately October 4, 1933. 284

22. Studies on the Gametogenesis of *Stenophylax stellatus*, Curt. (Trichoptera) — Oogenesis. By R. A. R. Gresson, Ph.D. (From the Department of Zoology, University of Edinburgh.) *Communicated by* Professor J. H. Ashworth, F.R.S. (With One Group Text-figures (1–14) and Two Plates) Issued separately October 30, 1933. 322

PAGE

23. The Mathematical Representation of the Energy Levels of the Secondary Spectrum of Hydrogen. By Ian Sandeman, Ph D, Late Carnegie Research Scholar, University of St Andrews Issued separately December 22, 1933, . 347

OBITUARY NOTICES. Francis Ernest Bradley, William Inglis Clark; Sir Dugald Clerk; William Grant Craib; John Cuthbertson, Rev Joseph Robert Fraser; William Hope Fowler; Frederick Gardiner, John Glaister; John Henderson; Theophilus Bulkeley Hyslop; William Jacob Holland; Robert T. A. Innes; Robert Jardine; John Edward Marr; Spencer Mort; Ralph Richardson, Johannes Schmidt; John Stephenson; John W Tait, James Edward Talmage; Sir J. Arthur Thomson, John Millar Thomson; William Arthur Williams, ... 354-384

APPENDIX—

Proceedings of the Statutory General Meeting, October 1932, 387
Proceedings of the Ordinary Meetings, Session 1932–1933, 390
Proceedings of Statutory General Meeting, October 1933, . . 396
The Keith, Makdougall-Brisbane, Neill, Gunning Victoria Jubilee, James Scott, Bruce, and David Anderson-Berry Prizes, and the Bruce-Preller Lecture Fund, ... 399
Awards, . . . 403
Accounts of the Society, Session 1932–1933, . . 411
Voluntary Contributors under Law VI (end of para. 3), . 419
The Council of the Society at October 23, 1933, 420
Fellows of the Society at October 23, 1933, . . . 421
Honorary Fellows of the Society at October 23, 1933, . 447
Changes in Fellowship during Session 1932–1933, . . . 449
Fellows of the Society elected during Session 1932–1933, . . 449
Laws of the Society, ... 450
Additions to the Library—Presentations, etc.—1932-1933, . 458
Index, ... 461
Index, under Authors' Names, of Papers published in *Transactions*, . . 464

* The thanks of the Society are due to the Carnegie Trust for the Universities of Scotland for grants towards the cost of the illustrations of these papers, and for similar grants for the following papers in *Proceedings*, Vol LII, 1931-32:—

Sir E. A Sharpey-Schafer and Mr. W. A. Bain . . No. 2.
Miss Evlyn Boyd . . No 10

PROCEEDINGS

OF THE

ROYAL SOCIETY OF EDINBURGH.

VOL. LIII. 1932–33.

..—On the Development of the Olfactory Organ in *Protopterus*.

By **Margaret H. Fullarton**, B.Sc., Assistant in the Zoology Department, University of Glasgow. *Communicated by* Professor J. Graham Kerr, F.R.S. (With Six Text-figures.)

(MS. received August 20, 1932. Read November 7, 1932.)

My thanks are due to Professor Graham Kerr for all the material used in this research and for much helpful criticism The sections used were already stained and mounted. The stages mentioned are named according to Keibel's *Normentafeln*, Heft x.

Observations.

The first definite olfactory rudiments, arising as two solid ingrowths of the deep layer of ectoderm on the under-side of the head, are to be seen in sections of Stage XXVII.

A section through the left olfactory rudiment of Stage XXVIII (fig. 1) shows the early structure of the organ clearly, with its ingrowing thickening of the deep ectodermal layer covered by the thin superficial ectoderm. The cells in both layers are heavily laden with yolk particles, as illustrated in the text-figure by black dots.

In the later stages, XXIX and XXX, the rudiment is similar; the cells near their outer ends are becoming progressively less yolky.

In Stage XXXI (fig. 2) transverse sections show that a cavity has been formed in the deep ectodermal layer by cytolysis. The super[illegible]al ectoderm still persists, stretching over this cavity, but it is seen to [illegible] degenerate. Yolk is still present in the cells. In an external view of [illegible] under-side of the head two rounded dimples are apparent, one on [illegible]ther side, and these indicate the rudimentary olfactory organs.

By Stage XXXII (fig. 3) transverse sections show that the superficial ectoderm has broken down over the rudiment, and that the cells contain only a little yolk. In external view the olfactory organs show little difference from Stage XXXI.

Transverse sections through the rudiment of Stage XXXIV show that the organ is open throughout its length and that a lateral diverticulum is growing out from its cavity. In external view the two dimples have elongated to form two slits, which pass outwards and backwards on each side of the under part of the head. An interesting point was observed at this stage where the organ is elongating with growth, as is indicated by the formation of the slit. Transverse sections through the anterior part of the organ showed that the cells in the region of the opening are, like other ectodermal cells at this stage, free from yolk. On examining sections through the posterior part of the slit, however, the cells were seen to be yolky and thus to resemble endodermal cells at this stage.

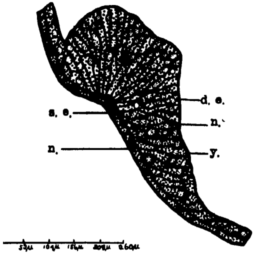


FIG 1.—Transverse section through the left olfactory organ of Stage XXVIII.

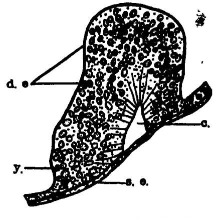


FIG. 2.—Transverse section through the right olfactory organ of Stage XXXI.

a n, anterior naris, *b c*, blood-cell; *b v.*, blood-vessel; *c*, cavity of the olfactory organ; *d e*, deep ectodermal layer; *l d.*, lateral diverticulum; *n*, nucleus; *p n.*, posterior naris; *s e*, superficial ectodermal layer; *y.*, yolk particles in the cell.

In Stage XXXV a well-developed lateral diverticulum is present. The anterior part of the opening is again "obviously" of ectodermal tissue in contrast to the cells of the posterior part of the opening which are to all appearances endodermal. Sagittal sections were used to make a reconstruction of the left olfactory organ of this stage (fig. 4).

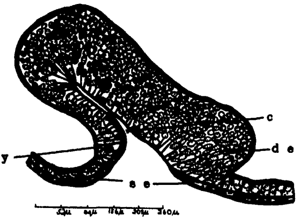


FIG 3 —Transverse section through the left olfactory organ of Stage XXXII

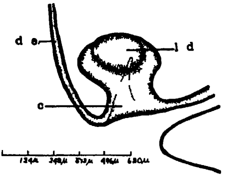


FIG 4 —Reconstruction of the left olfactory organ in Stage XXXV made from a series of sagittal sections

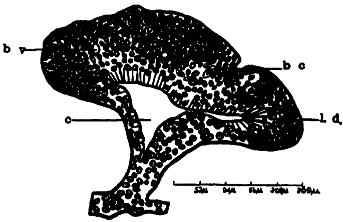


FIG 5 —Transverse section through the right olfactory organ of Stage XXXVI

The drawing shows the left olfactory organ viewed from the left side. The well-defined lateral diverticulum is seen to be about the middle of the organ, and the main cavity, represented by the dotted line, is seen to open to the exterior and to extend into the diverticulum. In external view the opening is seen to be of dumb-bell shape, a widening has arisen anteriorly and posteriorly, and a constriction is beginning to separate these two regions. This opening is only visible after part of the lower jaw has been removed. Professor Graham Kerr has illustrated these changes in shape of the olfactory opening in the *Quarterly Journal of Microscopical Science*, vol. liv.

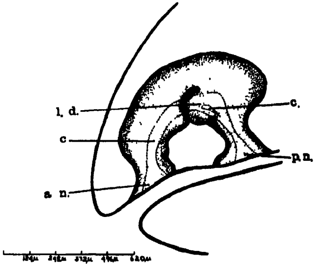


FIG. 6—Reconstruction of the left olfactory organ in Stage XXXVI, made from a series of sections parallel to the sagittal plane

Transverse sections through the organ in Stage XXXVI show that the anterior and posterior parts of the dumb-bell opening are now completely separated, by fusion of the intervening edges of the connecting slit, to form the well-separated anterior and posterior nares. The anterior naris opens to the exterior, and it is hidden by the overhanging anterior lip of the animal. The posterior naris opens into the buccal cavity. The now almost tubular cavity passes from the anterior opening to the posterior and contains much mucus. This mucus is formed by gland-cells which line the dorsal wall of the cavity. The lateral diverticulum is now situated towards the hind end of the organ; a section through it shows its persisting connection with the olfactory cavity (fig. 5). The gland-cells found in the dorsal wall of the main cavity appear to line the cavity of the external diverticulum. A reconstruction of this stage was made from a series of sagittal sections (fig. 6). This drawing shows the left olfactory organ viewed from the left side. In the actual specimen the two openings, now completely separated from each other, may be seen by removing most of the lower jaw.

REMARKS.

1. This is an unusual type of development. The organ first develops as a solid ingrowth of the deep layer of ectoderm, whose cavity develops secondarily and opens to the exterior; the external aperture elongates, then dilates anteriorly and posteriorly, and finally becomes closed again except at its dilated anterior and posterior ends

2. The origin of the two openings is probably different, the anterior being "ectodermal"—that is, with its richly protoplasmic cells free from yolk; and the posterior "endodermal"—that is, of yolky cells. This can be explained if one remembers that the posterior part of the wall of the buccal cavity in *Protopterus* is "endodermal" in origin; the slit of the embryonic olfactory opening extends outwards and backwards as it grows, thus penetrating the "endodermal" part of the buccal cavity so that posteriorly the opening comes to lie in "endodermal" tissue. Greil, working on *Ceratodus*, states that an ingrowth of ectoderm takes place, and that this ingrowth lines the buccal cavity. According to Professor Graham Kerr on the other hand, the main lining of the buccal cavity in the Dipnoi arises *in situ* by actual transformation of the originally yolk-laden cells; "it is as if an influence were spreading inwards from the external ectoderm, gradually transforming the original 'endodermal' yolk-laden cells into ectoderm like itself." He also notes that in the study of celloidin sections "there is frequently visible a quite broad zone of transition in which the richly protoplasmic 'ectoderm' cells pass by imperceptible gradations into the typical yolk-cells, there being no trace of the absolutely sharp boundary which must be present were the ingrowth hypothesis correct."

The "ectodermal" origin of the anterior nares and the "endodermal" origin of the posterior nares support the latter view.

3. The lateral diverticulum found in *Protopterus* embryos is found also in the embryonic *Lepidosiren*, whose olfactory organ is closely similar in its development. No such organ is mentioned by Semon as occurring in the development of *Ceratodus*. This diverticulum resembles the organ of Jacobson in being a little pocket-like outgrowth from the main nasal cavity, but in the higher vertebrates this organ arises from the medial

wall of the nasal cavity, and generally from the ventral part of the medial wall. In *Protopterus* and *Lepidosiren* the lateral diverticulum arises from the dorsal part of the lateral wall of the nasal cavity, and thus cannot be said to be homologous with the organ of Jacobson in the higher vertebrates.

Adult specimens have not been examined with the stages cited above, but Parker makes no mention of any structure resembling this lateral diverticulum in the adult *Protopterus*. The olfactory organ in the adult has many pockets developed from its cavity, and these would not be distinguishable from such a simple diverticulum as is found in the embryo.

4. The position of the anterior naris behind the upper lip would lead one to expect that both openings of the olfactory organ communicate with the buccal cavity. This has been shown by Huxley not to be the case, the anterior opening communicating with the exterior. Parker suggests that this lip is probably an adaptation in connection with the torpid state. Professor Graham Kerr considers the modification an adaptation for burrowing through the mud.

5. The possession of external and internal nares in *Protopterus* and *Lepidosiren* would lead one to expect that these fish, whose respiration is to such a great extent aerial, take in air through the nasal passages. Professor Graham Kerr, from his own observations, considers that the olfactory organ in the living fish is purely sensory, the animal depending upon this sense principally in its search for food.

BIBLIOGRAPHY.

GEGENBAUR, C., 1898. *Vergleichende Anatomie der Wirbeltiere*, Band i.

HERTWIG, O., 1906. *Handbuch der vergleichenden und experimentellen Entwickelungslehre der Wirbeltiere*, Band ii, Teil 2.

KEIBEL, F., 1909. *Normentafeln zur Entwicklungsgeschichte der Wirbeltiere*, Heft x.

KERR, J. GRAHAM, 1910. "On Certain Features in the Development of the Alimentary Canal in *Lepidosiren* and *Protopterus*," *Quart. Journ. Micr. Sci.*, vol. liv, pt. iv.

PARKER, W. N., 1892. "The Anatomy and Physiology of *Protopterus annectens*," *Trans. R.I.A.*, vol. xxx, pt. iii.

WIEDERSHEIM, R., 1909. *Grundriss der vergleichenden Anatomie der Wirbeltiere*, 7te Aufl.

(*Issued separately January* 16, 1933.)

II.—A Matrix Notation for Mendelian Populations. By Professor Lancelot Hogben, M.A., D.Sc., Department of Social Biology in the University of London.

(MS. received July 7, 1932 Read December 5, 1932.)

1. INTRODUCTION.

THOSE who have had experience of problems involving inbreeding or the correlation of relatives in a population with specified proportions of Mendelian genotypes will have realised what difficulties reside in the absence of a convenient notation. In the absence of such a notation difficulties arise more from the unwieldiness of the expressions obtained than the abstruseness of the problems encountered. The familiar chess-board diagrams of Mendelian hybridisation and the form of a contingency table for relatives alike suggest that a matrix notation might be generalised to take into account both sex-linked inheritance and the more usual type of transmission. A determinant form would meet some of the requirements of autosomal transmission, but is not adapted to the asymmetrical case, when the male sex * cannot be heterozygous. In extending to more remote relationships a previous inquiry (3) into fraternal and filial correlations involving sex linkage, the writer has found it useful to employ a system of operations applicable to other types of inquiry. The former investigation is not yet complete. In the meantime it seemed possible that the method used might prove suggestive to other workers. A preliminary account of the fundamental operations is here given in so far as they are relevant to single gene substitutions.

2. DEFINITION OF THE OPERATORS.

1. There are three types of matrices in the proposed notation. They are written thus:

(*a*) Population matrix (or P-matrix). $$Z_n = \begin{vmatrix} a_n & d_n \\ b_n & e_n \\ c_n & f_n \end{vmatrix}.$$

(*b*) The Product form. $$P_n = \begin{vmatrix} l_n & m_n & n_n \\ o_n & p_n & q_n \\ r_n & s_n & t_n \end{vmatrix}.$$

---

* Throughout the male is treated as the heterogametic sex. The reader will find no difficulty in making the trifling adjustments necessary when, as in birds and Lepidoptera, the female is the heterogametic sex.

(*c*) The Correlate. $$C_n = \begin{vmatrix} L_n & M_n & N_n \\ O_n & P_n & Q_n \\ R_n & S_n & T_n \end{vmatrix}.$$

The sum of the terms in each of the vertical columns of a P-matrix is unity and the sum of all the terms of a correlate or a product is unity.

2. The operation of ZERO SUBSTITUTION performed upon a P-matrix is written either as:

${}_0(Z_n)$ signifying that throughout any operation performed with the P-matrix all the terms in the left-hand column, *i.e.* a_n, b_n, c_n, become zero.

$(Z_n)_0$ signifying that throughout any operation performed with the P-matrix all the terms in the right-hand column, *i.e.* d_n, e_n, f_n, become zero.

In each case the remaining column is multiplied by two throughout.

3. Two operations defined as *multiplication* and *condensation* may be performed upon a pair of P-matrices. They will be defined in § 5 and § 10. Two operations denoted extension and extraction may be carried out with a Product form. These consist of expanding the product in a series of P-matrices or *minors* of fixed value. Either operation can be carried out "asymmetrically" or "symmetrically" according to whether one or another set of minors is used. The fixed values of the minors used in extraction or extension of the symmetrical or asymmetrical type are given below in § 7 and § 8.

4. The addition of any number of matrices of one and the same type follows the same rule, namely, that corresponding terms are added, thus

$$a \,.\, P_a + b \,.\, P_b + c \,.\, P_c + \;.\;.\;.\; n \,.\, P_n = \begin{vmatrix} \sum_a^n r \,.\, l_r & \sum_a^n r \,.\, m_r & \sum_a^n r \,.\, n_r \\ \sum_a^n r \,.\, o_r & \sum_a^n r \,.\, p_r & \sum_a^n r \,.\, q_r \\ \sum_a^n r \,.\, r_r & \sum_a^n r \,.\, s_r & \sum_a^n r \,.\, t_r \end{vmatrix}$$

where $(a + b + c + \;.\;.\;.\; n) = 1$.

5. The product of two P-matrices is defined as

$$P_{st} = Z_s \times Z_t = \begin{vmatrix} a_s & d_s \\ b_s & e_s \\ c_s & f_s \end{vmatrix} \times \begin{vmatrix} a_t & d_t \\ b_t & e_t \\ c_t & f_t \end{vmatrix}$$

$$= \begin{vmatrix} \frac{1}{2}(a_s d_t + a_t d_s) & . & \frac{1}{2}(b_s d_t + b_t d_s) & . & \frac{1}{2}(c_s d_t + c_t d_s) \\ \frac{1}{2}(a_s e_t + a_t e_s) & . & \frac{1}{2}(b_s e_t + b_t e_s) & . & \frac{1}{2}(c_s e_t + c_t e_s) \\ \frac{1}{2}(a_s f_t + a_t f_s) & . & \frac{1}{2}(b_s f_t + b_t f_s) & . & \frac{1}{2}(c_s f_t + c_t f_s) \end{vmatrix}.$$

Lemma

$$Z^2_n = \begin{vmatrix} a_n d_n & b_n d_n & c_n d_n \\ a_n e_n & b_n e_n & c_n e_n \\ a_n f_n & b_n f_n & c_n f_n \end{vmatrix}.$$

6. When multiplication is carried out between two P-matrices on which the operation of zero substitution has been carried out, the coefficient $\frac{1}{2}$ is eliminated throughout from each term of the product (see 2).

Lemma

$$_0(Z_s) \times {}_0(Z_t) = 0 = (Z_s)_0 \times (Z_t)_0$$

$$(Z_s)_0 \times {}_0(Z_t) = \begin{vmatrix} a_s d_t & b_s d_t & c_s d_t \\ a_s e_t & b_s e_t & c_s e_t \\ a_s f_t & b_s f_t & c_s f_t \end{vmatrix}$$

$$_0(Z_s) \times (Z_t)_0 = \begin{vmatrix} a_t d_s & b_t d_s & c_t d_s \\ a_t e_s & b_t e_s & c_t e_s \\ a_t f_s & b_t f_s & c_t f_s \end{vmatrix}$$

7. The operation of EXTENSION of a Product form is the formation of a P-matrix series of which every term is formed by multiplying one of the terms in the product with a P-matrix "minor" of fixed value for each locus in the general product form This is written

$$E\,.\,(P_n) = l_n\,.\,s_l + m_n\,.\,s_m + n_n\,.\,s_n\;.\;.\;.\;t_n\,.\,s_t,$$

or briefly,

$$E\,.\,(P_n) = y_n \quad s_y$$

(the summation sign being omitted).

The fixed values of the EXTENSORS are of two classes, according to whether the extension is "symmetrical" or asymmetrical. The symmetrical extensors are.

$$\begin{array}{cccccc} s_l & s_m & s_n & s_p & s_q & s_t \\ 1\;\;1 & \frac{1}{2}\;\;\frac{1}{2} & 0\;\;0 & \frac{1}{4}\;\;\frac{1}{4} & 0\;\;0 & 0\;\;0 \\ 0\;\;0 & \frac{1}{2}\;\;\frac{1}{2} & 1\;\;1 & \frac{1}{2}\;\;\frac{1}{2} & \frac{1}{2}\;\;\frac{1}{2} & 0\;\;0 \\ 0\;\;0 & 0\;\;0 & 0\;\;0 & \frac{1}{4}\;\;\frac{1}{4} & \frac{1}{2}\;\;\frac{1}{2} & 1\;\;1 \end{array} \qquad \begin{array}{c} s_0 = s_m \text{ and } s_r = s_n \\ s_s = s_q. \end{array}$$

The asymmetrical extensors are·

$$\begin{array}{cccc} s'_m & s'_n & s'_r & s'_s \\ \frac{1}{2}\;\;\frac{1}{2} & 0\;\;0 & 0\;\;1 & 0\;\;\frac{1}{2} \\ \frac{1}{2}\;\;0 & 1\;\;0 & 1\;\;0 & \frac{1}{2}\;\;0 \\ 0\;\;\frac{1}{2} & 0\;\;1 & 0\;\;0 & \frac{1}{2}\;\;\frac{1}{2} \end{array} \qquad \begin{array}{c} s'_l = s_l \text{ and } s'_t = s_t: \\ s'_0 = s'_p = s'_q = 0. \end{array}$$

8. An analogous operation to the extension of a product is called extraction, and is similar except that the fixed minors have different values. It is written

$$\xi(P_n) = y_n\,.\,s_y.$$

The minors of this operation are called extractors, and their values are:

$$\begin{array}{ccccccccc}
e_l & e_m & e_n & e_o & e_p & e_q & e_r & e_s & e_t \\
1\ 1 & 0\ 1 & 0\ 1 & 1\ 0 & 0\ 0 & 0\ 0 & 1\ 0 & 0\ 0 & 0\ 0 \\
0\ 0 & 1\ 0 & 0\ 0 & 0\ 1 & 1\ 1 & 0\ 1 & 0\ 0 & 1\ 0 & 0\ 0 \\
0\ 0 & 0\ 0 & 1\ 0 & 0\ 0 & 0\ 0 & 1\ 0 & 0\ 1 & 0\ 1 & 1\ 1
\end{array}$$

In asymmetrical extension the values of the extractors are the same for l, m, n, r, s, t; but the extractors for o, p, q are zero.

9. The P-matrix series obtained by operating upon a product with E or ξ may be summed as a P-matrix thus:

$$\xi(\mathrm{P}_n) = \left|\begin{array}{ccc} l_n+o_n+r_n & \cdot & l_n+m_n+n_n \\ m_n+p_n+s_n & & o_n+p_n+q_n \\ n_n+q_n+t_n & \cdot & r_n+s_n+t_n \end{array}\right|$$

$$\mathrm{E}.(\mathrm{P}_n) = \left|\begin{array}{ccc} \tfrac{1}{4}(4l_n+2m_n+2o_n+p_n) & \cdot & \tfrac{1}{4}(4l_n+2m_n+2o_n+p_n) \\ \tfrac{1}{4}(m_n+2n_n+o_n+p_n+q_n+2r_n+s_n) & \cdot & \tfrac{1}{4}(m_n+2n_n+o_n+p_n+q_n+2r_n+s_n) \\ \tfrac{1}{4}(p_n+2q_n+2s_n+4t_n) & \cdot & \tfrac{1}{4}(p_n+2q_n+2s_n+4t_n) \end{array}\right|$$

$$\mathrm{E}'.(\mathrm{P}_n) = \left|\begin{array}{ccc} \tfrac{1}{2}(2l_n+m_n) & \cdot & \tfrac{1}{2}(2l_n+m_n+2r_n+s_n) \\ \tfrac{1}{2}(m_n+2n_n+2r_n+s_n) & \cdot & 0 \\ \tfrac{1}{2}(s_n+2t_n) & \cdot & \tfrac{1}{2}(m_n+2n_n+s_n+2t_n) \end{array}\right|$$

$$\mathrm{E}(\mathrm{Z}_n)^2 = \left|\begin{array}{cc} \tfrac{1}{4}(2d_n+e_n)(2a_n+b_n) & \tfrac{1}{4}(2d_n+e_n)(2a_n+b_n) \\ \tfrac{1}{2}d_n(b_n+2c_n)+e_n+f_n(2a_n+b_n) & \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \\ \tfrac{1}{4}(e_n+2f_n)(b_n+2c_n) & \cdot\ \ \cdot\ \ \cdot \end{array}\right|$$

$$\mathrm{E}'(\mathrm{Z}_n)^2 = \left|\begin{array}{cc} \tfrac{1}{2}d_n(2a_n+b_n) & (d_n+f_n)(2a_n+b_n) \\ \tfrac{1}{2}d_n(b_n+2c_n)+f_n(2a_n+b_n) & 0 \\ \tfrac{1}{2}f_n(b_n+2c_n) & (d_n+f_n)(b_n+2c_n) \end{array}\right|.$$

10. The CONDENSATION of two P-matrices gives the CORRELATE defined by

$$\mathrm{Z}_{\mathrm{R}} \sim \mathrm{Z}_r \qquad \text{or} \qquad \left|\begin{array}{cc} \mathrm{A} & \mathrm{D} \\ \mathrm{B} & \mathrm{E} \\ \mathrm{C} & \mathrm{F} \end{array}\right| \sim \left|\begin{array}{cc} a & d \\ b & e \\ c & f \end{array}\right| =$$

$$\left|\begin{array}{ccc} \tfrac{1}{4}(\mathrm{A}+\mathrm{D})(a+d) & \tfrac{1}{4}(\mathrm{A}+\mathrm{D})(b+e) & \tfrac{1}{4}(\mathrm{A}+\mathrm{D})(c+f) \\ \tfrac{1}{4}(\mathrm{B}+\mathrm{E})(a+d) & \tfrac{1}{4}(\mathrm{B}+\mathrm{E})(b+e) & \tfrac{1}{4}(\mathrm{B}+\mathrm{E})(c+f) \\ \tfrac{1}{4}(\mathrm{C}+\mathrm{F})(a+d) & \tfrac{1}{4}(\mathrm{C}+\mathrm{F})(b+e) & \tfrac{1}{4}(\mathrm{C}+\mathrm{F})(c+f) \end{array}\right|.$$

11. ZERO SUBSTITUTION as follows can be performed in the condensation of two P-matrices.

$$(\mathrm{C}_{\mathrm{R}_r})_0 = \left|\begin{array}{ccc} \mathrm{A}a & \mathrm{A}b & \mathrm{A}c \\ \mathrm{B}a & \mathrm{B}b & \mathrm{B}c \\ \mathrm{C}a & \mathrm{C}b & \mathrm{C}c \end{array}\right|$$

$${}_0(\mathrm{C}_{\mathrm{R}_r}) = \left|\begin{array}{ccc} \mathrm{D}d & \mathrm{D}e & \mathrm{D}f \\ \mathrm{E}d & \mathrm{E}e & \mathrm{E}f \\ \mathrm{F}d & \mathrm{F}e & \mathrm{F}f \end{array}\right|$$

$$_0(C_{R_r})_0 = \begin{vmatrix} \frac{1}{2}(Ad+aD) & \frac{1}{4}(Bd+bD+Ea+eA) & \frac{1}{4}(Cd+cD+Fa+fA) \\ \frac{1}{4}(Ae+aE+Db+dB) & \frac{1}{2}(Be+bE) & \frac{1}{4}(Ce+cE+Fb+fB) \\ \frac{1}{4}(Af+aF+Dc+dC) & \frac{1}{4}(Bf+bF+Ec+eC) & \frac{1}{2}(Cf+cF) \end{vmatrix}$$

$$(C_R)_0 \sim {}_0(C_r) \begin{vmatrix} Ad & Ae & Af \\ Bd & Be & Bf \\ Cd & Ce & Cf \end{vmatrix} \qquad {}_0(C_R) \sim (C_r)_0 \begin{vmatrix} Da & Db & Dc \\ Ea & Eb & Ec \\ Fa & Fb & Fc \end{vmatrix}.$$

Lemma

$$\text{(i)}\quad Z_R \sim Z_R = \begin{vmatrix} \frac{1}{4}(A+D)^2 & \frac{1}{4}(B+E)(A+D) & \frac{1}{4}(C+F)(A+D) \\ \frac{1}{4}(A+D)(B+E) & \frac{1}{4}(B+E)^2 & \frac{1}{4}(C+F)(B+E) \\ \frac{1}{4}(A+D)(C+F) & \frac{1}{4}(B+E)(C+F) & \frac{1}{4}(C+F)^2 \end{vmatrix}$$

$$\text{(ii)}\qquad (C_{RR})_0 = \begin{vmatrix} A^2 & BA & CA \\ AB & B^2 & CB \\ AC & BC & C^2 \end{vmatrix}$$

$$\text{(iii)}\qquad {}_0(C_{RR}) = \begin{vmatrix} D^2 & ED & FD \\ DE & E^2 & FE \\ DF & EF & F^2 \end{vmatrix}$$

$$\text{(iv)}\qquad {}_0(C_{RR})_0 = \begin{vmatrix} AD & BD & CD \\ AE & BE & CE \\ AF & BF & CF \end{vmatrix}.$$

3. THE BIOLOGICAL SIGNIFICANCE OF THE OPERATIONS.

The following definitions will be sufficient for using the foregoing operations in problems of inbreeding and the correlation of relatives.

(*a*) A P-matrix represents a population of which the terms of the left column are the frequencies of the female genotypes and the terms of the right-hand column are the male genotypes. The order in which they are to be read is as follows:—

$$Z = \begin{vmatrix} R & R \\ H & H \\ D & D \end{vmatrix} \qquad Z' = \begin{vmatrix} R & R \\ H & O \\ D & D \end{vmatrix}.$$

The latter type or asymmetrical form is used in sex-linked inheritance. In a system of random mating the P-matrices for the population are respectively:

$$Z_R = \begin{vmatrix} p^2 & p^2 \\ 2pq & 2pq \\ q^2 & q^2 \end{vmatrix} \qquad Z'_R = \begin{vmatrix} p^2 & p \\ 2pq & 0 \\ q^2 & q \end{vmatrix}.$$

Extractors and extensors represent pairs of parents or sibships. The extractors are pairs of parents, and the extensors being sibships. The order is the same.

(*b*) The product of two P-matrices represents the mating system formed by pairing the individuals with one population (or sibship, etc.) with individuals of another. The order of the terms is taken to be

$$P = \begin{vmatrix} RR & RH & RD \\ RH & HH & HD \\ RD & DH & DD \end{vmatrix}; \quad P' = \begin{vmatrix} RR\,.\,Ry & H\,.\,Ry & DD\,.\,Ry \\ \dots & \dots & \dots \\ RR\,.\,Dy & H\,.\,Dy & DD\,.\,Dy \end{vmatrix}.$$

(*c*) It follows that the square of a P-matrix represents the mating system resulting from mating the genotypes (represented in their respective frequencies by the given P-matrix) ***inter se.***

(*d*) The operation denoted by E or E′ (latter for sex-linked gene substitutions) represents the population of offspring resulting from a mating system corresponding to the product on which the operation is performed

(*e*) The operation ξ represents the genotype frequencies of the population whose mating relations are represented by the product on which the operation is performed.

(*f*) The operation of condensation corresponds to the formation of a contingency table by taking all possible pairs of individuals, one member of each pair being taken from one population or P-matrix, the other from the other population or P-matrix.

(*g*) By zero substitution the mating of two populations may be made to involve the males of one and the females of the other or *vice versa.*

(*h*) By zero substitution the condensation of two population groups may be taken to mean taking pairs of males only, taking pairs of females only, or taking a male from one and a female from the other. Without substitution the operation of condensation represents all possible pairs taken simultaneously.

(*j*) The order of the terms in a correlate is given by

$$\begin{vmatrix} RR & RH & RD \\ RH & HH & HD \\ RD & DH & DD \end{vmatrix}.$$

(*k*) A system of inbreeding is specified by the product forms corresponding to the $(n-1)$th and the nth generations together with the P-matrices of the nth and the $(n+1)$th generations.

4. TABLES OF P-MATRICES.

It will be seen in what follows that certain products constantly recur in problems of inbreeding or correlation in a system of random mating. The values of the P-matrices formed by extending these products is given in Tables I–III. The form $(s_y) \times (Z_R)$, etc., are used for whole relationships.

TABLE I.—SYMMETRICAL P-MATRICES (WHOLE AND HALF RELATIONSHIPS).

$s_y{}^* =$	l		$m=o.$		$n=r=p.$		$q=s.$		$t.$	
	p	p	$\frac{3}{4}p$	$\frac{3}{4}p$	$\frac{1}{2}p$	$\frac{1}{2}p$	$\frac{1}{4}p$	$\frac{1}{4}p$	0	0
$\mathrm{E}(s_y \times \mathrm{Za})$ . .	q	q	$\frac{1}{4}(2q+1)$	$\frac{1}{4}(2q+1)$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}(2p+1)$	$\frac{1}{4}(2p+1)$	p	p
	0	0	$\frac{1}{4}q$	$\frac{1}{4}q$	$\frac{1}{2}q$	$\frac{1}{2}q$	$\frac{3}{4}q$	$\frac{3}{4}q$	q	q
	$\frac{1}{2}p(p+1)$	$\frac{1}{2}p(p+1)$	$\frac{1}{8}p(4p+3)$	$\frac{1}{8}p(4p+3)$	$\frac{1}{4}p(2p+1)$	$\frac{1}{4}p(2p+1)$	$\frac{1}{8}p(4p+1)$	$\frac{1}{8}p(4p+1)$	$\frac{1}{2}p^2$	$\frac{1}{2}p^2$
$\mathrm{E}\{\mathrm{Za} \times \mathrm{E}(s_y \times \mathrm{Za})\}$	$\frac{1}{2}q(2p+1)$	$\frac{1}{2}q(2p+1)$	$\frac{1}{8}(8pq+2q+1)$	$\frac{1}{8}(8pq+2q+1)$	$\frac{1}{4}(4pq+1)$	$\frac{1}{4}(4pq+1)$	$\frac{1}{8}(8pq+2p+1)$	$\frac{1}{8}(8pq+2p+1)$	$\frac{1}{2}p(2q+1)$	$\frac{1}{2}p(2q+1)$
	$\frac{1}{2}q^2$	$\frac{1}{2}q^2$	$\frac{1}{8}q(4q+1)$	$\frac{1}{8}q(4q+1)$	$\frac{1}{4}q(2q+1)$	$\frac{1}{4}q(2q+1)$	$\frac{1}{8}q(4q+3)$	$\frac{1}{8}q(4q+3)$	$\frac{1}{2}q(q+1)$	$\frac{1}{2}q(q+1)$
$e_y =$			$l=m=n.$		$o=p=q$		$r=s=t.$			
			p	p	$\frac{1}{2}p$	$\frac{1}{2}p$	0	0		
$\mathrm{E}\{_e(e_y) \times (\mathrm{Za})_e\}$			q	q	$\frac{1}{2}$	$\frac{1}{2}$	p	p		
			0	0	$\frac{1}{2}q$	$\frac{1}{2}q$	q	q		

[* The values of y in the Product form for a system of random mating are: $l=p^4$; $(m+o)=4p^3q$; $(n+r+p)=6p^2q^2$; $(q+s)=4pq^3$; $t=q^4$; and for the bottom row $(l+m+n)=p^2$; $(o+p+q)=2pq$; and $(r+s+t)=q^2$.]

TABLE II.—ASYMMETRICAL P-MATRICES (WHOLE RELATIONSHIPS).

	l.	*m*.	*n*.	*r*	*s*.	*t*.
$E(s_y \times Z_R)$ . . .	$\begin{matrix} p & \frac{1}{2}(p+1) \\ q & 0 \\ 0 & \frac{1}{2}q \end{matrix}$	$\begin{matrix} \frac{5}{8}p & \frac{1}{8}(4p+3) \\ \frac{1}{8}(2q+3) & 0 \\ \frac{3}{8}q & \frac{1}{8}(4q+1) \end{matrix}$	$\begin{matrix} \frac{1}{4}p & \frac{1}{4}(2p+1) \\ \frac{1}{4}(2p+1) & 0 \\ \frac{3}{4}q & \frac{1}{4}(2q+1) \end{matrix}$	$\begin{matrix} \frac{3}{4}p & \frac{1}{4}(2p+1) \\ \frac{1}{4}(2q+1) & 0 \\ \frac{1}{4}q & \frac{1}{4}(2q+1) \end{matrix}$	$\begin{matrix} \frac{3}{8}p & \frac{1}{8}(4p+1) \\ \frac{1}{8}(2p+3) & 0 \\ \frac{5}{8}q & \frac{1}{8}(4q+3) \end{matrix}$	$\begin{matrix} 0 & \frac{1}{2}p \\ p & 0 \\ q & \frac{1}{2}(q+1) \end{matrix}$
$E[(s_y)_s \times {}_s(Z_R)]$.	$\begin{matrix} p & 1 \\ q & 0 \\ 0 & 0 \end{matrix}$	$\begin{matrix} \frac{3}{4}p & \frac{3}{4} \\ \frac{1}{4}(2q+1) & 0 \\ \frac{1}{4}q & \frac{1}{4} \end{matrix}$	$\begin{matrix} \frac{1}{2}p & \frac{1}{2} \\ \frac{1}{2} & 0 \\ \frac{1}{2}q & \frac{1}{2} \end{matrix}$	$\begin{matrix} \frac{1}{2}p & \frac{1}{2} \\ \frac{1}{2} & 0 \\ \frac{1}{2}q & \frac{1}{2} \end{matrix}$	$\begin{matrix} \frac{1}{4}p & \frac{1}{4} \\ \frac{1}{4}(2p+1) & 0 \\ \frac{3}{4}q & \frac{3}{4} \end{matrix}$	$\begin{matrix} 0 & 0 \\ p & 0 \\ q & 1 \end{matrix}$
$E[{}_s(s_y) \times (Z_R)_s]$ . .	$\begin{matrix} p & p \\ q & 0 \\ 0 & q \end{matrix}$	$\begin{matrix} \frac{1}{2}p & p \\ \frac{1}{2} & 0 \\ \frac{1}{2}q & q \end{matrix}$	$\begin{matrix} 0 & p \\ p & 0 \\ q & q \end{matrix}$	$\begin{matrix} p & p \\ q & 0 \\ 0 & q \end{matrix}$	$\begin{matrix} \frac{1}{2}p & p \\ \frac{1}{2} & 0 \\ \frac{1}{2}q & q \end{matrix}$	$\begin{matrix} 0 & p \\ p & 0 \\ q & q \end{matrix}$
$E\{Z_R \times E(s_y \times Z_R)\}$	$\begin{matrix} \frac{1}{2}p(p+1) & \frac{1}{4}(3p+1) \\ \frac{1}{2}q(2p+1) & 0 \\ \frac{1}{2}q^2 & \frac{3}{4}q \end{matrix}$	$\begin{matrix} \frac{1}{32}p(16p+11) & \frac{1}{32}(24p+5) \\ \frac{1}{32}(32pq+6q+5) & 0 \\ \frac{1}{32}q(16q+5) & \frac{1}{32}(24q+3) \end{matrix}$	$\begin{matrix} \frac{p}{16}(8p+3) & \frac{1}{16}(12p+1) \\ \frac{1}{16}(16pq+2p+3) & 0 \\ \frac{q}{16}(8q+5) & \frac{1}{16}(12q+3) \end{matrix}$	$\begin{matrix} \frac{p}{16}(8p+5) & \frac{1}{16}(12p+3) \\ \frac{1}{16}(16pq+2q+3) & 0 \\ \frac{q}{16}(8q+3) & \frac{1}{16}(12q+1) \end{matrix}$	$\begin{matrix} \frac{1}{32}p(16p+5) & \frac{1}{32}(24p+3) \\ \frac{1}{32}(32pq+6p+5) & 0 \\ \frac{1}{32}q(16q+11) & \frac{1}{32}(24q+5) \end{matrix}$	$\begin{matrix} \frac{1}{2}p^2 & \frac{3}{4}p \\ \frac{1}{2}p(2q+1) & 0 \\ \frac{1}{2}q(q+1) & \frac{1}{4}(3q+1) \end{matrix}$
$E\{{}_s(Z_R) \times E[(s_y)_s \times (Z_R)]_s\}$	$\begin{matrix} \frac{1}{2}p(p+1) & \frac{1}{2}(p+1) \\ \frac{1}{2}q(2p+1) & 0 \\ \frac{1}{2}q^2 & \frac{1}{2}q \end{matrix}$	$\begin{matrix} \frac{1}{8}p(4p+3) & \frac{1}{8}(4p+3) \\ \frac{1}{8}(8pq+2q+1) & 0 \\ \frac{1}{8}q(4q+1) & \frac{1}{8}(4q+1) \end{matrix}$	$\begin{matrix} \frac{1}{4}p(2p+1) & \frac{1}{4}(2p+1) \\ \frac{1}{4}(4pq+1) & 0 \\ \frac{1}{4}q(2q+1) & \frac{1}{4}(2q+1) \end{matrix}$	$\begin{matrix} \frac{1}{4}p(2p+1) & \frac{1}{4}(2p+1) \\ \frac{1}{4}(4pq+1) & 0 \\ \frac{1}{4}q(2q+1) & \frac{1}{4}(2q+1) \end{matrix}$	$\begin{matrix} \frac{1}{8}p(4p+1) & \frac{1}{8}(4p+1) \\ \frac{1}{8}(8pq+2p+1) & 0 \\ \frac{1}{8}q(4q+3) & \frac{1}{8}(4q+3) \end{matrix}$	$\begin{matrix} \frac{1}{2}p^2 & \frac{1}{2}p \\ \frac{1}{2}p(2q+1) & 0 \\ \frac{1}{2}q(q+1) & \frac{1}{2}(q+1) \end{matrix}$
$E\{(Z_R)_s \times E_s[(s_y)_s \times {}_s(Z_R)]\}$	$\begin{matrix} p & p \\ q & 0 \\ 0 & q \end{matrix}$	$\begin{matrix} \frac{3}{4}p & p \\ \frac{1}{4}(2q+1) & 0 \\ \frac{1}{4}q & q \end{matrix}$	$\begin{matrix} \frac{1}{2}p & p \\ \frac{1}{2} & 0 \\ \frac{1}{2}q & q \end{matrix}$	$\begin{matrix} \frac{1}{2}p & p \\ \frac{1}{2} & 0 \\ \frac{1}{2}q & q \end{matrix}$	$\begin{matrix} \frac{1}{4}p & p \\ \frac{1}{4}(2p+1) & 0 \\ \frac{3}{4}q & q \end{matrix}$	$\begin{matrix} 0 & p \\ p & 0 \\ q & q \end{matrix}$
$E\{{}_s(Z_R) \times E[{}_s(s_y) \times (Z_R)_s]_s\}$	$\begin{matrix} \frac{1}{2}p(p+1) & \frac{1}{2}(p+1) \\ \frac{1}{2}q(2p+1) & 0 \\ \frac{1}{2}q^2 & \frac{1}{2}q \end{matrix}$	$\begin{matrix} \frac{1}{4}p(2p+1) & \frac{1}{4}(2p+1) \\ \frac{1}{4}(4pq+1) & 0 \\ \frac{1}{4}q(2q+1) & \frac{1}{4}(2q+1) \end{matrix}$	$\begin{matrix} \frac{1}{2}p^2 & \frac{1}{2}p \\ \frac{1}{2}p(2q+1) & 0 \\ \frac{1}{2}q(q+1) & \frac{1}{2}(q+1) \end{matrix}$	$\begin{matrix} \frac{1}{2}p(p+1) & \frac{1}{2}(p+1) \\ \frac{1}{2}q(2p+1) & 0 \\ \frac{1}{2}q^2 & \frac{1}{2}q \end{matrix}$	$\begin{matrix} \frac{1}{4}p(2p+1) & \frac{1}{4}(2p+1) \\ \frac{1}{4}(4pq+1) & 0 \\ \frac{1}{4}q(2q+1) & \frac{1}{4}(2q+1) \end{matrix}$	$\begin{matrix} \frac{1}{2}p^2 & \frac{1}{2}p \\ \frac{1}{2}p(2q+1) & 0 \\ \frac{1}{2}q(q+1) & \frac{1}{2}(q+1) \end{matrix}$
$E\{(Z_R)_s \times E_s[{}_s(s_y) \times (Z_R)_s]\}$	$\begin{matrix} p^2 & p \\ 2pq & 0 \\ q^2 & q \end{matrix}$	$\begin{matrix} p^2 & p \\ 2pq & 0 \\ q^2 & q \end{matrix}$	$\begin{matrix} p^2 & p \\ 2pq & 0 \\ q^2 & q \end{matrix}$	$\begin{matrix} p^2 & p \\ 2pq & 0 \\ q^2 & q \end{matrix}$	$\begin{matrix} p^2 & p \\ 2pq & 0 \\ q^2 & q \end{matrix}$	$\begin{matrix} p^2 & p \\ 2pq & 0 \\ q^2 & q \end{matrix}$

TABLE III.—ASYMMETRICAL P-MATRICES (HALF RELATIONSHIPS).

	l.	*m.*	*n.*	*r.*	*s.*	*t.*
$E\{(\epsilon_v)_o \times {}_o(Z_R)\}$ . .	$\begin{matrix} p & 1 \\ q & 0 \\ 0 & 0 \end{matrix}$	$\begin{matrix} \frac{1}{2}p & \frac{1}{2} \\ \frac{1}{2} & 0 \\ \frac{1}{2}q & \frac{1}{2} \end{matrix}$	$\begin{matrix} 0 & 0 \\ p & 0 \\ q & 1 \end{matrix}$	$\begin{matrix} p & p \\ q & 0 \\ 0 & q \end{matrix}$	$\begin{matrix} \frac{1}{2}p & \frac{1}{2} \\ \frac{1}{2} & 0 \\ \frac{1}{2}q & \frac{1}{2} \end{matrix}$	$\begin{matrix} p & 0 \\ p & 0 \\ q & 1 \end{matrix}$
$E\{{}_o(\epsilon_v) \times (Z_R)_o\}$ . .	$\begin{matrix} p & p \\ q & 0 \\ 0 & q \end{matrix}$	$\begin{matrix} p & p \\ q & 0 \\ 0 & q \end{matrix}$	$\begin{matrix} p & p \\ q & 0 \\ 0 & q \end{matrix}$	$\begin{matrix} 0 & p \\ p & 0 \\ q & q \end{matrix}$	$\begin{matrix} 0 & p \\ p & 0 \\ q & q \end{matrix}$	$\begin{matrix} 0 & p \\ p & 0 \\ q & q \end{matrix}$
$E\{E[(\epsilon_v)_o \times {}_o(Z_R)]_o \times {}_o(Z_R)\}$	$\begin{matrix} \frac{1}{2}p(p+1) & \frac{1}{2}(p+1) \\ \frac{1}{2}q(2p+1) & 0 \\ \frac{1}{2}q^2 & \frac{1}{2}q \end{matrix}$	$\begin{matrix} \frac{1}{4}p(2p+1) & \frac{1}{4}(2p+1) \\ \frac{1}{4}(4pq+1) & 0 \\ \frac{1}{4}q(2q+1) & \frac{1}{4}(2q-1) \end{matrix}$	$\begin{matrix} \frac{1}{2}p^2 & \frac{1}{2}p \\ \frac{1}{2}p(2q+1) & 0 \\ \frac{1}{2}q(q+1) & \frac{1}{2}(q+1) \end{matrix}$	$\begin{matrix} \frac{1}{2}p(p+1) & \frac{1}{2}(p+1) \\ \frac{1}{2}q(2p+1) & 0 \\ \frac{1}{2}q^2 & \frac{1}{2}q \end{matrix}$	$\begin{matrix} \frac{1}{4}p(2p+1) & \frac{1}{4}(2p+1) \\ \frac{1}{4}(4pq+1) & 0 \\ \frac{1}{4}q(2q+1) & \frac{1}{4}(2q+1) \end{matrix}$	$\begin{matrix} \frac{1}{2}p^2 & \frac{1}{2}p \\ \frac{1}{2}p(2q+1) & 0 \\ \frac{1}{2}q(q+1) & \frac{1}{2}(q+1) \end{matrix}$
$E\{E_o[(\epsilon_v)_o \times {}_o(Z_R)] \times (Z_R)_o\}$	$\begin{matrix} p & p \\ q & 0 \\ 0 & q \end{matrix}$	$\begin{matrix} \frac{1}{2}p & p \\ \frac{1}{2} & 0 \\ \frac{1}{2}q & q \end{matrix}$	$\begin{matrix} 0 & p \\ p & 0 \\ q & q \end{matrix}$	$\begin{matrix} p & p \\ q & 0 \\ 0 & q \end{matrix}$	$\begin{matrix} \frac{1}{2}p & p \\ \frac{1}{2} & 0 \\ \frac{1}{2}q & q \end{matrix}$	$\begin{matrix} 0 & p \\ p & 0 \\ q & q \end{matrix}$
$E\{E[{}_o(\epsilon_v) \times (Z_R)_o]_o \times {}_o(Z_R)\}$	$\begin{matrix} \frac{1}{2}p(p+1) & \frac{1}{2}(p+1) \\ \frac{1}{2}q(2p+1) & 0 \\ \frac{1}{2}q^2 & \frac{1}{2}q \end{matrix}$	$\begin{matrix} \frac{1}{2}p(p+1) & \frac{1}{2}(p+1) \\ \frac{1}{2}q(2p+1) & 0 \\ \frac{1}{2}q^2 & \frac{1}{2}q \end{matrix}$	$\begin{matrix} \frac{1}{2}p(p+1) & \frac{1}{2}(p+1) \\ \frac{1}{2}q(2p+1) & 0 \\ \frac{1}{2}q^2 & \frac{1}{2}q \end{matrix}$	$\begin{matrix} \frac{1}{2}p^2 & \frac{1}{2}p \\ \frac{1}{2}p(2q+1) & 0 \\ \frac{1}{2}q(q+1) & \frac{1}{2}(q+1) \end{matrix}$	$\begin{matrix} \frac{1}{2}p^2 & \frac{1}{2}p \\ \frac{1}{2}p(2q+1) & 0 \\ \frac{1}{2}q(q+1) & \frac{1}{2}(q+1) \end{matrix}$	$\begin{matrix} \frac{1}{2}p^2 & \frac{1}{2}p \\ \frac{1}{2}p(2q+1) & 0 \\ \frac{1}{2}q(q+1) & \frac{1}{2}(q+1) \end{matrix}$
$E\{E_o[{}_o(\epsilon_v) \times (Z_R)_o] \times (Z_R)_o\}$	$\begin{matrix} p^2 & p \\ 2pq & 0 \\ q^2 & q \end{matrix}$	$\begin{matrix} p^2 & p \\ 2pq & 0 \\ q^2 & q \end{matrix}$	$\begin{matrix} p^2 & p \\ 2pq & 0 \\ q^2 & q \end{matrix}$	$\begin{matrix} p^2 & p \\ 2pq & 0 \\ q^2 & q \end{matrix}$	$\begin{matrix} p^2 & p \\ 2pq & 0 \\ q^2 & q \end{matrix}$	$\begin{matrix} p^2 & p \\ 2pq & 0 \\ q^2 & q \end{matrix}$

The form $(e_v)_0 \times {}_0(\mathrm{Z_R})$ or ${}_0(e_v) \times (\mathrm{Z_R})_0$ for half relationships. In Table I for symmetrical forms it is irrelevant whether we use either type of zero substitution for the latter, except that, if the female parent is mated to the population at random, the P-matrices corresponding to *l*, *m*, *r* are equal, and equivalent for those corresponding to *l*, *m*, *n*, etc., for the case when the male is mated with the population at random. The values of $\mathrm{E}\{\mathrm{E}_0[{}_0(e_v) \times (\mathrm{Z_R})_0] \times (\mathrm{Z_R})_0\}$ are not tabulated since they can be derived at once from the identities in the $(s_v) \times (\mathrm{Z_R})$ and ${}_0(e_v) \times (\mathrm{Z_R})_0$ rows. The P-matrix $\mathrm{Z_R}$ has already been defined (§ 3 (*a*)) as representing a population mating at random.

5. APPLICATION TO SYSTEMS OF MATING AND THE CORRELATION OF RELATIVES.

In what follows, illustrative examples of the use of the notation proposed and the operations defined in the preceding sections will be given. Several of the examples given are theorems for which particular solutions are given by Jennings (3) in his classical memoir Others derive results due to Dahlberg (1), Fisher (2), and the writer.

(*a*) Random mating—autosomal gene substitution

A system is in equilibrium when

$$\mathrm{Z}_n = \mathrm{E}(\mathrm{Z}_n)^2.$$

From (2[9]), taking the second term in the left-hand column,

$$b_n = b_{n+1} = \tfrac{1}{2}\{d_n(b_n + 2c_n) + e_n + f_n(2a_n + b_n)\}.$$

When the distribution of the sexes is symmetrical this becomes

$$b_n = \tfrac{1}{2}\{a_n(b_n + 2c_n) + b_n + c_n(2a_n + b_n)\}.$$

Whence

$$b^2 = 4ac$$

or

$$b = 2\sqrt{ac}$$

(Pearson-Hardy law.)

(*b*) Random mating for sex-linked genes.

Let the initial population be

$$\mathrm{Z}_n = \begin{vmatrix} p & p \\ 0 & 0 \\ q & q \end{vmatrix}.$$

The mating system is $(\mathrm{Z}_n)^2$

$$\mathrm{P}_n = \begin{vmatrix} p^2 & 0 & pq \\ pq & 0 & q^2 \end{vmatrix}.$$

Extending this asymmetrically:

$$Z_{n+1}=\begin{vmatrix} p^2 & p \\ 2pq & 0 \\ q^2 & q \end{vmatrix}$$

Mating this population *inter se* is represented by $(Z_{n+1})^2$, whence

$$P_{n+1}=\begin{vmatrix} p^3 & 2p^2q & pq^2 \\ p^2q & 2pq^2 & q^3 \end{vmatrix}.$$

Extending this, we find that $Z_{n+1}=Z_{n+2}$, *i.e.* for the initial condition specified above equilibrium is attained after one generation of random mating (Jennings, p. 80).

If, instead of starting with a population of which the males and females belong to the same genotypes in corresponding proportions, all females belong to one (DD) and all males to the other (Ry), we proceed as follows. The P-matrix obtained by extending the Product-form of the general asymmetrical P-matrix when squared may be simplified by the relation $(d+f)=1$

$$E'\begin{vmatrix} a & d \\ b & 0 \\ c & f \end{vmatrix}^2 = \begin{vmatrix} \frac{1}{2}d(2a+b) & \frac{1}{2}(2a+b) \\ \frac{1}{2}b+(cd+af) & 0 \\ \frac{1}{2}f(b+2c) & \frac{1}{2}(b+2c) \end{vmatrix}.$$

Whence we obtain the zero groups for successive generations.

$$Z_0=\begin{vmatrix} 0 & 1 \\ 0 & 0 \\ 1 & 0 \end{vmatrix}\quad Z_1=\begin{vmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{vmatrix}\quad Z_2=\begin{vmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} \end{vmatrix}\quad Z_3=\begin{vmatrix} \frac{1}{8} & \frac{1}{4} \\ \frac{1}{2} & 0 \\ \frac{3}{8} & \frac{3}{4} \end{vmatrix}\quad Z_4=\begin{vmatrix} \frac{3}{32} & \frac{3}{8} \\ \frac{14}{32} & 0 \\ \frac{15}{32} & \frac{5}{8} \end{vmatrix}\quad Z_5=\begin{vmatrix} \frac{15}{128} & \frac{5}{16} \\ \frac{58}{128} & 0 \\ \frac{55}{128} & \frac{11}{16} \end{vmatrix}$$

This leads to the series

Ry	0	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{5}{16}$	. . . . .
Dy	1	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{5}{8}$	$\frac{11}{16}$	. . . . .
RR	0	0	$\frac{1}{8}$	$\frac{3}{32}$	$\frac{15}{128}$	
RD	1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{14}{32}$	$\frac{58}{128}$	
DD	0	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{15}{32}$	$\frac{55}{128}$	

(Jennings, p. 81.)

(*c*) Brother-sister mating for autosomal genes.

The mating system of the $(n-1)$th generation is denoted:

$$P_{n-1}=\begin{vmatrix} l & m & n \\ o & p & q \\ r & s & t \end{vmatrix}.$$

Then the constitution of the nth generation is the P-matrix

$$Z_n=E\,.\,(P_{n-1}).$$

Extending we find, since the distribution of the sexes is symmetrical so that $m=0$, $n=r$, and $q=s$:

$$Z_n = \left|\begin{array}{cc} \frac{1}{4}(4l+4m+p) & \frac{1}{4}(4l+4m+p) \\ \frac{1}{2}(2m+4n+p+2q) & \frac{1}{2}(2m+4n+p+2q) \\ \frac{1}{4}(p+4q+4l) & \frac{1}{4}(p+4q+4l) \end{array}\right| .$$

The mating system in the nth generation of fraternal inbreeding is the P-matrix series:

$$l\,.\,(s_l)^2 + 2m(s_m)^2 + 2n\,.\,(s_n)^2 + p\,.\,(s_p)^2 + 2q\,.\,(s_q)^2 + t\,.\,(s_t)^2.$$

This gives the product

$$P_n = \left|\begin{array}{ccc} \frac{1}{16}(16l+8m+p) & \frac{1}{8}(4m+p) & \frac{1}{16}p \\ \frac{1}{8}(4m+p) & \frac{1}{4}(2m+8n+p+2q) & \frac{1}{8}(4q+p) \\ \frac{1}{16}p & \frac{1}{8}(4q+p) & \frac{1}{16}(p+8q+16t) \end{array}\right| .$$

Since $Z_{n+1} = E\,.\,(P_n)$,

$$Z_{n+1} = \left|\begin{array}{cc} \frac{1}{16}(16l+18m+8n+4p+2q) & \dots\dots \\ \frac{1}{8}(6m+8n+4p+6q) & \dots\dots \\ \frac{1}{16}(2m+8n+4p+18q+16t) & \dots\dots \end{array}\right| .$$

If we start from the mating system in the F.I. generation of a monohybrid experiment,

$$P_1 = \left|\begin{array}{ccc} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right| .$$

This gives

$$P_2 = \left|\begin{array}{ccc} \frac{1}{16} & \frac{1}{8} & \frac{1}{16} \\ \frac{1}{8} & \frac{1}{4} & \frac{1}{8} \\ \frac{1}{16} & \frac{1}{8} & \frac{1}{16} \end{array}\right| \qquad P_3 = \left|\begin{array}{ccc} \frac{9}{64} & \frac{3}{32} & \frac{1}{64} \\ \frac{3}{32} & \frac{5}{16} & \frac{3}{32} \\ \frac{1}{64} & \frac{3}{32} & \frac{9}{64} \end{array}\right| \text{, etc.}$$

Extending according to the relations given below, we obtain the series:

R	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{11}{32}$	$\frac{3}{8}$	. . . . . .
H	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{5}{16}$	$\frac{1}{4}$	. . . . . .
D	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{11}{32}$	$\frac{3}{8}$	. . . . . .

(Jennings, p. 71.)

(*d*) Brother-sister mating for sex linkage.

As before we put:

$$P_{n-1} = \left|\begin{array}{ccc} l & m & n \\ r & s & t \end{array}\right| .$$

Extending asymmetrically we have:

$$Z_n = \left|\begin{array}{cc} \frac{1}{2}(2l+m) & \frac{1}{2}(2l+m+2r+s) \\ \frac{1}{2}(m+2n+2r+s) & 0 \\ \frac{1}{2}(s+2t) & \frac{1}{2}(m+2n+s+2t) \end{array}\right| .$$

The mating system is the product series:

$$l\,.\,(s_l)^2 + m\,.\,(s_m)^2 + n\,.\,(s_n)^2 + \,.\,.\,.\,.\,.\,.\,.\; t(s_t)^2.$$

The summation of this series gives:

$$P_n = \begin{vmatrix} \frac{1}{4}(4l+m) & \frac{1}{4}(m+4r+s) & \frac{1}{4}s \\ \frac{1}{4}m & \frac{1}{4}(m+4n+s) & \frac{1}{4}(s+4t) \end{vmatrix}.$$

The constitution of the $(n+1)$th generation is given by

$$Z_{n+1} = E\,.\,P(_n).$$

The value of this is:

$$Z_{n-1} = \begin{vmatrix} \dfrac{(8l+3m+4r+s)}{8} & \dfrac{(8l+6m+4n+2s)}{8} \\ \dfrac{(4m+4n+4r+4s)}{8} & 0 \\ \dfrac{(m+4n+3s+8t)}{8} & \dfrac{(2m+4n+4r+6s+8t)}{8} \end{vmatrix}.$$

Let the original cross be DD × R_y, then

$$P_{n-1} = \begin{vmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{vmatrix}$$

$$Z_n = \begin{vmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{vmatrix} \qquad P_n = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} \qquad Z_{n+1} = \begin{vmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} \end{vmatrix} \qquad P_{n+1} = \begin{vmatrix} 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} \end{vmatrix}$$

$$Z_{n+2} = \begin{vmatrix} \frac{1}{8} & \frac{1}{4} \\ \frac{1}{2} & 0 \\ \frac{3}{8} & \frac{3}{4} \end{vmatrix} \qquad Z_{n+3} = \begin{vmatrix} \frac{1}{8} & \frac{3}{8} \\ \frac{3}{8} & 0 \\ \frac{1}{2} & \frac{5}{8} \end{vmatrix}, \text{ etc.}$$

Giving:

R*y*	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{8}$	. . . ., etc.
D*y*	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{5}{8}$	. . . .
RR	0	$\frac{1}{8}$	$\frac{1}{8}$	. . . .
RD	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{8}$	. . . . .
DD	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{1}{2}$	. . . .

(Jennings, *op. cit.*, p. 85.)

(*e*) Parent-child mating for sex linkage: daughter mated back to father.

As before, we put:

$$P_{n-1} = \begin{vmatrix} l & m & n \\ r & s & t \end{vmatrix}$$

and

$$Z_n = \begin{vmatrix} \frac{1}{2}(2l+m) & \frac{1}{2}(2l+m+2r+s) \\ \frac{1}{2}(m+2n+2r+s) & 0 \\ \frac{1}{2}(s+2t) & \frac{1}{2}(m+2n+s+2t) \end{vmatrix}.$$

The mating system is now:

$$l\,.\,{}_0(e_l)\times(s_l)_0+m\,.\,{}_0(e_m)\times(s_m)_0+\quad\ldots\ldots\;t\,.\,{}_0(e_t)\times(s_t)_0.$$

The summation is the product:

$$P_n=\begin{vmatrix}\tfrac{1}{2}(2l+m) & \tfrac{1}{2}(2n+m) & 0\\ 0 & \tfrac{1}{2}(2r+s) & \tfrac{1}{2}(s+2t)\end{vmatrix}.$$

Whence, by extension:

$$Z_{n+1}=\begin{vmatrix}\tfrac{1}{4}(4l+3m+2n) & \tfrac{1}{4}(4l+3m+2n+2r+s)\\ \tfrac{1}{4}(m+2n+2r+s) & 0\\ \tfrac{1}{4}(2r+3s+4t) & \tfrac{1}{4}(m+2n+2r+3s+4t)\end{vmatrix}$$

Starting with the original parents $D_y\times RR$, we may write:

$$P_{n-1}=\begin{vmatrix}0&0&0\\ 1&0&0\end{vmatrix}\quad Z_n=\begin{vmatrix}0&1\\ 1&0\\ 0&0\end{vmatrix}\quad P_n=\begin{vmatrix}0&0&0\\ 0&1&0\end{vmatrix}\quad Z_{n+1}=\begin{vmatrix}0&\tfrac{1}{2}\\ \tfrac{1}{2}&0\\ \tfrac{1}{2}&\tfrac{1}{2}\end{vmatrix}\quad P_{n+1}=\begin{vmatrix}0&0&0\\ 0&\tfrac{1}{2}&\tfrac{1}{2}\end{vmatrix}$$

$$Z_{n+2}=\begin{vmatrix}0&\tfrac{1}{4}\\ \tfrac{1}{4}&0\\ \tfrac{3}{4}&\tfrac{3}{4}\end{vmatrix}\qquad Z_{n+3}=\begin{vmatrix}0&\tfrac{1}{8}\\ \tfrac{1}{8}&0\\ \tfrac{7}{8}&\tfrac{7}{8}\end{vmatrix}.$$

This gives the series:

Ry	1	$\tfrac{1}{2}$	$\tfrac{1}{4}$	$\tfrac{1}{8}$	. . . . .
Dy	0	$\tfrac{1}{2}$	$\tfrac{3}{4}$	$\tfrac{7}{8}$	. . .
RR	0	0	0	0	. . . .
RD	1	$\tfrac{1}{2}$	$\tfrac{1}{4}$	$\tfrac{1}{8}$	. . .
DD	0	$\tfrac{1}{2}$	$\tfrac{3}{4}$	$\tfrac{7}{8}$	. . . .

(Jennings, p. 86.)

(*f*) Correlation of relatives—case of father-son with sex-linked transmission.

This is represented by the correlate sum of the correlates

$$y_n\,.\,{}_0(e_n)\sim{}_0(s_n).$$

The coefficient y_n being the terms of the product

$$\begin{vmatrix}p^2 & 2p^2q & pq^2\\ p^2q & 2pq^2 & q^3\end{vmatrix}.$$

If mating in the population occurs at random. On condensing the series, we obtain the following:—

	R	D
R	p^2	pq
D	pq	q^2

From which, by the product moment method, $r=0$.

(*g*) Correlation of sisters with sex-linked inheritance.

The corresponding series of correlates is now:

$$y_n \,.\, (s_n)_0 \sim (s_n)_0.$$

This yields the table:

	R	H	D
R	$\frac{1}{2}p^2(1+p)$	$\frac{1}{2}p^2q$	0
H	$\frac{1}{2}p^2q$	$\frac{3}{2}pq$	$\frac{1}{2}pq^2$
D	0	$\frac{1}{2}pq^2$	$\frac{1}{2}q^2(1+q)$

When the heterozygote is intermediate $r=0{\cdot}75$ (Hogben, *op. cit*).

(*h*) Marriages of first cousins for autosomal genes.

The next illustration will be an expression for a problem of which Dahlberg has given an *ad hoc* solution. The problem is the concentration of recessives among the offspring of cousins in a system of random mating.

The mating system of the grandparents is:

$$P_R = \begin{vmatrix} p^4 & 2p^3q & p^2q^2 \\ 2p^3q & 4p^2q^2 & 2pq^3 \\ p^2q^2 & 2pq^3 & q^4 \end{vmatrix}.$$

The general term of this product matrix is denoted y in what ensues. The offspring of the generation of grandparents is:

$$y \,.\, s_y.$$

If these offspring mate at random with the population the mating system of the parents of cousins is

$$y \,.\, (s_y \times Z_R).$$

The generation of cousins is found by extension of this product,

$$y \,.\, E \,.\, (s_y \times Z_R).$$

If each group of cousins produced by a similar pair of grandparents mated *inter se*, the mating system is:

$$y \,.\, [E \,.\, (s_y \times Z_R)]^2.$$

The offspring of cousins are therefore:

$$y \,.\, E \,.\, \{[E \,.\, (s_y \times Z_R)]^2\}.$$

Tables of $E(s_y \times Z_R)$ are given in the preceding section and the evaluation of $E\{[E(P)]^2\}$ is given in section 2 (9). From this it is found that all powers higher than p^2 vanish in the first term of the zero group so derived. The exact expression for the concentration of recessives among offspring of first cousins is thus the value given by Dahlberg (1),

$$\tfrac{1}{16}(p+15p^2).$$

(*j*) Correlation of cousins—autosomal case.

Using the same symbolism as in the previous example, the generation of cousins is:

$$y \,.\, \mathrm{E}(s_y \times \mathrm{Z_R}).$$

The contingency table is formed by summating:

$$y \,.\, [\mathrm{E}(s_y \times \mathrm{Z_R}) \sim \mathrm{E}(s_y \times \mathrm{Z_R})].$$

This at once gives the table:

	R	H	D
R	$\tfrac{1}{4}p^3(3p+1)$	$\tfrac{1}{4}p^2q(6p+1)$	$\tfrac{3}{4}p^2q^2$
H	$\tfrac{1}{4}p^2q(6p+1)$	$\tfrac{1}{4}pq(12pq+1)$	$\tfrac{1}{4}pq^2(6q+1)$
D	$\tfrac{3}{4}p^2q^2$	$\tfrac{1}{4}pq^2(6q+1)$	$\tfrac{1}{4}q^3(3q+1)$

When the heterozygote is intermediate this gives $r=0{\cdot}125$ in agreement with Fisher's (2) finding and contrary to the value given by Snow (5).

(*k*) Assortative mating—the sex-linked case.

We shall first consider the case of assortative mating, when transmission is sex-linked, R males mating with R females and D males with R or H females. Clearly we may write for the mating system

$$\mathrm{P}_n = \begin{vmatrix} x & 0 & 0 \\ 0 & y & z \end{vmatrix}.$$

Now

$$\mathrm{Z}_n = \xi(\mathrm{P}_n) = \begin{vmatrix} x & x \\ y & 0 \\ z & y+z \end{vmatrix}$$

and

$$\mathrm{Z}_{n+1} = \mathrm{E}(\mathrm{P}_n) = \begin{vmatrix} x & x+\tfrac{1}{2}y \\ \tfrac{1}{2}y & 0 \\ \tfrac{1}{2}y+z & \tfrac{1}{2}y+z \end{vmatrix}.$$

Let the original cross be RR × Ry. This gives H and Ry which, when mated, give H and R females and D and R males. Assortative mating under the above restriction now begins:

$$Z_n = \begin{vmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{vmatrix} \quad P_n = \begin{vmatrix} \frac{1}{2} & 0 & 0 \\ \\ 0 & \frac{1}{2} & 0 \end{vmatrix} \quad Z_{n+1} = \begin{vmatrix} \frac{1}{2} & \frac{3}{4} \\ \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} \end{vmatrix} \quad P_{n+1} = \begin{vmatrix} \frac{1}{2} & 0 & 0 \\ \\ 0 & \frac{1}{4} & \frac{1}{4} \end{vmatrix}$$

$$Z_{n+2} = \begin{vmatrix} \frac{1}{2} & \frac{5}{8} \\ \frac{1}{8} & 0 \\ \frac{3}{8} & \frac{3}{8} \end{vmatrix} \quad P_{n+2} = \begin{vmatrix} \frac{1}{2} & 0 & 0 \\ \\ 0 & \frac{1}{8} & \frac{3}{8} \end{vmatrix} \quad Z_{n+3} = \begin{vmatrix} \frac{1}{2} & \frac{9}{16} \\ \frac{1}{16} & 0 \\ \frac{7}{16} & \frac{7}{16} \end{vmatrix} \quad P_{n+3} = \begin{vmatrix} \frac{1}{2} & 0 & 0 \\ \\ 0 & \frac{1}{16} & \frac{7}{16} \end{vmatrix}$$

$$Z_{n+4} = \begin{vmatrix} \frac{1}{2} & \frac{17}{32} \\ \frac{1}{32} & 0 \\ \frac{15}{32} & \frac{15}{32} \end{vmatrix}.$$

We thus obtain the series given by Jennings (pp. 83–84). If transmission is not sex linked, and the only restriction is that R mates with R, the relevant matrices are:

$$Z_n = \begin{vmatrix} x & x \\ y & y \\ z & z \end{vmatrix}$$

$$P_n = \begin{vmatrix} x & 0 & 0 \\ 0 & \dfrac{y^2}{y+z} & \dfrac{zy}{y+z} \\ 0 & \dfrac{zy}{y+z} & \dfrac{z^2}{y+z} \end{vmatrix}.$$

Whence, by extension,

$$Z_{n+1} = \begin{vmatrix} x + \dfrac{y^2}{4(1-x)} & \cdots\cdots \\ \dfrac{1}{1-x}\{\frac{1}{2}y^2 + zy\} & \cdots\cdots \\ \dfrac{1}{1-x}\{z^2 + zy + \frac{1}{4}y^2\} & \cdots \end{vmatrix}.$$

The last is equivalent to the general expression given by Jennings (p. 67).

The identity of the correlate and product form for the symmetrical cases renders the proposed notation suitable to deal with systems of assortative mating involving a known correlation of parental genotypes.

6. CONCENTRATION OF RECESSIVES AMONG OFFSPRING OF RELATIVES IN A SYSTEM OF RANDOM MATING.

The foregoing illustrations will, it is hoped, suffice to illustrate the use of the notation proposed and to justify its validity. A summary of the matrix formulæ for offspring of consanguineous unions will now be given, and evaluated for the terms *a* and *d* in the P-matrix representing the offspring of parents of the specified degree of relationship in a population mating at random. Similar results have already been arrived at by

Professor J. B. S. Haldane, F.R.S., using a different method. Professor Haldane kindly forwarded to me a summary of his own findings, which have not yet been published.

(*a*) Autosomal genes.

Relationship of Parents.	Matrix Formula.	Recessives.
Unrelated	$E\,.\,(Z_R)^2$	p^2
Parent-child	$y\,.\,E(e_y \times s_y)$	$\frac{1}{4}(p+3p^2)$
Brother-sister	$y\,.\,E(s_y)^2$	$\frac{1}{4}(p+3p^2)$
Uncle or aunt with niece or nephew.	$y\,.\,E[E(s_y \times Z_R) \times s_y]$	$\frac{1}{8}(p+7p^2)$
First cousins	$y\,.\,E\{[E(s_y \times Z_R)]^2\}$	$\frac{1}{16}(p+15p^2)$
Second cousins	$y\,.\,E\{E[E(s_y \times Z_R) \times Z_R]\}^2$	$\frac{1}{64}(p+63p^2)$
Grandparent and grandchild.	$y\,.\,E\{e_y \times E(s_y \times Z_R)\}$	$\frac{1}{8}(p+7p^2)$

(*b*) Sex-linked genes.

Relationship of Parents.	Matrix Formula	Recessive Females.
Unrelated	$E(Z_R)^2$	p^2
Parent-child	$y\,.\,E(s_y \times e_y)$	$\frac{1}{2}(p+p^2)$
Mother-son	$y\,.\,E[{}_0(s_y) \times (e_y)_0]$	$\frac{1}{2}(p+p^2)$
Father-daughter	$y\,.\,E[(s_y)_0 \times {}_0(e_y)]$	$\frac{1}{2}(p+p^2)$
Brother-sister	$y\,.\,E(s_y)^2$	$\frac{1}{4}(p+3p^2)$

Maternal cousins (male with mother's sister's daughter, or female with mother's sister's son)

$$y\,.\,E\{[E(s_y)_0 \times {}_0(Z_R)]^2\} \qquad \frac{1}{16}(3p+13p^2)$$

Paternal cousins (male with father's brother's daughter, or female with father's brother's son)

$$y\,.\,E\{[E_0(s_y) \times (Z_R)_0]^2\} \qquad p^2$$

Cousins (female with mother's brother's son, or male with father's sister's daughter)

$$y\,.\,E\{E[(s_y)_0 \times {}_0(Z_R)]_0 \times {}_0E[{}_0(s_y) \times (Z_R)_0]\} \qquad p^2$$

Cousins (male with mother's brother's daughter, or female with father's sister's son)

$$y\,.\,E\{{}_0E[(s_y)_0 \times {}_0(Z_R)] \times E[{}_0(s_y) \times (Z_R)_0]_0\} \qquad \frac{1}{8}(p+7p^2)$$

Male to paternal aunt

$$y\,.\,E\{(s_y)_0 \times {}_0E[{}_0(s_y) \times (Z_R)_0]\} \qquad p^2$$

Male to maternal aunt

$$y\,.\,E\{(s_y)_0 \times {}_0E[(s_y)_0 \times {}_0(Z_R)]\} \qquad \frac{1}{8}(3p+5p^2)$$

Female to paternal uncle

$$y\,.\,E\{{}_0(s_y) \times E[{}_0(s_y) \times (Z_R)_0]_0\} \qquad \frac{1}{4}(p+3p^2).$$

Female to maternal uncle

$$y\,.\,E\{{}_0(s_y) \times E[(s_y)_0 \times {}_0(Z_R)]_0\} \qquad \frac{1}{8}(p+7p^2).$$

In all cases the frequency of males is the same as in random mating in general, *i.e.* $d=p$.

(*c*) Half relationships.

To apply the notation employed throughout this communication to half-sibs, half-uncle or -aunt and nephew or niece, half-cousins, etc., it is only necessary to substitute throughout the P-matrix

$$E\{{}_0(e_y)\times(Z_R)_0\} \quad \text{or} \quad E\{(e_y)_0\times{}_0(Z_H)\} \quad \text{for} \quad s_y.$$

One example will suffice. For the concentration of recessives in offspring of half-brethren, when transmission is autosomal we take the term a in the P-matrix,

$$E\,.\,[E\,.\,(e_y)_0\times{}_0(Z_H)]^2.$$

This gives:

$$\tfrac{1}{8}(p+7p^2).$$

7. SUMMARY

A matrix notation suitable to determine the characteristics of different systems of inbreeding and the correlation of relatives in Mendelian populations and applicable to single autosomal or sex-linked gene substitutions is given. Certain expressions constantly recur in problems of inbreeding or correlation With the aid of the proposed notation, tables of matrices which embody these recurring expressions can be drawn up once and for all. Such tables are given herewith. The operators defined reduce the unwieldiness of the expressions obtained and lead to general expressions from which a number of established results are easily derived, and some new theorems of inbreeding have been deduced.

I am indebted to Miss P. Moshinsky, B.Sc., Mr E. A. G. Shrimpton, B.Sc., and Mr Cecil Gordon, M.Sc., for checking the results given; and also to Professor J. B. S. Haldane, F.R.S., who has kindly read the typescript and made useful criticisms.

REFERENCES.

(1) DAHLBERG, 1930. "Inbreeding in Man," *Genetics*, vol. xxx.

(2) FISHER, 1918. "The Correlations between Relatives on the Supposition of Mendelian Inheritance," *Trans. Roy. Soc. Edin*, vol. lii, pp 399-433.

(3) JENNINGS, 1917. "Numerical Results of Diverse Systems of Breeding," *Genetics*, vol. i.

(4) HOGBEN, 1932. "Filial and Fraternal Correlations in Sex-linked Inheritance," *Proc. Roy. Soc. Edin.*, vol. lii, pp. 331-336.

(5) SNOW, 1910. "On the Determination of the Chief Correlations between Collaterals," *Proc. Roy. Soc.*, B, vol. lxxxiii, p. 37.

(*Issued separately January* 16, 1933.)

III.—Supernumerary Pectoral Fins in *Raia circularis* Loudon.

By **W. F. Harper**, Ph.D., M.B., Department of Anatomy, University College (University of St Andrews), Dundee. *Communicated by* Professor A. D. PEACOCK.

(MS. received May 30, 1932 Amended MS. received August 2, 1932.
Read November 7, 1932.)

A MALE Sandy Ray (*Raia circularis* Loudon) caught in the North Sea, off Aberdeen, measured 47·5 cm. broad by 43 cm. long (excluding the tail) and exhibited an abnormality which is of very rare occurrence. This consisted of two well-developed fins situated ventrally and associated with the pectoral girdle and the last two branchial arches The measurements of the fins in the formalin-hardened specimen were as follows: length, 8 cm.; breadth at base, 1·3 cm.; breadth at free end, 2 cm. (left), 3·2 cm. (right); thickness at base, 0·7 cm. The integument covering the lateral and medial borders of the proximal portion of each fin was continuous with that over the fourth and fifth gill clefts respectively. In addition, a strong fold of integument bound the proximal fourth of each fin to the ventral aspect of the body of the fish (fig. 1). A brownish-red pigment was present on the ventral aspect of the terminal third of each, and a less marked pigmentation of the same colour on the dorsal aspect of the middle third. Small denticles, similar to those found elsewhere on the fins, were numerous on the distal third, but were more abundant on the dorsal aspect.

Broadly oval in transverse section in their proximal two-thirds, each fin flattens out and expands somewhat towards its free end, this being specially marked on the right side. A single stout bar of cartilage forms the basal support of each fin and extends two-thirds of its length before articulating with its radial elements. The terminal portion of each fin is supported by a series of six more or less parallel jointed radialia. A strong fibro-elastic band, springing from the anterior part of the coracoid, in front of, and medial to, the anterior facet for the propterygium, binds the basal cartilage to the pectoral girdle. Several fibres of this band are prolonged downwards and backwards to the margin of the anterior scapular fontanelle. From this attachment the fin, in the proximal part of its course, passes downwards and backwards in front of the articulation between coracoid and propterygium, and this buried portion is bent almost at right angles to the main body of the fin.

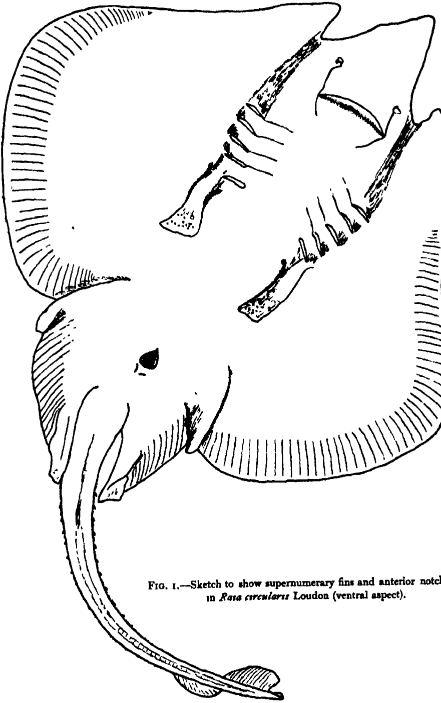


FIG. 1.—Sketch to show supernumerary fins and anterior notching in *Raia circularis* Loudon (ventral aspect).

The pectoral girdle and pterygial elements were normal except that there was a small lateral facet on the coracoid where it articulated with the basal cartilage of each fin.

The main muscular mass of the fin has a double origin: by an outer head from the medial surface of the propterygium for approximately 1 inch anterior to its junction with coracoid (fig. 2, A), and by a smaller head from the anterior aspect of the coracoid (fig. 2, B). Numerous muscle fibres, derived from the muscles of the fourth and fifth branchial arches, enter the outer head. From this double origin the two heads rapidly unite and pass directly on to the basal cartilage, around which they form a soft and fleshy mass. The basal cartilage along the proximal fourth of its lateral border is also firmly adherent to a strong fascial layer reflected on to it from the inferior margin of the propterygium.

FIG 2.—Dissection of pectoral girdle of *Raia circularis* Loudon, to show attachments of right fin (ventral aspect)

This fascial interval lies between the origin of the outer head of the fin muscle and the large pectoral muscle (fig. 2, C).

Each fin is innervated by a nerve derived from the third and fourth spinal nerves. The nerve reaches the medial aspect of the base of the fin by passing through a foramen in the coracoid. Here it bifurcates, one division continuing along the medial aspect of the fin, the other passing deeply in front of its attachment, to the lateral side

In addition to the abnormality described, the fish had a deep notch in each pectoral fin near the snout, and abnormally small copulatory claspers (fig. 1).

The development of such supernumerary fins in this situation is difficult to explain. Professor E. S. Goodrich, who kindly examined photographs of the specimen, suggested that they may be derivatives of the large pectorals The latter were therefore carefully examined and found to have three fewer radials on the right side and two fewer on the left side, as compared with a normal specimen of the same species and sex It is possible, therefore, that during development these extra fins have been squeezed out of the large pectorals, and, if so, the nerve supply indicates that they are derivatives of their anterior region. It should be borne in mind, however, that the number of radial elements is probably not constant within the same species.

The only reference to a similar abnormality appears to be a short note by Johnstone (3) on the occurrence of a unilateral ventral pectoral fin in *Raia maculata.* The fin described by him was situated on the right side and measured 5–6 cm. long. The fish was approximately half the length of the present specimen, and that author also noted an abnormal notch on the right side of the pectoral fin in front, indicating incomplete fusion with the anterior region of the trunk.

Less unusual is the occurrence of a supernumerary pectoral fin on the dorsal surface, an abnormality which was first noted by Lacépède (4), who described the specimen as a distinct species under the name of *Raia cuvieri.* Gervais (2) and Rennie (5) recorded somewhat similar abnormalities. Bateson (1), in his *Materials for the Study of Variation,* quotes several cases of extra fins in *Raiidæ* which appear to be of the same nature.

I am indebted to Dr H. C Williamson for the specimen from which the above description was made, and to Professor D Rutherford Dow, in whose department this paper was prepared.

[REFERENCES

REFERENCES.

(1) Bateson, W., 1894. *Materials for the Study of Variation*, p. 540.

(2) Gervais, P., 1864. "Cas de polymélie (membres surnuméraires) observé sur un *Batracien* du genre *Pelobates* et sur une espèce du genre *Raia*," *Compt. Rend. Acad. Sci. Paris*, vol. iii, p. 802.

(3) Johnstone, J., 1919. "On the Occurrence of an Additional Pectoral Fin in *Raia maculata*," *Report on Lancashire Sea-fisheries*, vol. xxviii, p. 31.

(4) Lacépède, E., 1798. *Hist. Nat. des Poiss.*, vol. i, p. 141.

(5) Rennie, J., 1906. "Accessory Fins in *Raia batis*," *Anat. Anz.*, vol. xxviii, p. 428.

(*Issued separately January* 16, 1933)

IV.—On Properties of Null Geodesics, and their Application to the Theory of Radiation. By **W. O. Kermack**, M.A., D.Sc., **W. H. M'Crea**, M.A., Ph.D., and **Professor E. T. Whittaker**, F.R.S.

(MS. received June 27, 1932 Read December 5, 1932)

1. *Introduction.*—In a previous paper * an expression was given by one of us for the Doppler effect in a de Sitter universe. In the present communication an attempt is made to obtain a more general expression for the Doppler effect, and for this purpose certain results have been derived relating to the properties of null geodesics in a general Riemannian space. In the paper referred to, the problem of defining spatial distance in general space was discussed and certain formulæ suggested which satisfy the essential conditions associated with the idea of spatial distance. This problem has now been approached from a different standpoint, and a new formula of more general form is now put forward. In the final section of the present paper a physical interpretation is given of the theorem relating to null geodesics which is proved in the first section. This involves a discussion of the concept of energy in general relativity, in order that a decision may be arrived at as to what law governs the transference of energy by radiation from one point to another. It appears that when the simplest and most natural assumptions are made for this purpose, then these are precisely the assumptions which are necessary to ensure the general consistency of relativity with the principles of quantum theory. This is illustrated by an application to a statical universe, and to the reddening of light in an expanding universe

I. THEORY OF NULL GEODESICS.

2. *Transport Vectors along Null Geodesics.*—Consider any Riemannian space V_n whose metric is specified by the equation

$$ds^2 = \sum_{pq} g_{pq} dx^p dx^q. \qquad (2.1)$$

Let Γ be a null geodesic in V_n. At any point C of Γ take a null vector η^p which has the direction of the null geodesic Γ at C. This fixes η^p save for a constant multiplier which will be chosen arbitrarily at the point C. Now let the vector η^p, thus completely specified at C, be carried from C along the null geodesic Γ by Levi-Civita parallel transport, so that a null vector η^p is thus specified at every point of Γ; it will evidently be directed along the tangent to Γ at every point. We shall call it a ***transport vector*** belonging to the null geodesic.

---

* E. T. Whittaker, *Proc. Roy. Soc.*, A, vol. cxxxiii, 1931, pp. 93–105.

Write now

$$\eta^p = \frac{dx^p}{d\lambda}, \qquad \text{(2 2)}$$

where dx^p corresponds to a displacement along Γ. This introduces a new variable λ, which in the theory of null geodesics plays much the same part as the interval "*s*" plays in the theory of ordinary geodesics.

Since the transport vector is carried from one point of Γ to another by parallel transport, we have

$$\frac{d\eta^l}{d\lambda} + \sum_{pq} \begin{Bmatrix} pq \\ l \end{Bmatrix} \eta^q \frac{dx^p}{d\lambda} = 0$$

or

$$\frac{d^2x^l}{d\lambda^2} + \sum_{pq} \begin{Bmatrix} pq \\ l \end{Bmatrix} \frac{dx^p}{d\lambda}\frac{dx^q}{d\lambda} = 0. \qquad \text{(2.3)}$$

Thus the equations of the null geodesics have precisely the same form when λ is taken as independent variable, as the equations of the ordinary geodesics have when the interval s *is taken as independent variable.* Hence we readily see that if we write

$$T = \tfrac{1}{2}\sum_{pq} g_{pq}\frac{dx^p}{d\lambda}\frac{dx^q}{d\lambda} \equiv \tfrac{1}{2}\sum_{pq} g_{pq}\dot{x}^p\dot{x}^q, \qquad \text{(2.4)}$$

then the equations of the null geodesics may also be written

$$\frac{d}{d\lambda}\left(\frac{\partial T}{\partial \dot{x}^r}\right) - \frac{\partial T}{\partial x^r} = 0. \qquad \text{(2.5)}$$

A Theorem on Null Geodesics.—Let Γ, Γ′ be neighbouring null geodesics in V_n, C being a point on Γ, and C′ a point near C on Γ′. Let η^p be a transport vector along Γ, and let η_p be its covariant form. Write

$$J = \sum_p \eta_p \delta\xi^p, \qquad \text{(2.6)}$$

where η_p denotes the value of this transport vector at C, and $\delta\xi^p$ denotes the vector CC′. Then we shall prove that J *is independent of the choice of the direction* CC′, *and is also independent of the position of the point* C *on the null geodesic* Γ, so that J depends only on the two geodesics as a whole and not on any particular points on them. (Theorem I.)

From (22) we have at C

$$\sum_{pq} g_{pq}\frac{dx^p}{d\lambda}\frac{dx^q}{d\lambda} = 0. \qquad \text{(2.71)}$$

To the neighbouring point C′ on the null geodesic Γ′ may be attributed co-ordinates $x'^p = x^p + \delta x^p$, and the parameter $\lambda' = \lambda + \delta\lambda$. But we shall have at C′

$$\sum_{pq} g'_{pq} \frac{dx'^p}{d\lambda'} \frac{dx'^q}{d\lambda'} = 0. \quad . \quad . \quad . \quad . \quad (2.72)$$

Substituting then for x'^p, etc., and expanding to the first order, and subtracting equation (2.71) we have

$$\sum_{pqr} \frac{\partial g_{pq}}{\partial x^r} \cdot \frac{dx^p}{d\lambda} \cdot \frac{dx^q}{d\lambda} \delta x^r + 2\sum_{pq} g_{pq} \frac{d\delta x^p}{d\lambda} \cdot \frac{dx^q}{d\lambda} - 2\sum_{pq} g_{pq} \frac{dx^p}{d\lambda} \cdot \frac{dx^q}{d\lambda} \cdot \frac{d\delta\lambda}{d\lambda} = 0. \quad (2.8)$$

The last term vanishes by (2.71). Also we have from (2.5)

$$\tfrac{1}{2}\sum_{pq} \frac{\partial g_{pq}}{\partial x^r} \cdot \frac{dx^p}{d\lambda} \cdot \frac{dx^q}{d\lambda} = \frac{d}{d\lambda}\Big(\sum_q g_{rq} \frac{dx^q}{d\lambda}\Big).$$

Thus (2.8) becomes

$$\sum_r \frac{d}{d\lambda}\Big(\sum_q g_{rq} \frac{dx^q}{d\lambda}\Big) \cdot \delta x^r + \sum_{pq} g_{pq} \frac{d\delta x^p}{d\lambda} \cdot \frac{dx^q}{d\lambda} = 0,$$

or, changing the dummy suffix,

$$\sum_{pq} \frac{d}{d\lambda}\Big(g_{pq} \frac{dx^q}{d\lambda}\Big) \cdot \delta x^p + \sum_{pq} g_{pq} \frac{d\delta x^p}{d\lambda} \cdot \frac{dx^q}{d\lambda} = 0,$$

or

$$\sum_p \frac{d\eta_p}{d\lambda} \cdot \delta x^p + \sum_p \eta_p \frac{d\delta x^p}{d\lambda} = 0,$$

or

$$\frac{d}{d\lambda}\sum_p \eta_p \delta x^p = 0,$$

or

$$\sum_p \eta_p \delta x^p = \text{constant},$$

which proves the theorem.*

3. *An Extension of the Theorem.*—Consider the null geodesic Γ passing through the point C, and consider all the neighbouring null geodesics which near C are parallel to Γ, of which there are ∞^{n-1}. Let η^p be a null vector at C directed along Γ, and form the scalar J corresponding to this η^p and any one of the neighbouring parallel null geodesics. Corresponding to any particular value of J there will be ∞^{n-2} of these geodesics. *Then this set of* ∞^{n-2} *null geodesics lies in a* V_{n-1} *which*

---

* This theorem is a generalisation of a theorem proved by J. L. Synge and A. J. M'Connell, *Phil Mag.*, (7), vol. v, 1928, pp. 241–263. Their theorem (equation (5 5) of their paper) may be obtained as a particular case of the one given here by supposing that the points C, C′ are correlated to each other by the condition that a certain parameter is to have the same value at C′ as it has at C. Reference may also be made to a paper by C. Lanczos, *Zs. f. Physik*, vol. xvii, 1923, pp. 168–189.

intersects a local flat subspace S_{n-1} *of* $(n-1)$ *dimensions at* C *in a local flat subspace* S_{n-2} *of* $(n-2)$ *dimensions. This* S_{n-2} *is perpendicular to the projection of* Γ *in* S_{n-1}. (Theorem II.)

Since this theorem refers only to the neighbourhood of C it is sufficient to prove it for the case of a local flat space S_n. For we shall assume there is no singularity at C, or in some neighbourhood of C.

Choose then any set of local rectangular axes at the point C, and let the direction ratios of Γ be $(\lambda_1, \lambda_2, \ldots, \lambda_n)$. Then

$$\lambda_1^2+\lambda_2^2+\ldots+\lambda_n^2=0. \quad . \quad . \quad . \quad . \quad (3.1)$$

The vector η^p will have components $(\eta\lambda_1, \eta\lambda_2, \ldots, \eta\lambda_n)$, where η is some constant. Any point on a parallel null geodesic through the point $(x_1, x_2, \ldots, x_n)$ will have co-ordinates $(x_1+\rho\lambda_1, x_2+\rho\lambda_2, \ldots, x_n+\rho\lambda_n)$, where ρ is a variable parameter. Hence, since we may take the vector $\delta\xi^p$ to be $(x_1, x_2, \ldots, x_n)$, we have

$$J=(x_1\lambda_1+x_2\lambda_2+\ldots+x_n\lambda_n)\eta. \quad . \quad . \quad . \quad (3.2)$$

Therefore J remains constant as long as $(x_1, x_2, \ldots, x_n)$ lies in the hyperplane

$$x_1\lambda_1+x_2\lambda_2+\ldots+x_n\lambda_n=J/\eta. \quad . \quad . \quad . \quad (3.3)$$

Further, any one of this family of parallel geodesics which passes through one point of (3.3) will lie wholly in this hyperplane, as is seen at once by substituting $(x_1+\rho\lambda_1, x_2+\rho\lambda_2, \ldots, x_n+\rho\lambda_n)$ for $(x_1, x_2, \ldots, x_n)$ in (3.3) and using (3.1). It follows at once that all points in which these null geodesics cut any local subspace S_{n-1} lie in a local subspace S_{n-2} Furthermore, this S_{n-2} is perpendicular to the projection of the original null geodesic in this S_{n-1}. For, take this S_{n-1} to be $x_1=0$, the projection of the geodesic is given by

$$\frac{x_2}{\lambda_2}=\frac{x_3}{\lambda_3}=\ldots=\frac{x_n}{\lambda_n}, \quad . \quad . \quad . \quad (3.4)$$

so that the direction cosines of the projection are $(\lambda_2/\sigma, \lambda_3/\sigma, \ldots, \lambda_n/\sigma)$, where $\sigma=i\lambda_1$. The S_{n-2} is

$$\lambda_2x_2+\lambda_3x_3+\ldots+\lambda_nx_n=J/\eta, \quad . \quad . \quad . \quad (3.5)$$

which is obviously perpendicular to (3.4).

If now we consider in the original V_n all the ∞^{n-1} null geodesics which near C are parallel to Γ, then at a distant point C_1 along Γ they will no longer be strictly parallel to Γ. But by taking the neighbouring geodesics sufficiently close together at C, they can be made as nearly parallel as desired at C_1. For we shall assume that there is no singularity on the part CC_1 of Γ, or in that neighbourhood of CC_1 which contains the relevant portions of the neighbouring geodesics.

Now we can class these geodesics according to the value of J at C. But by Theorem I all the ∞^{n-2} null geodesics which yield a particular value of J for one point of Γ still yield the same value for any other point of Γ. It follows that the class of ∞^{n-2} null geodesics, which lie in a flat subspace S_{n-1} in the neighbourhood of one point on Γ, will continue to lie, to a first approximation, in a flat subspace (another S_{n-1}) in the immediate neighbourhood of a second point on Γ, even when this is distant from the first point. Clearly, this group of null geodesics lies in a null hypersurface V_{n-1}, to which the local flat subspaces S_{n-1} are tangential. It may be noted that this V_{n-1} has a fundamental form

$$ds^2 = a_{ij}dx^i dx^j \qquad (i, j = 1, 2, \ldots, n-2) \qquad . \qquad . \qquad (3.6)$$

where there are only $(n-2)$, and not $(n-1)$, x's; and where a_{ij} is a function of these x's, and a parameter λ serving as a curvilinear co-ordinate along the null geodesics. Also for the particular value $J=0$ the original geodesic Γ lies in this V_{n-1}.

II. SPATIAL DISTANCE.

4. *A New Definition of Spatial Distance in General Relativity.*—We shall now consider the special case where the V_n is a space-time manifold of four dimensions, and the null geodesics give the histories of light pulses.

It has been pointed out * that spatial distance in its simplest aspect is a function of two point-events in space-time, which exists only when the two point-events are on the same null geodesic, *i.e.* when light emitted at one reaches the other.

We take a "star" A and an observer B whose world-lines intersect a null geodesic Γ at points C_1, C_0 respectively; so that C_1, C_0 are the point-events whose distance apart we wish to determine. Let a variable λ be determined on Γ (save for a multiplicative constant) by equations (2.3). Then we may define *the element of spatial distance* along the null geodesic Γ to be $d\lambda$. The multiplicative constant is to be fixed by the condition that, in the case where the star A and observer B are very near each other, this definition of spatial distance is to agree with the usual definition of spatial distance, which holds only for objects in B's infinitesimally small, instantaneous, three-dimensional space.

Let the line-element from the observer B at the instant of reception of the light, to the star A at the instant of emission of the light, when they are infinitesimally close to each other, be (dx^0, dx^1, dx^2, dx^3), and let $(\varpi_0, \varpi_1, \varpi_2, \varpi_3)$ be the covariant components of a unit vector in the direction of the observer B's world-line. Then the distance AB, in B's instantaneous space, is measured by the interval of B's proper time between the emission of the light and its reception—that is, the projection of the element (dx^0, dx^1, dx^2, dx^3) on B's world-line, or

---

* E. T. Whittaker, *Proc. Roy. Soc.*, A, vol. cxxxiii, 1931, pp. 93–105.

$$\varpi_0 dx^0 + \varpi_1 dx^1 + \varpi_2 dx^2 + \varpi_3 dx^3. \quad . \quad . \quad . \quad (4.1)$$

The element of spatial distance, $d\lambda$, must reduce therefore to this value at the observer's world-point. But from the definition of λ we know that

$$\frac{dx^0}{d\lambda} = kY(x^0, x^1, x^2, x^3), \quad . \quad . \quad . \quad . \quad . \quad (4.2)$$

where k is an unknown constant, and Y is a known function of (x^0, x^1, x^2, x^3). Hence at the observer's world-point $C_0(x_0{}^0, x_0{}^1, x_0{}^2, x_0{}^3)$ we have

$$\varpi_0 dx^0 + \varpi_1 dx^1 + \varpi_2 dx^2 + \varpi_3 dx^3 = \frac{dx^0}{kY(x_0{}^0, x_0{}^1, x_0{}^2, x_0{}^3)}. \quad . \quad (4.3)$$

Now the spatial distance Δ is to be given by

$$\Delta = \int_{C_1}^{C_0} d\lambda,$$

or, from (4.2),

$$\Delta = \int_{C_1}^{C_0} \frac{dx^0}{kY(x^0, x^1, x^2, x^3)}, \quad . \quad . \quad (4.4)$$

or, from (4 3),

$$\Delta = Y(x_0{}^0, x_0{}^1, x_0{}^2, x_0{}^3)\left[\varpi_0 + \varpi_1\left(\frac{dx^1}{dx^0}\right)_0 + \varpi_2\left(\frac{dx^2}{dx^0}\right)_0 + \varpi_3\left(\frac{dx^3}{dx^0}\right)_0\right]\int_{C_1}^{C_0} \frac{dx^0}{Y(x^0, x^1, x^2, x^3)}, \quad (4.5$$

where

$$\left(\frac{dx^1}{dx^0}\right)_0, \quad \left(\frac{dx^2}{dx^0}\right)_0, \quad \left(\frac{dx^3}{dx^0}\right)_0$$

relate to the direction of the null geodesic Γ at C_0. *This is a new general formula for spatial distance in any kind of curved space.*[*]

5. *Spatial Distance in the de Sitter World.*—The de Sitter world is defined by the metric

$$\frac{ds^2}{R^2} = \frac{du^2 - dx^2 - dy^2 - dz^2}{1 + x^2 + y^2 + z^2 - u^2} + \frac{(udu - xdx - ydy - zdz)^2}{(1 + x^2 + y^2 + z^2 - u^2)^2}. \quad . \quad (5.1)$$

It can be shown that the transport vector η^p along a null geodesic in this space-time manifold is, apart from a constant factor,

$$\eta^0 = X, \quad \eta^1 = X\frac{dx}{du}, \quad \eta^2 = X\frac{dy}{du}, \quad \eta^3 = X\frac{dz}{du} \quad . \quad . \quad (5.2)$$

---

[*] In a forthcoming paper it is proved by H. S. Ruse that this definition of spatial distance is precisely equivalent to that previously given by him, *Proc. Roy. Soc. Edin.*, vol. lii, 1932, pp. 183–194.

where $X = 1 + x^2 + y^2 + z^2 - u^2$. Hence from (4.2)

$$k\mathrm{Y}(u, x, y, z) = \frac{du}{d\lambda} = \eta^0 = 1 + x^2 + y^2 + z^2 - u^2 \quad . \quad . \quad (5.3)$$

Therefore (4.5) gives

$$\Delta = (1 + x_0^2 + y_0^2 + z_0^2 - u_0^2)$$

$$\left[\varpi_0 + \varpi_1\left(\frac{dx}{du}\right)_0 + \varpi_2\left(\frac{dy}{du}\right)_0 + \varpi_3\left(\frac{dz}{du}\right)_0\right]\int_{C_1}^{C_0}\frac{du}{1 + x^2 + y^2 + z^2 - u^2} \ . \quad (5.4)$$

and the integration is taken along the null geodesic

$$\frac{u - u_0}{u_0 - u_1} = \frac{x - x_0}{x_0 - x_1} = \frac{y - y_0}{y_0 - y_1} = \frac{z - z_0}{z_0 - z_1}. \quad . \quad (5.5)$$

Hence

$$\Delta = (1 + x_0^2 + y_0^2 + z_0^2 - u_0^2)$$

$$[\varpi_0(u_0 - u_1) + \varpi_1(x_0 - x_1) + \varpi_2(y_0 - y_1) + \varpi_3(z_0 - z_1)](u_0 - u_1)\int_{C_1}^{C_0}\frac{du}{\mathrm{A}u^2 + 2\mathrm{H}u + \mathrm{B}} \quad (5.6)$$

where

$$\begin{aligned}
\mathrm{A} &= (x_0 - x_1)^2 + (y_0 - y_1)^2 + (z_0 - z_1)^2 - (u_0 - u_1)^2 \\
\mathrm{H} &= (x_0 - x_1)(x_1u_0 - x_0u_1) + (y_0 - y_1)(y_1u_0 - y_0u_1) + (z_0 - z_1)(z_1u_0 - z_0u_1) \\
\mathrm{B} &= (x_1u_0 - x_0u_1)^2 + (y_1u_0 - y_0u_1)^2 + (z_1u_0 - z_0u_1)^2 + (u_0 - u_1)^2,
\end{aligned}$$

and, since C_0C_1 is a null geodesic, $\mathrm{AB} = \mathrm{H}^2$.

Performing the integration in (5 6) we have, after some reduction,

$$\Delta = \frac{\sqrt{1 + x_0^2 + y_0^2 + z_0^2 - u_0^2}}{\sqrt{1 + x_1^2 + y_1^2 + z_1^2 - u_1^2}}\{\varpi_0(u_0 - u_1) + \varpi_1(x_0 - x_1) + \varpi_2(y_0 - y_1) + \varpi_3(z_0 - z_1)\} \quad (5.7)$$

This is the formula for spatial distance in the de Sitter world according to the foregoing definition.

In the case previously treated by Whittaker (*loc cit.*, p. 101) B's world-line was restricted to be parallel to the axis of u, so that at C_0

$$\varpi_p = \frac{g_{p0}}{\sqrt{g_{00}}}.$$

Writing

$$\begin{aligned}
\xi_0 &= 1 + x_0^2 + y_0^2 + z_0^2 \\
\xi_1 &= 1 + x_1^2 + y_1^2 + z_1^2 \\
\theta &= 1 + x_0x_1 + y_0y_1 + z_0z_1
\end{aligned}$$

we find in this case

$$\Delta = \mathrm{R}\,.\,\frac{\sqrt{\xi_0\xi_1 - \theta^2}}{\sqrt{\xi_0}\sqrt{\xi_1 - u_1^2}}. \quad . \quad . \quad . \quad (5.8)$$

The formula found from Whittaker's previous definition of Δ was

$$\Delta = \mathrm{R}\,.\,\frac{\sqrt{\xi_0\xi_1 - \theta^2}}{\sqrt{\xi_1}\sqrt{\xi_0 - u_0^2}}. \quad . \quad . \quad . \quad . \quad (5.9)$$

The formula (5.8) can be shown by means of a suitable transformation to be equivalent to that given by an application of Ruse's definition to the de Sitter world.*

III. PROPAGATION OF LIGHT IN GENERAL RELATIVITY.

6. *Wave-fronts.*—We shall now apply the results of Section I to the theory of the propagation of light in general relativity. The V_n is again the space-time manifold. In Theorem II the S_{n-2} now corresponds to a wave-front which is, in the local space of three dimensions, determined by the motion of the local observer. The history of the wave-front will be represented by the aggregate of null geodesics passing through its points, and such that their projections in the local space are normal to the wave-front. This will be true if the set of geodesics is determined in the manner indicated above. Theorem II shows that if this condition is satisfied at one point, then it is satisfied at all points along the geodesics. This corresponds, therefore, to the fact that the history of a wave-front predicted by means of the null geodesics through its points is the same as that obtained by supposing the wave-front always to move forward at right angles to itself, with a velocity equal to the velocity of light as given by the geodesics. Two successive waves of light will be represented by two wave-fronts in a local space, or by two parallel sets of geodesics in the four-dimensional manifold. The actual wave-length determined by an observer will clearly be the perpendicular distance between the wave-fronts, corresponding to these geodesics, in his local instantaneous space. Alternatively, it is the interval between the points of intersection of the two S_{n-1}'s, given by the two sets of parallel geodesics, with the observer's world-line.

7. *The Doppler Effect in General Relativity.*—Consider two observers A, B at points C_1, C_0 respectively on a null geodesic Γ. Let η^ν be a transport vector along Γ, and let ${}_0\eta^\nu$, ${}_1\eta^\nu$ be its values at C_0, C_1 respectively. We shall regard Γ as belonging to some wave-front observed by A and B, and shall consider the set of neighbouring null geodesics which determine the succeeding wave-front. We have seen that these all correspond to the same value of the invariant J along Γ. Let the V_{n-1} determined by these geodesics cut A's world-line at a point C_1', and B's world-line at a point C_0'. Then, from the above discussion the interval C_1C_1' gives the wave-length λ_1 as observed by A, and C_0C_0' gives the wave-length λ_0 as observed by B. If ρ_ν, ϖ_ν are unit covariant vectors along the world-lines of A, B respectively, then the covariant components of the vectors C_1C_1', C_0C_0' are

$$(\lambda_1\rho_0, \lambda_1\rho_1, \lambda_1\rho_2, \lambda_1\rho_3) \quad \text{and} \quad (\lambda_0\varpi_0, \lambda_0\varpi_1, \lambda_0\varpi_2, \lambda_0\varpi_3).$$

---

* H. S. Ruse, *loc. cit.*, § 5.

Equating the values of the invariant J for the points C_0, C_1 we now have therefore

$$\lambda_0 \sum_p {}_0\eta^p \varpi_p = \lambda_1 \sum_p {}_1\eta^p \rho_p$$

or

$$\frac{\lambda_0}{\lambda_1} = \frac{{}_1\eta^0\rho_0 + {}_1\eta^1\rho_1 + {}_1\eta^2\rho_2 + {}_1\eta^3\rho_3}{{}_0\eta^0\varpi_0 + {}_0\eta^1\varpi_1 + {}_0\eta^2\varpi_2 + {}_0\eta^3\varpi_3}. \quad . \quad . \quad (7{\cdot}1)$$

This is the formula for the Doppler effect in general relativity. It is evidently symmetrical between the two observers.

We can give a general definition of "relative rest" by saying that A *is at rest relative to* B *if the vector* ρ_p *is obtained by the parallel transport of the vector* ϖ_p *along* Γ *from* C_0 *to* C_1. This is evidently the simplest generalisation of the classical notion of two particles moving with equal velocities in parallel straight lines.

Now the vector η^p is also carried along Γ by parallel transport. Hence, if A is at rest relative to B, the quantity $\sum_p \eta^p \varpi_p$ is carried along Γ by parallel transport, and so, being a scalar, it is a constant, *i.e.*

$$\sum_p {}_0\eta^p \varpi_p = \sum_p {}_1\eta^p \rho_p$$

or, in (7·1),

$$\lambda_0 = \lambda_1$$

Thus when A *is at rest relative to* B, in this general sense, *then there is no Doppler effect between* A *and* B. This result was also found by Lanczos (*loc. cit.*, p. 177).

The converse is not true. It does not follow that if the Doppler effect vanishes, then the observers are instantaneously at rest relative to each other. An example of this is provided by the case of special relativity.

It does *not* appear to be true that when A is at rest relative to B in the present sense, then the spatial distance of A from B as defined in this paper has a stationary value.

7.1. Attention must be drawn to two points in connection with the derivation of equation (7.1).

First, we have to assume that the two sets of null goedesics corresponding to two successive wave-fronts, which are parallel at C_1, are close enough together to be also nearly parallel, to a sufficient approximation, at the point C_0. If this is not so, Theorem II is not applicable. But, further, it will not then be possible in any case to interpret the phenomenon in terms of wave-length. For waves at C_1 will then become so distorted before they reach C_0 that they cannot be recognised as waves at all

Second, it must be pointed out why it was necessary to consider the value of J determined by the doubly infinite set of null geodesics passing through the points of the wave-front. It might appear that one geodesic neighbouring to Γ would serve to determine by its distance from Γ the wave-length of the corresponding light. But the fact is that if for one observer two light-quanta appear to be on the same "ray," so that in his neighbourhood they lie in a plane through his time axis, then they will not appear to be on the same "ray" for a different observer. So, though they may serve to determine a wave-length for the first observer, they cannot be so used by the second observer.

In purely geometrical optics in relativity it is sufficient to think of rays as determined by individual null geodesics by their projections on the instantaneous space of the observer concerned. For anything of the nature of physical optics, on the other hand, when we wish to think of simultaneous events *along* a ray in order to determine, for example, the wave-length of the light, then the ray is constituted by the intersection of a sheet of null geodesics with the instantaneous space. Such a "ray" will not transform into a ray for a second observer moving in a different manner. We must therefore bring in the idea of *wave-fronts*. We have shown how to find the wave-front for any observer once it has been determined for a given observer The corresponding "ray" for any observer is then a suitably drawn normal to the wave-front in his instantaneous space.

8. *The Doppler Effect in Special Relativity.*—We shall now deduce the formula for the Doppler effect in special relativity. We therefore take

$$ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2, \qquad . \qquad . \qquad (8.1)$$

so that

$$(\eta^0)^2 = \frac{1}{c^2}\{(\eta^1)^2 + (\eta^2)^2 + (\eta^3)^2\}$$

or

$$\eta^1 = cl\eta^0, \qquad \eta^2 = cm\eta^0, \qquad \eta^3 = cn\eta^0 \quad . \quad . \quad . \quad (8.2)$$

where η_0, l, m, n are absolute constants, and $l^2+m^2+n^2=1$. Now suppose the system of reference has been chosen so that the observer B is at rest, so that

$$\varpi_0 = 1, \qquad \varpi_1 = \varpi_2 = \varpi_3 = 0. \qquad . \quad . \quad . \quad . \quad (8.3)$$

Further, let A be on a "star" whose velocity is w, so that

$$(\rho_0, \rho_1, \rho_2, \rho_3) = \left(\frac{1}{\sqrt{1-\frac{w^2}{c^2}}}, \quad \frac{-\dot{x}}{c^2\sqrt{1-\frac{w^2}{c^2}}}, \quad \frac{-\dot{y}}{c^2\sqrt{1-\frac{w^2}{c^2}}}, \quad \frac{-\dot{z}}{c^2\sqrt{1-\frac{w^2}{c^2}}}\right) \qquad (8.4)$$

where $(\dot{x}, \dot{y}, \dot{z})$ are the three components of w.

Substituting then in (7.1) we find

$$\frac{\lambda_0}{\lambda_1}=\frac{1-\dfrac{1}{c}(l\dot{x}+m\dot{y}+n\dot{z})}{\sqrt{1-\dfrac{w^2}{c^2}}}$$

$$=\frac{1-\dfrac{w_r}{c}}{\sqrt{1-\dfrac{w^2}{c^2}}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (8.5)$$

where w_r is the radial component of w towards the observer. *This is the formula for the Doppler effect in special relativity, when the observer is assumed to be at rest.*

9. *The Doppler Effect in the de Sitter World.*—We shall now find the Doppler effect in the de Sitter world given by the metric (5.1). The transport vector η^p along a null geodesic is given by (5.2). Taking this geodesic Γ in the form

$$x=au+a', \qquad y=bu+b', \qquad z=cu+c' \quad . \quad . \quad (9.1)$$

it can be shown that the covariant form of η^p is given by

$$\eta_p=\frac{\gamma_p}{\sqrt{1+x^2+y^2+z^2-u^2}} \quad . \quad . \quad . \quad (9.2)$$

where

$$\begin{aligned}
\gamma_0&=R^2\sqrt{1+a'^2+b'^2+c'^2}\\
\gamma_1&=a'R^2\sqrt{a^2+b^2+c^2-1}-aR^2\sqrt{1+a'^2+b'^2+c'^2}\\
\gamma_2&=b'R^2\sqrt{a^2+b^2+c^2-1}-bR^2\sqrt{1+a'^2+b'^2+c'^2}\\
\gamma_3&=c'R^2\sqrt{a^2+b^2+c^2-1}-cR^2\sqrt{1+a'^2+b'^2+c'^2},
\end{aligned}$$

so that the γ_p depend only on the direction of Γ and not on the particular point $P(u, x, y, z)$ on it at which η_p is evaluated.

Thus the general formula (7.1) for the Doppler effect becomes, in the case of the de Sitter world,

$$\frac{\lambda_0}{\lambda_1}=\frac{\sqrt{1+x_0{}^2+y_0{}^2+z_0{}^2-u_0{}^2}}{\sqrt{1+x_1{}^2+y_1{}^2+z_1{}^2-u_1{}^2}}\,\frac{\rho^0\gamma_0+\rho^1\gamma_1+\rho^2\gamma_2+\rho^3\gamma_3}{\varpi^0\gamma_0+\varpi^1\gamma_1+\varpi^2\gamma_2+\varpi^3\gamma_3}, \quad . \quad (9.3)$$

where ρ^p is the unit contravariant vector in the direction of the motion of the observer A at $C_1(u_1, x_1, y_1, z_1)$, and ϖ^p is the unit contravariant vector in the direction of motion of the observer B at $C_0(u_0, x_0, y_0, z_0)$, and γ_p is a covariant vector in the direction of the null geodesic C_1C_0.

This formula is valid whatever be the position and motion of the two observers, and so is more general than the formula given in Whittaker's previous paper (*loc. cit.*, p. 104, equation (27)). In order to derive this last-mentioned formula, we suppose that the observer's and star's world-lines are both parallel to the axis of u, so that

$$\rho^1 = \rho^2 = \rho^3 = 0,$$

and therefore

$$\rho^0 = \mathrm{R}\,.\,\frac{1 + x_1^2 + y_1^2 + z_1^2 - u_1^2}{\sqrt{1 + x_1^2 + y_1^2 + z_1^2}},$$

and similarly

$$\varpi^1 = \varpi^2 = \varpi^3 = 0, \qquad \varpi^0 = \mathrm{R}\frac{1 + x_0^2 + y_0^2 + z_0^2 - u_0^2}{\sqrt{1 + x_0^2 + y_0^2 + z_0^2}}.$$

Thus the formula (9.3) becomes in this case

$$\frac{\lambda_0}{\lambda_1} = \frac{\sqrt{1 + x_1^2 + y_1^2 + z_1^2 - u_1^2}}{\sqrt{1 + x_0^2 + y_0^2 + z_0^2 - u_0^2}}\;\frac{\sqrt{1 + x_0^2 + y_0^2 + z_0^2}}{\sqrt{1 + x_1^2 + y_1^2 + z_1^2}}, \qquad (9.4)$$

which is at once seen to be equivalent to the formula of the previous paper, namely,

$$\frac{\lambda_0}{\lambda_1} = \frac{\cos\sigma}{\cos(\sigma + \rho)},$$

since we have

$$\cos\sigma = \frac{\sqrt{1 + x_1^2 + y_1^2 + z_1^2 - u_1^2}}{\sqrt{1 + x_1^2 + y_1^2 + z_1^2}}, \qquad \cos(\sigma + \rho) = \frac{\sqrt{1 + x_0^2 + y_0^2 + z_0^2 - u_0^2}}{\sqrt{1 + x_0^2 + y_0^2 + z_0^2}}.$$

IV. ENERGY TRANSPORT IN GENERAL RELATIVITY.

10. *The Energy of a Material Particle and of a Photon.*—The most important property of energy and momentum is that they are *conserved*. Now in general relativity it turns out that a certain tensor, T^{pq}, has the property that its divergence vanishes, *i.e* $(\mathrm{T}^{pq})_q = 0$. This may be interpreted as the condition that it should represent some physical magnitude which is *conserved*, and it is, in fact, well known that this tensor represents the momentum and energy-density at any point in the universe. If now we consider a single isolated particle represented by a narrow tube in this universe, it may be shown * that the above result leads to the equation

$$\frac{d}{ds}\left(m_0\frac{dx^p}{ds}\right) = -\sum_{qr} m_0 \begin{Bmatrix} qr \\ p \end{Bmatrix} \frac{dx^q}{ds}\cdot\frac{dx^r}{ds} \qquad (10.1)$$

where m_0 is a constant, and s is the interval measured along the narrow tube. Hence the motion of a single isolated particle is necessarily a geodesic. But, further, equation (10 1) shows that the vector $m_0\frac{dx^p}{ds}$ is carried along the world-line of the particle by parallel transport, so that this vector is physically *conserved* during the history of the particle. This vector then appropriately represents the energy and momentum of the particle. It may readily be shown that in a local Euclidean space the components of this vector do, in fact, represent the three components of momentum, once the fourth component along the time-axis represents the energy relative to an observer at rest in this space-time framework. Thus the energy as measured by any observer is the product of this momentum-energy vector, with the unit vector directed along his world-line.

---

* Eddington, *Mathematical Theory of Relativity*, 1924 (2nd ed.), § 56.

It is convenient in passing to distinguish, and appreciate the relationship between, the three senses in which the general term "energy" may be employed. First, the energy density in space is given by a tensor of rank two; second, the energy of a single system is given by a tensor of rank one (a vector); while, third, the energy actually measured by any observer is a scalar found in the way indicated.

Now in the present paper we are dealing with light pulses whose world-lines are null geodesics, so that the energy of a pulse is appropriately represented by a vector just as the energy of a particle is represented. The vector will, however, in this case be a null vector. The principle that it must be *conserved* along the null geodesic leads to its identification as a transport vector along this geodesic.

It may readily be verified that if at any point E represents the energy of a light pulse passing that point, then the components of the covariant vector $(E, lE/c, mE/c, nE/c)$ give the energy, together with the three components of momenta in the directions of the three spatial axes in the local space of the point; and it is a null vector in this space.

We are now in a position to show that Theorem I has an interesting physical interpretation.

11. *Comparison of Relativity Theory and Quantum Theory.*—Suppose again that we have two observers, A, B, situated at different points, C_0, C_1, on a null geodesic. A light-quantum is emitted at C_0 and observed at C_1. Suppose that in A's local co-ordinates this light-quantum has energy E. There are two distinct methods of calculating the wave-length as observed by B. According to the Quantum Theory, the frequency as observed by A will be E/h, and the wave-length therefore hc/E. This, then, determines the distance between two wave-fronts in A's space. The corresponding families of geodesics will then determine, in the manner described above, the wave-fronts in B's space. This space is fixed by a knowledge of B's world-line, and hence the wave-length λ as observed by him can be derived. But, alternatively, we may have regard only to the *energy* emitted at C_0 and transferred to C_1, where it is observed by B in his space in accordance with the quantum conditions. In this way

he will ascribe to the quantum a wave-length λ', say. Clearly, if the theories of relativity and quantum mechanics are to be consistent, we must have $\lambda = \lambda'$.

Let us now form the invariant J at the point C_1. The point C_1' will be taken in the local space of B, so that it lies in the second wave-front and so that C_1C_1' is perpendicular to the wave-front in this space. Then C_1C_1' is the wave-length determined by the geodesics passing through the wave-front, which has been determined initially at the point C_0 as in the first method. Thus $C_1C_1' = \lambda_1$.

Let (l_1, m_1, n_1) be the direction cosines of C_1C_1' in B's local space. Then, by what has been proved above, (l_1, m_1, n_1) are the direction cosines of the projection in this space of the geodesic, along which the light pulse is travelling. Thus the energy-momentum vector is $(E_1, l_1E_1/c, m_1E_1/c, n_1E_1/c)$, where E_1 is the energy as observed by B. Whence

$$\begin{aligned} J_1 &= \lambda_1 l_1^2 E_1/c + \lambda_1 m_1^2 E_1/c + \lambda_1 n_1^2 E_1/c \\ &= \lambda_1 E_1/c = \lambda_1 p_1 \qquad \text{(11.1)} \end{aligned}$$

where J_1, λ_1, p_1 are the values of J, λ, and the momentum p, as measured by B. Since J is invariant and its value J_0 at C_0 has been chosen to agree with $\lambda p = h$ at that point, then

$$J_0 = J_1 = h.$$

But by the second method the wave-length λ' is determined at C_1 by the equation

$$\lambda_1' p_1 = h \qquad \text{(11.2)}$$

where now h is the value of Planck's constant for B, which, however, we shall assume to be the same as for A.

Hence it follows that

$$\lambda_1 = \lambda_1'. \qquad \text{(11.3)}$$

This proves that the wave-length determined by these two methods is the same, provided the energy-momentum vector is carried by parallel transport; or, alternatively, that a necessary condition for the congruity of the results is that the energy-momentum vector must be carried as a transport vector.

This result means that *the Doppler effect in general relativity as calculated from the wave theory of light agrees always with the Doppler effect as calculated from the energy of the associated light-quanta.*

It may be instructive to give one or two examples of this result which can be treated independently of the general theory of this paper.

12. *The Statical Universe.*—Suppose that space-time is statical so that its metric can be specified by an equation,

$$ds^2 = g_{00}dt^2 - dl^2 \quad \text{where} \quad dl^2 = \sum_{p,q=1}^{3} g_{pq}dx^p dx^q \quad . \quad . \quad (12.1)$$

where the g's are functions of the spatial co-ordinates (x^1, x^2, x^3) alone.

Consider a set of observers who are "at rest" in this statical world, so that their world-lines are

$$x^1 = \text{const.}, \qquad x^2 = \text{const.}, \qquad x^3 = \text{const.},$$

and therefore the unit vector we have denoted by ϖ^r is given by

$$\varpi^r = \left(\frac{dt}{ds}, 0, 0, 0\right) = \left(\frac{1}{\sqrt{g_{00}}}, 0, 0, 0\right),$$

or

$$\varpi_r = (\sqrt{g_{00}}, 0, 0, 0). \quad . \quad . \quad (12.2)$$

Let now η^r be a transport vector along the geodesic which gives the path of either a material particle or a photon. Then, in accordance with the identification of this vector with the energy-momentum vector, *the energy* E *of the particle or photon as measured by one of these observers* is proportional to the projection of η^p on his world-line, *i.e.*

$$\begin{aligned} \mathrm{E} &= \text{const.} \sum_r \eta^r \varpi_r = \text{const.} \sqrt{g_{00}}\eta^0 \\ &= \text{const.} \sqrt{g_{00}}\frac{dt}{ds}\left(\text{or } \frac{dt}{d\lambda}\right) \quad . \quad . \quad (12.3) \end{aligned}$$

where $\frac{dt}{ds}\left(\text{or } \frac{dt}{d\lambda}\right)$ is taken along the geodesic (or null geodesic). But in the metric (12.1) the equations to the geodesics possess the integral

$$g_{00}\frac{dt}{ds} = \text{const.} \quad \text{or} \quad g_{00}\frac{dt}{d\lambda} = \text{const.} \quad . \quad . \quad . \quad . \quad (12.4)$$

Hence from (12.3), (12.4) we have

$$\mathrm{E} = \frac{\text{const.}}{\sqrt{g_{00}}} \quad . \quad . \quad . \quad (12.5)$$

Now, to treat the question from the standpoint of the wave theory, we notice that for an atom at rest at a point P_1, if $d\tau_1$ is the element of proper time as measured by an observer at rest at P_1, we have

$$d\tau_1^2 = (g_{00})_1 dt^2, \quad . \quad . \quad . \quad . \quad (12.61)$$

while for an observer at rest at a point P_2

$$d\tau_2^2 = (g_{00})_2 dt^2. \quad . \quad . \quad . \quad . \quad . \quad (12.62)$$

From the integral (12.4) it follows that the system admits the transformation which changes t into $t+\text{const.}$ Therefore, if an atom is at rest at P_1 and an observer at rest at P_2, so that their co-ordinates (x^1, x^2, x^3)

do not vary, then if two consecutive signals are emitted by the atom at values of t differing by ϵ (say), these will arrive at the observer at values of t which also differ by ϵ. So the period of the radiation as measured at its place of emission is to its period as measured by the observer in the ratio which $d\tau/dt$ at P_1 bears to $d\tau/dt$ at P_2, *i.e.*

$$\frac{\nu\ (\text{atom})}{\nu\ (\text{observer})}=\frac{\text{period (observer)}}{\text{period (atom)}}=\frac{(\sqrt{g_{00}})_2}{(\sqrt{g_{00}})_1}. \qquad (12.7)$$

It then follows from a comparison of (12 5), (12.7) that *the frequency of radiation as measured by any observer is proportional to the energy of the associated corpuscles* (particles or photons) *as measured by him.*

13. *Expanding Universe.*—We may follow the recent work of Heckmann * and consider an expanding universe for which the metric is

$$ds^2=dt^2-R^2dl^2, \qquad dl^2=\sum_{p,\,q=1}^{3} g_{pq}dx^p dx^q \qquad (13.1)$$

where the g_{pq} are functions of (x^1, x^2, x^3) alone, and R is a function of t alone. If the universe is homogeneous, and if there is no exchange of energy between matter and radiation, then Heckmann shows that the energy-density of radiation ρ_s is given by

$$\rho_s=B\,.\,R^{-4} \quad (\text{B a positive constant}). \qquad (13.2)$$

Thus the energy in the volume-element corresponding to the intervals x^1 to x^1+dx^1, x^2 to x^2+dx^2, x^3 to x^3+dx^3 is

$$\rho_s\ R^3\sqrt{\gamma}dx^1dx^2dx^3=B\,.\,R^{-1}\sqrt{\gamma}dx^1dx^2dx^3 \qquad (13.3)$$

where $\gamma=|g_{pq}|$, (p, $q=1$, 2, 3).

Now, if the number of light-quanta in the universe remains the same, then the number associated with the volume-element must remain constant and so be given by, say,

$$N\sqrt{\gamma}dx^1dx^2dx^3\,. \qquad (13.4)$$

where N is constant. Hence if ϵ is the mean energy of a quantum we have

$$N\epsilon=B\,.\,R^{-1},$$

or, if ν is the frequency, supposed the same for all quanta so that $\epsilon=h\nu$, then

$$\nu\propto R^{-1} \qquad (13.5)$$

provided Planck's constant h does not change.

Now the path of any quantum is given by a null geodesic for which

$$0=dt^2-R^2dl^2,$$

---

* O. Heckmann, *Göttingen Nachr.* (Math. Phys.), 1931, pp. 126–130.

or

$$dl = \frac{dt}{\mathrm{R}}.$$

Hence the distance along the track of the quantum between two points, P_1, P_2, fixed in the (x^1, x^2, x^3) co-ordinate system is

$$l = \int_{\mathrm{P}_1}^{\mathrm{P}_2} \frac{dt}{\mathrm{R}}. \qquad (13.6)$$

If we vary this, keeping l fixed, we obtain

$$0 = \frac{\delta t_2}{\mathrm{R}(t_2)} - \frac{\delta t_1}{\mathrm{R}(t_1)}, \qquad (13.7)$$

so that if δt_1 is the proper time between the passage of successive wave-crests at time t_1 past an observer stationed at P_1, then δt_2 will be the proper time between the passage of the wave-crests at time t_2 past an observer stationed at P_2. Hence if ν_1, ν_2 are the frequencies observed by the observers at P_1, P_2 respectively, we have

$$\frac{\nu_1}{\nu_2} = \frac{\delta t_2}{\delta t_1} = \frac{\mathrm{R}(t_2)}{\mathrm{R}(t_1)}. \qquad (13.8)$$

Thus from the wave point of view we have

$$\nu \propto \mathrm{R}^{-1} \qquad (13.9)$$

where R has the value for the time t at which the observation is made. *Hence the two ways of measuring the frequency lead to the same result*, provided the number of quanta is conserved, and provided Planck's constant h does not vary. Alternatively, assuming the agreement of the two ways of arriving at the frequency, we have verified that the number of light-quanta is conserved in this type of expanding universe.

(Issued separately January 24, 1933.)

V.—Expansion and Contraction of Coal caused respectively by the Sorption and Discharge of Gas. By Professor Henry Briggs, D.Sc., Ph.D., and R. P. Sinha, B.Sc., Assoc. H.-W. Coll.

(MS. received October 28, 1932 Read December 5, 1932)

So far as we are aware, the only measurements that have been made of the linear extension of a colloidal medium due to gaseous sorption are those of Dr F. T. Meehan, published in 1927.* Meehan studied the response of charcoal to carbon dioxide at atmospheric pressure and different temperatures, and found it to expand almost equally along and across the grain when it took in the gas. No tests of this kind appear to have been carried out with other rigid gels or with other gases, and none with any gas under pressure.

As firedamp (mainly methane) occurs in coal seams under pressure of several hundred pounds per square inch, and as its rate of exudation into mine workings is a function of that pressure and of the permeability of the seam, it is important as a practical problem to ascertain whether the permeability is increased by the shrinkage of the coal as the gas is being emitted, and the experiments here described were undertaken to elucidate this neglected aspect of adsorption.

MATERIALS.

The experiments were conducted with different varieties of coal, and with both firedamp and carbon dioxide, over a range of pressure of from zero (atmospheric) to 300 lb. per square inch (gauge). The temperature was that of the laboratory; temperatures were noted, but, as their variation above and below a mean value of 18·6° C. was insufficient to affect the results, they are omitted from this record.

The firedamp was obtained, compressed in steel cylinders, from the Cymmer Colliery, South Wales; its composition is: methane, 96·4; ethane, nil; ethylene, 0·2; hydrogen, nil; carbon monoxide, 0·2; carbon dioxide, 0·9; nitrogen, 2·3 per cent.† The carbon dioxide was discharged from "J" Sparklet bulbs by means of a simple apparatus already described.‡

---

* *Proc. Roy. Soc.*, A, vol. cxv, p. 119.

† *Explosions in Mines Committee*, 6th Report, 1914, Cd. 7638, p. 4.

‡ Briggs, *Proc. Roy. Soc. Edin.*, vol. xliv, 1924, p. 202.

The coals were of the following characters:—

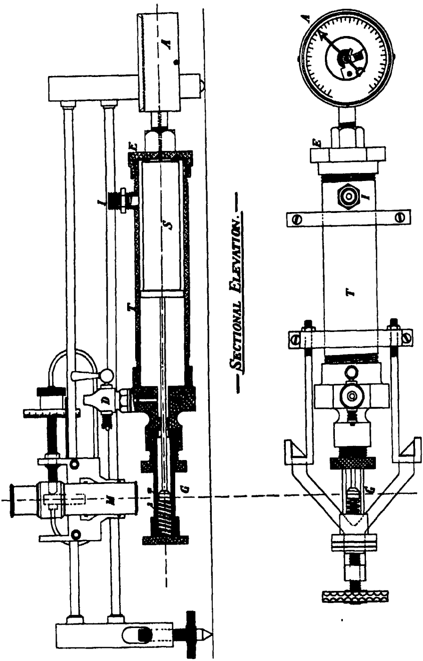
Description.	Source.	Volatile Matter per cent	Ash per cent.	Moisture (expelled at 105° C.) per cent.
Anthracite I .	Stanllyd Vein, Blaina Colliery, Carmarthenshire.	6·1	1·5	3·2
Anthracite II .	Polmaise Colliery, Stirlingshire.	6 5	1·5	3·5
Durain I .	Peacock Splint, Loanhead, Midlothian.	44·2	2·0	3·3
Durain II .	Great Seam, Easthouses Colliery, Midlothian.	34·2	4·5	7·3
Clarain I .	Humph Coal, Lanarkshire.	33·3	1·6	8·5
Clarain II .	Great Seam, Easthouses Colliery, Midlothian.	33·8	3·3	9·7
Clarain III .	Niddrie and Benhar Coal Co, Midlothian (seam uncertain)	42·5	1 0	7·4
Cannel . .	Kittlepurse Seam, Roslin Colliery, Midlothian.	29·2	41·5	3·4

The test specimens (average length, 77 mm.; average cross-section, 19·5 mm. square) were sawn from lumps that, with two exceptions, had stood in the laboratory for at least a year. The exceptions were the Easthouses coals, which were tested two to five days after leaving the pit.

In the table above, "clarain" is used as a convenient term indicating a finely-laminated intergrowth of bright and dull coal. The bright bands (vitrain) predominated, and "Clarain I" consisted almost altogether of vitrain. The clarains were intersected by cleavage planes (cleats); on the other hand, the specimens of anthracite, durain, and cannel were compact and (to the eye) homogeneous and free from cracks.

APPARATUS.

The apparatus used to measure the expansion and contraction of the specimens is shown in fig. 1. The barrel, *T*, is a stout brass tube fitted with a calibrated pressure gauge, *A*. Gas was admitted through the cock, *I*, while a second cock, *D*, allowed the pressure to be released and facilitated the preliminary operation of flushing the apparatus to expel air. The prism of coal, *S*, was maintained under longitudinal pressure by means of the spring, *s*. Elongation or shrinkage of the coal was observed by aid of a microscope, *M*, sighted through a short piece of boiler-gauge tube, *G*, on a very fine mark at *m*. Suitable illumination



—SECTIONAL ELEVATION.—

—PLAN—

FIG. 1.—Apparatus used for the Measurement of Expansion and Contraction of Coal.

enabled the mark to be seen as a sharp bright line crossing a dull yellow background. Movements were measured by the micrometer.

Blank experiments with dummy (metal) specimens showed that there was no correction to be applied to the readings in consequence of the mechanical stress imposed on the apparatus itself by raising the internal pressure to 300 lb. per square inch.

The average uncertainty of reading affecting a single measurement (expansion or contraction) was ascertained to be ±0·0021 mm.

After taking an initial reading, the usual procedure was to admit gas to a pressure of 300 lb., to maintain that pressure until expansion had apparently ceased, and to record the progress of the linear elongation by means of numerous readings. Then the pressure was released and the ensuing contraction ascertained in like manner.

RESULTS.

The experiments showed that coals expand slightly when they take up gas and that the process is approximately reversible.

The chart, fig. 2, depicts the relation between time and the percentage elongation, gas at 300 lb. pressure being admitted in each instance at zero on the time scale. Treated in this way with carbon dioxide, a specimen of Anthracite I (for example), cut with the long axis of the prism parallel with the bedding, began to expand after an interval of nearly 4 hours. The rate of expansion appreciably diminished after 46 hours and virtually ceased after 93 hours. The maximum recorded extension was 0·577 per cent. The gas was released and the pressure returned to atmospheric after 117·5 hours had expired. A rapid shrinkage occurring during the next 4 hours or so was followed by contraction at a reduced rate. The experiment was brought to an end 190 hours after its commencement, the coal showing no further sign of shrinkage, though the specimen was, at that time, 0·14 per cent. greater than its original length.

Unlike a specimen elongated by tension applied externally, coal expands in all directions when it adsorbs gas. Judging from tests on Clarain I, the amount of expansion varies in different directions, being greater across than along the bedding. To ascertain to what degree simple elastic compression affected the results, a durain prism was coated all over to retard the access of gas to the coal and then tested in the apparatus under 300 lb. pressure. The length slightly diminished as pressure was applied, the movement being of the order of that due to the compression of an elastic solid. Later, after the gas had had time

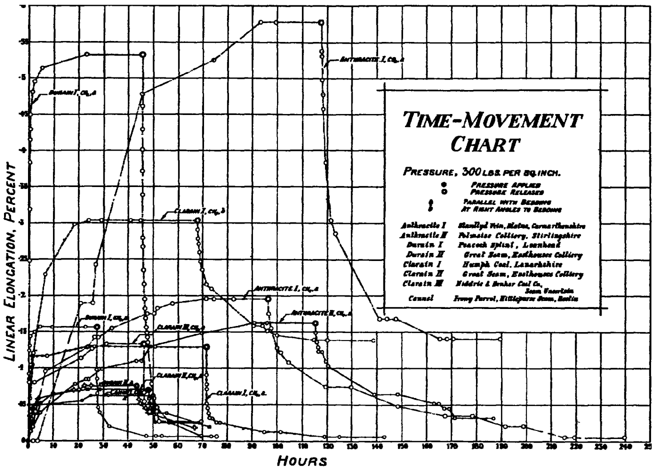

TIME-MOVEMENT CHART
PRESSURE, 300 LBS. PER SQ. INCH.
● PRESSURE APPLIED
○ PRESSURE RELEASED
PARALLEL WITH BEDDING
AT RIGHT ANGLES TO BEDDING
Anthracite I Stanllyd Vein, Blaina, Carmarthenshire
Anthracite II Polmaise Colliery, Stirlingshire
Durain I Peacock Splint, Loanhead
Durain II Great Seam, Easthouses Colliery
Clarain I Humph Coal, Lanarkshire
Clarain II Great Seam, Easthouses Colliery
Clarain III Niddrie & Benhar Coal Co., Seam Uncertain
Cannel Frosty Parrot, Killiejoun Seam, Bathie
ANTHRACITE I, CH. 2
DURAIN I, CH. 2
CLARAIN I, CH. 3
ANTHRACITE I, CH. 4
ANTHRACITE II, CH. 4
DURAIN I, CH. 2
CLARAIN III, CH. 2
CLARAIN II, CH. 2
DURAIN II
CANNEL
CLARAIN I, CH. 2
LINEAR ELONGATION, PERCENT
0
·05
·1
·15
·2
·25
·3
·35
·4
·45
·5
·55
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
210
220
230
240
250
HOURS

FIG. 2.

to penetrate the colloidal covering of the prism, the expansion due to sorption took place. The movements with which we are concerned are much larger in magnitude and of the opposite sign to those of simple elastic strain imposed by the gas pressure. Actually, the increase in linear dimension that was measured was the effect of sorption diminished by that of elastic compression.

The coals invariably reacted less strongly to methane than to carbon dioxide. For instance, the behaviour of Anthracite I on firedamp was similar to that on CO_2, but the maximum movement was only 0·195 per cent.

Special interest attaches to Durain I on account of the rapidity of its reaction, whether on CO_2 or on CH_4: almost the whole of the recorded movement took place within two hours of the gas being admitted or released.

The Easthouses coals, being freshly wrought, contained, no doubt, a considerable amount of adsorbed moisture, and for that reason were less responsive to firedamp.

Coal adsorbs moisture with greater avidity than firedamp, and if a coal that is charged with gas is placed in contact with water much of the gas is expelled and water taken up. A wet coal seam is never gassy. It is known that the amount of gas a coal can adsorb at any pressure stands in an inverse ratio to the amount of uncombined water the coal contains; it may therefore be inferred that the expansion consequent upon gas being taken in is similarly affected by the content of free moisture.

The expansion caused by a dry coal adsorbing moisture, either alone or with gas in different concentrations, has not been determined. The problem will be investigated in the more extended study we hope to make.

A coal, even when reduced to a fine powder, takes a long time to become saturated with gas, and the relatively large pieces used in these tests cannot have been fully charged in the few days they were subjected to pressure. Without ruling out the possibility of further slight movement had the treatment under pressure been continued, there is good reason to believe that expansion is one of the earliest consequences of gaseous sorption.

The cost of the apparatus and materials was defrayed by a grant from the Tait Fund for Mining Research, a fund administered by the Governors of the Heriot-Watt College, Edinburgh.

(*Issued separately January* 30, 1933.)

VI.—On the Graduation of Data by the Orthogonal Polynomials of Least Squares. By A. C. Aitken, M.A., D.Sc.

(MS. received October 31, 1932. Revised December 3, 1932. Read December 5, 1932.)

1. INTRODUCTORY.

THE problem of fitting a polynomial to data by Least Squares has engaged the attention of many writers. The methods of approach have been many and various. Continued fractions, determinants, the calculus of finite differences and sums, the method of moments, the linear combination of data, the use of the orthogonal polynomials of Legendre and Tchebychef, these and doubtless other instruments of analysis have been pressed into service. At the end of the present paper is given a selective bibliography, which we hope on a future occasion to complete and to supplement by adding brief indications of the standpoint and achievement of each investigator.

In the following pages we expound afresh the method of orthogonal polynomials, and supply a missing analogy by deriving new forms for these polynomials, in terms of the central and mean central factorials which occur, for example, in the Newton-Bessel and Newton-Stirling formulæ of interpolation by central and mean central differences. At the same time, the aim of the paper being entirely practical, we introduce new methods of arithmetical fitting. The first method, which is essentially a modification of R. A. Fisher's method, is non-symmetrical and involves the use of reduced descending factorial moments, easily obtained by successive summation, and the terminal values and differences of the Tchebychef polynomials used twice in a manner permitting of several checks. The second method is symmetrical and involves central and mean central factorial moments, and the central and mean central values and differences of the polynomials. Tables are exhibited of both central and mean central and terminal values and differences, in a form convenient for use, for values of n, the number of data, up to 25.

The advantages attending the use of orthogonal polynomials have been sufficiently emphasised by earlier writers. The chief advantage is that if, for example, a cubic polynomial has been fitted and it is desired to proceed to the trying of a quartic, the work does not have to be done all over *ab initio*, but requires only the finding of a new coefficient, which does not affect the earlier ones. In the method we adopt, it requires only the computation of an additional moment and the bringing into play of an additional column in the table which is used. Another advantage is that the summed squares of residuals can be computed in a simple manner before ever we proceed to the actual fitting, the residual variance being estimated according to the number of "degrees of freedom."

In the present communication we consider exclusively equidistant and unweighted and uncorrelated data. It is proposed to follow up the work by a further paper communicating systematised methods dealing with the excluded cases of non-equidistant, weighted, and correlated data.

2. USEFUL FORMULÆ. ORDINARY AND CENTRAL DIFFERENCES, SUMS AND FACTORIAL POLYNOMIALS.

In the problem of fitting polynomials to discontinuous data, the employment of powers of x almost invariably leads to difficulty and complication. On the other hand, the consistent use of the operations and polynomials of finite differences brings about a great simplification, not only in theory but in numerical application; and for this reason we have thought it worth while to include here for the convenience of the reader a brief list of the principal definitions and formulæ we shall have occasion to use. At the end of the paper * we also give references to passages in standard texts and memoirs bearing on these points.

(1) *Operations.*

(i) Increasing the variable: $\mathrm{E}u_x = u_{x+1}$.
(ii) Averaging adjacent entries: $\mu u_x = \frac{1}{2}(u_{x+\frac{1}{2}} + u_{x-\frac{1}{2}})$.
(iii) Advancing differences: $\Delta u_x = u_{x+1} - u_x$.
(iv) Central differences: $\delta u_x = u_{x+\frac{1}{2}} - u_{x-\frac{1}{2}}$.
(v) Symbolic relations: $\mathrm{E} = 1 + \Delta$. $\delta = \mathrm{E}^{\frac{1}{2}} - \mathrm{E}^{-\frac{1}{2}} = \Delta \mathrm{E}^{-\frac{1}{2}}$. $\mu = \frac{1}{2}(\mathrm{E}^{\frac{1}{2}} + \mathrm{E}^{-\frac{1}{2}})$.
(vi) Indefinite sums: $\sum u_x = \ldots + u_{x-3} + u_{x-2} + u_{x-1}$.

The lower limit here is arbitrary, but throughout the present paper it will be taken as $x=0$. The upper limit will usually refer to $x=n$, the final term being then u_{n-1}; but where it is necessary to distinguish we shall write explicitly $\sum\limits_{x}$ and $\sum\limits_{n}$.

(vii) Central sums: $\hat{\sum} u_x = \ldots + u_{x-\frac{5}{2}} + u_{x-\frac{3}{2}} + u_{x-\frac{1}{2}}$.
(viii) Mean central sums: $\mu \hat{\sum} u_x = \ldots + u_{x-2} + u_{x-1} + \frac{1}{2}u_x$.
(ix) Symbolic relations: $\sum = \Delta^{-1}$. $\hat{\sum} = \delta^{-1}$.

---

* List A.

(2) *Factorial Notations and Properties.*

(x) Descending factorials: $x^{(r)} = x(x-1)(x-2) \ldots (x-r+1)$.

(xi) Reduced descending factorials: $x_{(r)} = x^{(r)}/r!$.

(xii) Properties: $\Delta x^{(r)} = rx^{(r-1)}$. $\Delta x_{(r)} = x_{(r-1)}$. $\sum x^{(r)} = x^{(r+1)}/(r+1)$.

$$\sum x_{(r)} = x_{(r+1)}.$$

(xiii) Central* factorials: $x^{\{r\}} = (x + \tfrac{1}{2}\overline{r-1})^{(r)}$.

Examples: $x^{\{2s\}} = (x^2 - \tfrac{1}{4})(x^2 - \tfrac{9}{4}) \ldots (x^2 - \overline{s - \tfrac{1}{2}}^2)$.

$$x^{\{2s+1\}} = x(x^2-1)(x^2-4) \ldots (x^2 - s^2).$$

(xiv) Mean central factorials: $\mu x^{\{r\}} = x^{\{r-1\}}. x = x^{\{r-1\}+1}$.

Examples: $x^{\{2s\}+1} = x(x^2 - \tfrac{1}{4})(x^2 - \tfrac{9}{4}) \ldots (x^2 - \overline{s - \tfrac{1}{2}}^2)$.

$$x^{\{2s+1\}+1} = x^2(x^2-1)(x^2-4) \ldots (x^2 - s^2).$$

(xv) Reduced central factorials: $x_{\{r\}} = x^{\{r\}}/r!$, etc.

(xvi) Properties: $\delta x^{\{r\}} = rx^{\{r-1\}}$. $\delta x_{\{r\}} = x_{\{r-1\}}$. $\delta x^{\{r-1\}+1} = rx^{\{r-2\}+1}$.

$$\hat{\sum} x^{\{r\}} = x^{\{r+1\}}/(r+1). \quad \hat{\sum} x_{\{r\}} = x_{\{r+1\}}.$$

The polynomials that occur in the Newton-Stirling and the Newton-Bessel formulæ of interpolation are instances of central and mean central factorials alternately.

(3) *Operations on a Product.*

(xvii) Advancing difference: $\Delta(u_x v_x) = v_x \Delta u_x + u_{x+1} \Delta v_x$.

(xviii) Central difference: $\delta(u_x v_x) = \mu v_x \delta u_x + \mu u_x \delta v_x$.

(xix) Summation "by parts":

$$\sum_n (u_x v_x) = u_{n-1} \sum v_x - \Delta u_{n-2} \sum^2 v_x + \Delta u_{n-3} \sum^3 v_x - \ldots .$$

(xx) Alternative form: $\sum_n (u_x \Delta v_x) = (u_n v_n - u_0 v_0) - \sum_n (v_{x+1} \Delta u_x)$.

(4) *Identities in Factorial Polynomials.*

(xxi) $(x+m)^{(r)} = x^{(r)} + rx^{(r-1)}m + r_{(2)}x^{(r-2)}m^{(2)} + \ldots + m^{(r)}$.

(xxii) $(x+m)_{(r)} = x_{(r)} + x_{(r-1)}m + x_{(r-2)}m_{(2)} + \ldots + m_{(r)}$.

(xxiii) $(x-m)^{(r)} = x^{(r)} - rx^{(r-1)}m + r_{(2)}x^{(r-2)}(m+1)^{(2)} - \ldots + (-)^r(m+r-1)^{(r)}$.

(xxiv) $(x-m)_{(r)} = x_{(r)} - x_{(r-1)}m + x_{(r-2)}(m+1)_{(2)} - \ldots + (-)^r(m+r-1)_{(r)}$.

The above give various forms of Vandermonde's familiar algebraic identity, and may also be regarded as simple cases of the Gregory-Newton formula of interpolation. In central factorials there are less familiar identities of the following type, *e.g.*:—

---

* We would have used the notation $x^{[r]}$ for this, but Steffensen has adopted that notation for what he defines as a central factorial, but what we should prefer to regard as a mean central factorial. Reference A (5).

$$\text{(xxv)}\quad x^2-q^2=(x^2-\tfrac{1}{4})-(q^2-\tfrac{1}{4}).$$

$$\begin{aligned}\text{(xxvi)}\quad (x^2-\overline{q+\tfrac{1}{2}}^2)(x^2-\overline{q-\tfrac{1}{2}}^2)&=x^2(x^2-1)-2(q^2-\tfrac{1}{4})(x^2-1)+(q^2-\tfrac{1}{4})(q^2-\tfrac{9}{4})\\&=(x^2-\tfrac{1}{4})(x^2-\tfrac{9}{4})-2(q^2-1)(x^2-\tfrac{1}{4})+q^2(q^2-1).\end{aligned}$$

$$\begin{aligned}\text{(xxvii)}\quad &(x^2-\overline{q+1}^2)(x^2-q^2)(x^2-\overline{q-1}^2)\\&=x^2(x^2-1)(x^2-4)-3(q^2-1)x^2(x^2-1)+3q^2(q^2-1)(x^2-1)-q^2(q^2-1)(q^2-4)\\&=(x^2-\tfrac{1}{4})(x^2-\tfrac{9}{4})(x^2-\tfrac{25}{4})-3(q^2-\tfrac{9}{4})(x^2-\tfrac{1}{4})(x^2-\tfrac{9}{4})+3(q^2-\tfrac{1}{4})(q^2-\tfrac{9}{4})(x^2-\tfrac{9}{4})\\&\qquad-(q^2-\tfrac{1}{4})(q^2-\tfrac{9}{4})(q^2-\tfrac{25}{4}),\end{aligned}$$

and similarly for higher orders.

The expressions on either side here are symmetrical, or else alternating, in x^2 and q^2. The identities are special cases of Newton's divided difference formula of interpolation as applied to the quasi-central factorial on the left. For example, the last of the identities which we have just written might be rendered, in the customary notation for divided differences, as

$$f(q^2)=f(\tfrac{9}{4})+(q^2-\tfrac{9}{4})f(\tfrac{1}{4},\tfrac{9}{4})+(q^2-\tfrac{1}{4})(q^2-\tfrac{9}{4})f(\tfrac{1}{4},\tfrac{9}{4},\tfrac{25}{4})+\ .\ .\ .\ ,$$

where q^2 is regarded as the variable. That the numerical coefficients on the right of the identities must be binomial coefficients is seen at once by comparison with the limiting case $(x^2-q^2)^r$; indeed, the present identities bear to this limiting case exactly the relation which Vandermonde's identity for factorials bears to the ordinary binomial expansion.

We shall find that these expansions in central factorials enable a perfect analogy to be set up between the Tchebychef polynomials in central form and the Legendre polynomials.

3. THE ORTHOGONAL POLYNOMIALS OF TCHEBYCHEF.

With a view to the convenient use of indefinite summation the n equidistant data will be denoted by $u_0, u_1, \ldots, u_{n-1}$. The problem is to fit to these data a polynomial $U(x\,;\,n)$ of degree $k<n-1$ by the principle of Least Squares. Unless otherwise specified, the degree and the number of data will be understood to be k and n, and we shall write the polynomial briefly as $U(x)$, or U_x. If then

$$(1)\qquad U(x)=a_0+a_1T_1(x)+a_2T_2(x)+\ .\ .\ .\ +a_kT_k(x),$$

the basic polynomials $T_r(x\,;\,n)\equiv T_r(x)\equiv T_r$ being chosen later to satisfy suitable conditions, we have to minimise

$$(2)\qquad S^2=\sum\{u_x-a_0-a_1T_1(x)-\ .\ .\ .\ -a_kT_k(x)\}^2.$$

Great advantage will be secured if in this summation all product sums such as $\sum T_r T_s$ vanish for $r \neq s$. Hence, we impose on the polynomials T_r the orthogonal relations

$$(3) \qquad \sum T_r^2 \neq 0, \quad \sum T_r T_s = 0, \quad r > s,$$

or, what amounts to the same thing,

$$(4) \qquad \sum \phi_r(x) T_r(x) \neq 0, \quad \sum \phi_s(x) T_r(x) = 0, \quad r > s,$$

where $\phi_i(x)$ is an arbitrary polynomial of degree i. Then the orthogonal relations (3), together with the minimal conditions,

$$(5) \qquad \frac{\partial S^2}{\partial a_i} = 0, \qquad i = 0, 1, 2, \dots, k,$$

yield

$$(6) \qquad a_r \sum T_r^2 = \sum u_x T_r(x) = \sum u T_r,$$

from which the coefficients a_r in the graduating polynomial U_x may be found, as soon as we have determined the form and properties of the polynomials $T_r(x)$. At the same time, substitution of the relation (3) in (2) gives for the minimal value of S^2 the expression

$$(7) \qquad S^2 = \sum (u^2 - a_0^2 - a_1^2 T_1^2 - \dots - a_k^2 T_k^2),$$

or equally well, by (6),

$$(8) \qquad S^2 = \sum u^2 - a_0 \sum u - a_1 \sum u T_1 - \dots - a_k \sum u T_k,$$

a useful formula by means of which, side by side with the computation of each a_r from (6), we may also compute $a_r \sum u T_r$ and, by subtracting the results progressively from $\sum u^2$, evaluate S^2 as we go along.

We proceed to determine the polynomials $T_r(x)$ In (4) we shall choose $\phi_s(x)$ as $(x+1)(x+2) \dots (x+s)/s!$—that is, $(x+s)_{(s)}$. Then, applying the formula, § 2 (xix), for repeated summation by parts we obtain, using also § 2 (xii), the $r+1$ relations

$$(9) \qquad (n+s-1)_{(s)} \sum T_r - (n+s-2)_{(s-1)} \sum^2 T_r + \dots + (-)^s \sum^{s+1} T_r = 0, \quad s = 0, 1, \dots, r-1, \qquad \neq 0, \quad s = r.$$

Taking these in order, $s = 0, 1, \dots, r-1$, we see that we must have

$$(10) \qquad \sum T_r(x) = 0, \quad \sum^2 T_r(x) = 0, \dots, \quad \sum^r T_r(x) = 0, \quad r > 0,$$

and so, for $s = r$,

$$(11) \qquad \sum T_r^2(x) = (-)^r c \sum^{r+1} T_r(x).$$

Hence, if we denote for a moment $\sum_x^r T_r(x)$ by $G_{2r}(x)$, it follows from (10) that

$$T_r(x) = \Delta^r G_{2r}(x),$$

where $G_{2r}(x)$ is such that it itself and its first $r-1$ advancing differences vanish together at the lower limit $x=0$, and hence also at the upper, $x=n$. Hence, $G_{2r}(x)$ must itself vanish for $x=0, 1, 2, \ldots, r-1$, and also for $x=n, n+1, n+2, \ldots, n+r-1$. Thus we obtain the $2r$ linear factors of $G_{2r}(x)$, and, fixing the numerical coefficient by a convention to be explained in a moment, we have now

$$(12) \quad \ldots \quad T_r(x) = \Delta^r\{x_{(r)}(x-n)_{(r)}\},$$

a formula which is the analogue of Rodrigues' formula for the Legendre polynomials,

$$(13) \quad \ldots \quad P_r(x) = \left(\frac{d}{dx}\right)^r (x^2-1)^r/(r\,!)^2.$$

To obtain the explicit form of $T_r(x)$ we express $(x-n)_{(r)}$ in terms of $(x-r)_{(r)}$, $(x-r)_{(r-1)}$, and so on by means of § 2 (xxiv), substitute in (12) and perform Δ^r. The final result is

$$(14) \quad T_r(x) = (2r)_{(r)}x_{(r)} - (2r-1)_{(r)}(n-r)x_{(r-1)} + (2r-2)_{(r)}(n-r+1)_{(2)}x_{(r-2)} \\ - \ldots + (-)^r(n-1)_{(r)}.$$

From this formula the expression for any difference $\Delta^s T_r(x)$ can be written down at once by putting $x_{(i-s)}$ for each $x_{(i)}$. Again, since the formula (14) is simply a Gregory-Newton formula, the coefficients of $1, x, x_{(2)}, \ldots, x_{(s)}, \ldots$ are simply the *terminal* values and differences $T_r(0)$, $\Delta T_r(0)$, $\Delta^2 T_r(0)$, $\ldots$, $\Delta^s T_r(0)$, $\ldots$. This simple remark is of great computational value (§ 5, *infra*).

We may also note the useful fact that the numerical coefficients, apart from the factorial in n, of $x_{(s)}$ in (14) are the same as occur in the Legendre polynomial of degree r taken for the range $(0, n)$ of integration, namely,

$$P_r(x) = \left(\frac{d}{dx}\right)^r \{x(x-n)\}^r/(r\,!)^2.$$

There are various recurrence relations between the polynomials $T_r(x)$ and a difference equation of the second order resembling the differential equation satisfied by the Legendre polynomials, but these properties, which can be established by following the standard procedure for the Legendre polynomials, lie apart from our purpose.

In regard to the choice of numerical coefficient in (14), at least four conventions* already exist, but the one that we have adopted gives smaller values for $r > 3$ than any of them, values which, moreover, are integers if x is an integer, since $T_r(x)$ appears as an r^{th} difference of products of binomial coefficients.

---

* Adopted by Tchebychef, Fisher, Jordan, Chotimsky, and Allan. References B (1), (2), (6), (11), (18).

It is necessary next to derive an expression for $\sum T_r^2$, in order to compute a_r by means of (6). With the convention adopted, (11) gives

$$\sum T_r^2(x) = (-)^r \frac{(2r)!}{(r!)^2} \sum^{r+1} T_r(x)$$

$$= (-)^r \frac{(2r)!}{(r!)^2} \sum \{x_{(r)} \,.\, (x-n)_{(r)}\}. \tag{15}$$

Applying summation by parts to this, in the form § 2 (xx), and observing that at each step one or other of the factors in the partial integrate vanishes at $x=0$, or at $x=n$, we obtain after r steps,

$$\sum T_r^2(x) = (-)^{2r} \frac{(2r)!}{(r!)^2} \sum (x-n+r)_{(2r)}$$

$$= \frac{(2r)!}{(r!)^2} \Big[(x-n+r)_{(2r+1)} \Big]_0^n \qquad (r < n)$$

$$= n(n^2-1)(n^2-4) \ldots (n^2-r^2)/\{(2r+1) \,.\, (r!)^2\}, \tag{16}$$

which is the desired result.

4. TABLES OF THE TERMINAL VALUES AND DIFFERENCES OF $T_r(x)$, WITH VALUES OF $\sum T_r^2$.

The tables below, which are of great use in practice, were computed by special devices based on (14) and (16) of the preceding section. The values of $T_r(0)$, $\Delta T_r(0)$, $\Delta^2 T_r(0)$, . . . are entered in the rows of a triangular scheme, which will be understood from the illustrative example given below, the values of $\sum T_r^2$ being entered at the feet of the appropriate columns. It will be shown in the next section how such a triangular scheme of coefficients, for a given n, may be used *twice* upon the factorial moments of u_t, first by columns and then by rows, yielding with ease, at the first stage, the coefficients a_t of the orthogonal expansion U_m, from which the residual variance may be calculated; and at the second stage the terminal values U(0) and their differences, from which all the graduated values may be built up. The same table serves whatever the degree of the polynomial fitted.

SPECIMEN TABLE, $n=7$.

r	0	1	2	3	4	5
$T(0)$	1	−6	15	−20	15	−6
$\Delta T(0)$		2	−15	40	−50	30
$\Delta^2 T(0)$			6	−40	90	−84
$\Delta^3 T(0)$				20	−105	168
$\Delta^4 T(0)$					70	−252
$\Delta^5 T(0)$						252
$\sum T^2$	7	112	756	2400	3850	3024

A table such as the above, besides being a practical aid to computation, as will be seen in the next section, provides information such as the following:—

$$T_3(0) = -20, \quad \Delta^3 T_4(0) = -105, \text{ etc.}$$
$$T_3(x;7) = 20x_{(3)} - 40x_{(2)} + 40x - 20,$$
$$\Delta T_3(x;7) = 20x_{(2)} - 40x + 40, \text{ etc., etc.}$$

The values of $\sum T_r^2$ were computed by the writer in two ways, first in succession by unit increases in n, keeping r constant, then by unit increases in r, keeping n constant; finally, the last set of values, for $n=25$, was checked by § 3 (16). The triangular part of any table is easily constructed by the aid of tables of binomial coefficients. Thus for $n=7$ we juxtapose portions of two Pascal triangles of binomial coefficients,

$$\begin{array}{rrrrrr} 1 & -1 & 1 & -1 & 1 & -1 \\ & 2 & -3 & 4 & -5 & 6 \\ & & 6 & -10 & 15 & -21 \\ & & & 20 & -35 & 56 \\ & & & & 70 & -126 \\ & & & & & 252 \end{array} \quad \text{and} \quad \begin{array}{c} \\ n-1=6. \\ \begin{array}{rrrrrr} 1 & 6 & 15 & 20 & 15 & 6 \\ & 1 & 5 & 10 & 10 & 5 \\ & & 1 & 4 & 6 & 4 \\ & & & 1 & 3 & 3 \\ & & & & 1 & 2 \\ & & & & & 1, \end{array} \end{array}$$

the first being fixed, the second depending on n, and we multiply corresponding elements. The second triangle consists simply of the binomial coefficients of orders $n-1$, $n-2$, . . . , arranged by rows in the manner shown.

To save space in the tables below, we have omitted superscriptions, the specimen table above serving for elucidation. Also, to cope with large numbers, as $\sum T_r^2$ increases with n and r, we write, *e.g.*, $629028^{(9)}$ to indicate a certain number of *nine* digits (it is actually 629028288) cut down for our purpose to *six* significant digits.

$n=4$

1	−3	3
	2	−6
		6
4	20	36

$n=5$

1	−4	6	−4
	2	−9	12
		6	−20
			20
5	40	126	160

$n=6.$

1	−5	10	−10	5
	2	−12	24	−20
		6	−30	45
			20	−70
				70
6	70	336	720	700

$n=7.$

1	−6	15	−20	15	−6
	2	−15	40	−50	30
		6	−40	90	−84
			20	−105	168
				70	−252
					252
7	112	756	2400	3850	3024

$n=8.$

1	−7	21	−35	35	−21
	2	−18	60	−100	90
		6	−50	150	−210
			20	−140	336
				70	−378
					252
8	168	1512	6600	15400	19656

=9.

1	-8	28	-56	70	-56
	2	-21	84	-175	210
		6	-60	225	-420
			20	-175	560
				70	-504
					252
9	240	2772	15840	50050	91728

$n=10.$

1	-9	36	-84	126	-126
	2	-24	112	-280	420
		6	-70	315	-735
			20	-210	840
				70	-630
					252
10	330	4752	34320	140140	343980

$n=11.$

1	-10	45	-120	210	-252
	2	-27	144	-420	756
		6	-80	420	-1176
			20	-245	1176
				70	-756
					252
11	440	7722	68640	350350	$110074^{(17}$

$n=12.$

1	-11	55	-165	330	-462
	2	-30	180	-600	1260
		6	-90	540	-1764
			20	-280	1568
				70	-882
					252
12	572	12012	128700	800800	$311875^{(17}$

$n=13.$

1	-12	66	-220	495	-792
	2	-33	220	-825	1980
		6	-100	675	-2520
			20	-315	2016
				70	-1008
					252
13	728	18018	228800	$170170^{(17}$	$801965^{(17}$

$n=14.$

1	-13	78	-286	715	-1287
	2	-36	264	-1100	2970
		6	-110	825	-3465
			20	-350	2520
				70	-1134
					252
14	910	26208	388960	$340340^{(17}$	$190467^{(18}$

$n=15.$

1	-14	91	-364	1001	-2002
	2	-39	312	-1430	4290
		6	-120	990	-4620
			20	-385	3080
				70	-1260
					252
15	1120	37128	636480	$646646^{(17}$	$423259^{(18}$

$n=16.$

1	-15	105	-455	1365	-3003
	2	-42	364	-1820	6006
		6	-130	1170	-6006
			20	-420	3696
				70	-1386
					252
16	1360	51408	$100776^{(17}$	$117572^{(18}$	$888844^{(18}$

$n=17.$

1	-16	120	-560	1820	-4368
	2	-45	420	-2275	8190
		6	-140	1365	-7644
			20	-455	4368
				70	-1512
					252
17	1632	69768	$155040^{(17}$	$205751^{(18}$	$177769^{(19}$

$n=18.$

1	-17	136	-680	2380	-6188
	2	-48	480	-2800	10920
		6	-150	1575	-9555
			20	-490	5096
				70	-1638
					252
18	1938	93024	$232560^{(17}$	$348194^{(18}$	$340724^{(19}$

$n=19.$

1	−18	153	−816	3060	−8568
	2	−51	544	−3400	14280
		6	−160	1800	−11760
			20	−525	5880
				70	−1764
					252
19	2280	122094	341088$^{(17}$	572033$^{(18}$	629028$^{(19}$

$n=20.$

1	−19	171	−969	3876	−11628
	2	−54	612	−4080	18360
		6	−170	2040	−14280
			20	−560	6720
				70	−1890
					252
20	2660	158004	490314$^{(17}$	915253$^{(18}$	1123265$^{(110}$

$n=21.$

1	−20	190	−1140	4845	−15504
	2	−57	684	−4845	23256
		6	−180	2295	−17136
			20	−595	7616
				70	−2016
					252
21	3080	201894	692208$^{(17}$	143008$^{(18}$	194699$^{(110}$

$n=22.$

1	−21	210	−1330	5985	−20349
	2	−60	760	−5700	29070
		6	−190	2565	−20349
			20	−630	8568
				70	−2142
					252
22	3542	255024	961400$^{(17}$	2187185$^{(19}$	328555$^{(110}$

$n=23.$

1	−22	231	−1540	7315	−26334
	2	−63	840	−6650	35910
		6	−200	2850	−23940
			20	−665	9576
				70	−2268
					252
23	4048	318780	131560$^{(18}$	328078$^{(19}$	541149$^{(110}$

$n=24.$

1	−23	253	−1771	8855	−33649
	2	−66	924	−7700	43890
		6	−210	3150	−27930
			20	−700	10640
				70	−2394
					252
24	4600	394680	177606$^{(18}$	483483$^{(19}$	871852$^{(110}$

$n=25.$

1	−24	276	−2024	10626	−42504
	2	−69	1012	−8855	53130
		6	−220	3465	−32340
			20	−735	11760
				70	−2520
					252
25	5200	484380	236808$^{(18}$	701050$^{(19}$	137661$^{(111}$

EXACT VALUES OF $\sum T_r^2$ FOR $n=11, 12, \ldots, 25.$

n \ r	0.	1.	2.	3.	4.	5.
11	11	440	7722	68640	350350	1100736
12	12	572	12012	128700	800800	3118752
13	13	728	18018	228800	1701700	8019648
14	14	910	26208	388960	3403400	19046664
15	15	1120	37128	636480	6466460	42325920
16	16	1360	51408	1007760	11757200	88884432
17	17	1632	69768	1550400	20575100	177768864
18	18	1938	93024	2325600	34819400	340723656
19	19	2280	122094	3410880	57203300	629028288
20	20	2660	158004	4903140	91525280	1123264800
21	21	3080	201894	6922080	143008250	1946992320
22	22	3542	255024	9614000	218718500	3285549540
23	23	4048	318780	13156000	328077750	5411493360
24	24	4600	394680	17760600	483483000	8718517080
25	25	5200	484380	23680800	701050350	13766079600

$$\sum_{n} T_0^2 = n. \quad \sum_{n} T_r^2(x\,;\,n) = \frac{n+r}{n-r-1}\sum_{n-1} T_r^2(x\,;\,n-1) = \frac{(2r-1)(n^2-r^2)}{(2r+1)r^2}\sum_{n} T_{r-1}^2(x\,;\,n).$$

5. NUMERICAL PROCESS OF GRADUATION BY USE OF FACTORIAL MOMENTS WITH THE TRIANGLE OF COEFFICIENTS.

Let $m_{(r)}$ denote the r^{th} "reduced factorial moment" of u_x, that is,

$$(1) \quad . \quad . \quad . \quad . \quad . \quad . \quad m_{(r)} = \sum x_{(r)} u_x .$$

Substituting in § 2 (6) the values of the Tchebychef polynomials as given by § 3 (14), and summing, we derive at once

$$(2) \quad a_r \sum \mathrm{T}_r^2 = (2r)_{(r)} m_{(r)} - (2r-1)_{(r)}(n-r) m_{(r-1)} + (2r-2)_{(r)}(n-r+1)_{(2)} m_{(r-2)} \\ - \ . \ . \ . \ + (-)^r (n-1)_{(r)} m_{(0)} .$$

In particular, for the first few values of r,

$$(3) \quad \begin{aligned} n a_0 &= m_{(0)}, \\ a_1 \sum \mathrm{T}_1^2 &= -(n-1) m_{(0)} + 2 m_{(1)}, \\ a_2 \sum \mathrm{T}_2^2 &= (n-1)_{(2)} m_{(0)} - 3(n-2) m_{(1)} + 6 m_{(2)}, \\ a_3 \sum \mathrm{T}_3^2 &= -(n-1)_{(3)} m_{(0)} + 4(n-2)_{(2)} m_{(1)} - 10(n-3) m_{(2)} + 20 m_{(3)}. \end{aligned}$$

Now the set of coefficients in triangular array used here upon the factorial moments $m_{(i)}$ consists of the coefficients of $x_{(i)}$ in the Tchebychef polynomials; and these, by the remark we made in respect of § 3 (14), are also the *terminal values and differences* of the polynomials, which we must use upon the a_i, when found, in order to obtain U_0, $\Delta \mathrm{U}_0$, $\Delta^2 \mathrm{U}_0$, . . . , by means of

$$(4) \quad . \quad . \quad \begin{aligned} \mathrm{U}_0 &= a_0 + a_1 \mathrm{T}_1(0) + a_2 \mathrm{T}_2(0) + a_3 \mathrm{T}_3(0) + \ . \ . \ . \ , \\ \Delta \mathrm{U}_0 &= \qquad a_1 \Delta \mathrm{T}_1(0) + a_2 \Delta \mathrm{T}_2(0) + a_3 \Delta \mathrm{T}_3(0) + \ . \ . \ . \ , \\ \Delta^2 \mathrm{U}_0 &= \qquad\qquad a_2 \Delta^2 \mathrm{T}_2(0) + a_3 \Delta^2 \mathrm{T}_3(0) + \ . \ . \ . \ , \\ \Delta^3 \mathrm{U}_0 &= \qquad\qquad\qquad a_3 \Delta^3 \mathrm{T}_3(0) + \ . \ . \ . \ , \text{ etc.} \end{aligned}$$

It is seen therefore that the triangle of coefficients of the $m_{(i)}$ on the right side of (3) above is just the triangle of coefficients of the a_i on the right of (4), *with rows and columns interchanged.* Hence, the practical work of graduation falls into four easy steps: (i) to compute by repeated summation the reduced factorial moments of u_i; (ii) to combine these according to multipliers in the *columns* of our tables of § 4, thus obtaining the a_i, which we enter at the heads of columns; (iii) to combine these a_i according to the multipliers in the *rows* of the same table, thus obtaining U_0, $\Delta \mathrm{U}_0$, $\Delta^2 \mathrm{U}_0$, . . . , from which (iv) the other graduated values U_1, U_2, . . . , U_{n-1} can be built up by summation.

A useful check is afforded by a symmetrical or alternating property of $\mathrm{T}_r(x)$ and its differences, namely,

$$(5) \quad . \quad \mathrm{T}_r(x) = (-)^r \mathrm{T}_r(n-x-1); \quad \Delta^s \mathrm{T}_r(x) = (-)^{r+s} \Delta^s \mathrm{T}_r(n-x-s-1).$$

This property, which implies that the Tchebychef polynomials and their differences have symmetrical numerical values about the central value of x, namely $x=\frac{1}{2}(n-1)$, is an immediate consequence of § 6 (2), ***infra***; it is also almost intuitive, from the consideration that graduation of an arbitrary set of data ***in reversed order*** ought to yield identical results. In virtue of it we find

$$(6) \quad \begin{aligned} U_{n-1} &= a_0 - a_1T_1(0) + a_2T_2(0) - a_3T_3(0) + \ldots, \\ \Delta U_{n-2} &= a_1\Delta T_1(0) - a_2\Delta T_2(0) + a_3\Delta T_3(0) - \ldots, \\ \cdot \quad & \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad , \end{aligned}$$

which shows that, instead of calculating U_0, ΔU_0, Δ^2U_0, . . . at stage (iii) of the computation, we might, by reading all the entries in our tables ***with positive sign***, have computed the ***remote*** terminal values and differences U_{n-1}, ΔU_{n-2}, Δ^2U_{n-3}, ... It is well in any case to compute one of these, U_{n-1}, as a check.

While engaged in computing the a_i at stage (ii), we shall also compute the residual variance thus. The ***numerator*** of the expression for a_i being $\sum uT_i$, we may multiply the a_i, when found, into this numerator once again, thus obtaining $a_i\sum uT_i$, and these, by § 3 (8), are the quantities which, subtracted one at a time from $\sum u^2$, give the progressive values of S^2. Dividing S^2 by $n-i-1$, the number of remaining degrees of freedom,* we estimate the mean individual squared error, or variance, of residuals, and thus decide whether or not it is necessary to take another moment and another column of the table, in order to compute the a_{i+1} for the next higher polynomial.

As for the factorial moments, it is well known (A 3 and A 5) that these can be obtained by successive summation of the u_i in columns, summing from the last value u_{n-1} upwards and, at each stage after the first, ***discarding the highest item***, as illustrated in the numerical example which will now follow.

EXAMPLE: $n=10$.

x.	u.	$\Sigma=m_{(0)}$	$\Sigma^2=m_{(1)}$	$\Sigma^3=m_{(2)}$.	$\Sigma^4=m_{(3)}$.	$\Sigma^5=m_{(4)}$.	u^2.
0	17	*750*	..	.	..	..	289
1	40	733	*4412*	.	.	..	1600
2	47	693	3679	*13140*	..	..	2209
3	49	646	2986	9461	*24244*	..	2401
4	52	597	2340	6475	14783	*29747*	2704
5	69	545	1743	4135	8308	14964	4761
6	111	476	1198	2392	4173	6656	12321
7	123	365	722	1194	1781	2483	15129
8	127	242	357	472	587	702	16129
9	115	115	115	115	115	115	13225
							$\Sigma=70768$

---

* According to precepts well established and justified. *Cf.* B 6 (*a*), (*b*), (*c*).

The triangular table of coefficients for $n=10$ is then brought into use, and the computation is arranged around the sides of it as follows:—

			— a —							
	75.	6·285.	−0·0101.	−0·1100.	−0·07642	U_0.	ΔU_0.	$\Delta^2 U_0$.	$\Delta^3 U_0$.	$\Delta^4 U_0$.
750	1	−9	36	−84	126	17·68				
4412		2	−24	112	−280		21·89			
u 13140			6	−70	315			−16·43		
24244				20	−210				13·85	
29747					70					−5·35
$\sum T_r^2$	10	330	4752	34320	140140	$U_9 = 112·3$.				

Thus, *e.g.*,

$$a_2 = (750 \times 36 - 4412 \times 24 + 13140 \times 6)/4752 = -\frac{48}{4752} = -0·01010.$$

$$\Delta^2 U_0 = -0·0101 \times 6 - 0·1100 \times (-70) - 0·07642 \times 315 = -16·43.$$

$$U_9 = 75 \times 1 + 6·285 \times 9 - 0·0101 \times 36 - 0·1100 \times 84 - 0·07642 \times 126 = 112·3.$$

While engaged upon the a_i we estimate the residual variance thus:—

	$a_i \Sigma u T_i$	$\Sigma u^2 = 70768$.	S^2.	$S^2/(n-i-1)$.
$a_0 = 750/10 = 75$.	750×75	$= 56250$	14518	1613
$a_1 = 2074/330 = 6·285$	$2074 \times 6·285$	$= 13035$	1483	185
$a_2 = -48/4752 = -0·01010$.	$-48 \times -0·0101$	$=$ 0	1483	212
$a_3 = -3776/34320 = -0·1100$.	$-3776 \times -0·1100$	$=$ 415	1068	178
$a_4 = -10710/140140 = -0·07642$.	$-10710 \times -0·0764$	$=$ 818	250	50

The above brings out the fact that, when the loss of independence of parameters due to using moments of the data is put in terms of degrees of freedom, the parabola of fit is actually inferior * to the straight line, while the cubic polynomial is not much better. The quartic polynomial shows a marked reduction.

From the terminal values and differences we may build up a difference table in the ordinary way and arrive at the graduation:—

x.	0	1	2.	3	4.	5.	6.	7.	8.	9.	Σ.
u	17	40	47	49	52	69	111	123	127	115	750
U	17·7	39·6	45·0	47·9	56·7	74·6	99·3	123·4	133·9	112·5	750·6
$U-u$	0·7	0·4	2·0	1·1	4·7	5·6	11·7	0·4	6·9	2·5	R^2
$(U-u)^2$	0·5	0·2	4·0	1·2	22·1	31·4	136·9	0·2	47·6	6·3	250·4

Three checks corroborate the results. In the first place, the other terminal value 112·5 agrees well with the value 112·3 computed by means of the tables; then again, the sums of observed and computed values are in good agreement; while the value of R^2, 250, is also in excellent agreement with the other computation. The work of graduation is thus complete.

---

* This example was constructed from a curve with inflexions, in order to bring out such a feature; but the fact that the reduction in S^2 produced by the parabolic term $a_2 T_2$ is almost zero was an unanticipated accident.

6. CENTRAL FORMULÆ AND METHODS.

The formulæ for the Tchebychef polynomials assume a more compact shape when the data are referred to a central origin and when central or mean central factorial polynomials are used. It is convenient here, though by no means indispensable, to make a distinction between the cases when n is even and when n is odd, just as is done in the theory of central polynomial interpolation, and the results will be expressed in two different forms, one involving the factorials which appear in the Newton-Bessel, the other the factorials which appear in the Newton-Stirling interpolation formula.

We take the defining formula § 3 (12) in the shape

$$(1) \quad . \quad . \quad . \quad \mathrm{T}_r(\xi) = \Delta^r\{\xi^{(r)} . (\xi - n)^{(r)}\}/(r!)^2,$$

alter the origin to the centre of the data by taking a new x such that $x = \xi - \frac{1}{2}(n-1)$, and at the same time translate ordinary into central differences by means of the relation $\Delta u_x = \delta u_{x+\frac{1}{2}}$. Thus, adopting the notation $\hat{\mathrm{T}}_r(x)$ for the centralised Tchebychef polynomials, and putting $n = 2q$, we derive readily

$$(2) \quad \hat{\mathrm{T}}_r(x) = \delta^r\left\{\left(x^2 - \overline{q + \frac{r-1}{2}}^{\,2}\right)\left(x^2 - \overline{q + \frac{r-3}{2}}^{\,2}\right) \ . \ . \ . \ \left(x^2 - \overline{q - \frac{r-1}{2}}^{\,2}\right)\right\} \Big/ (r!)^2,$$

which is a more convincing analogue of Rodrigues's formula when the latter is put in the shape

$$(3) \quad . \quad . \quad . \quad . \quad \mathrm{P}_r(x) = \left(\frac{d}{dx}\right)^r (x^2 - q^2)^r/(r!)^2$$

Now, expressing the operand on the right of (2) by means of the central factorial identities of § 2 (xxvii), and applying central differencing to the results, using § 2 (xvi), we derive the following formulæ, which have been placed opposite to the expressions for the Legendre polynomials for the range $(-q, q)$ by way of comparison. The general result can easily be inferred, and written, if desired, in terms of reduced central and mean central factorials:—

	(i) *Legendre Polynomials for $(-q, q)$.*	(ii) *Centred Tchebychef Polynomials.*
(4)	$1.$	$1.$
	$2x.$	$2x.$
	$6x^2/2! - q^2.$	$6x^2/2! - (q^2 - \frac{1}{4}),$ or $6(x^2 - \frac{1}{4})/2! - (q^2 - 1).$
	$20x^3/3! - 2q^2x.$	$20x(x^2 - 1)/3! - 2(q^2 - \frac{9}{4})x,$ or $20x(x^2 - \frac{1}{4})/3! - 2(q^2 - 1)x.$
	$70x^4/4! - 5q^2x^2/2! + \frac{1}{4}q^4.$	$70x^2(x^2 - 1)/4! - 5(q^2 - \frac{9}{4})x^2/2! + \frac{1}{4}(q^2 - \frac{1}{4})(q^2 - \frac{9}{4}),$ or $70(x^2 - \frac{1}{4})(x^2 - \frac{9}{4})/4! - 5(q^2 - 4)(x^2 - \frac{1}{4})/2! + \frac{1}{4}(q^2 - 1)(q^2 - 4),$

and so on,* the numerical coefficients being readily found from Rodrigues's formula. We also note that

$$(5) \quad . \quad \hat{T}_{2r+1}(0)=0, \quad \hat{T}_{2r}(0)=(-)^r(q^2-\tfrac{1}{4})(q^2-\tfrac{9}{4}) \dots (q^2-\overline{r-\tfrac{1}{2}}^2)/(r!)^2,$$

and more generally that since the polynomials are, as they stand, in the shape of Newton-Stirling or Newton-Bessel formulæ, their central or mean central values and differences at the centre $x=0$ are simply (*cf.* the remark on § 3 (14)) ***the coefficients of the central and mean central factorials in* x *which appear in the polynomials*** in (4) (ii) above. This fact enables us, as before, to construct triangular tables conveying double information, and capable, if we use the appropriate ***central factorial*** moments, of serving a similar practical purpose. These tables, depending as they do on purely even or purely odd functions, will have zeros in alternate places, and so will have effectively only about half the number of entries in the former tables, which makes for speed.

There are recurrence relations and a central difference equation of the second order comparable as before with the differential equation of the Legendre polynomials, but these do not serve us here. They are easy to obtain, and not remarkably interesting.

7. TABLES OF CENTRAL VALUES AND DIFFERENCES OF THE TCHEBYCHEF POLYNOMIALS, WITH VALUES OF $\sum T_r^2$.

SPECIMEN TABLES, $n=7$ AND $n=8$.

$r.$	0.	1.	2.	3.	4.	5.
$\hat{T}(0)$	1	.	−12	.	30	.
$\mu\delta\hat{T}(0)$		2	.	−20	.	30
$\delta^2\hat{T}(0)$			6	.	−50	.
$\mu\delta^3\hat{T}(0)$				20	.	−84
$\delta^4\hat{T}(0)$					70	.
$\mu\delta^5\hat{T}(0)$						252
$\sum\hat{T}_r^2$	7	112	756	2400	3850	3024

$r.$	0.	1.	2.	3.	4.	5.
$\mu\hat{T}(0)$	1	.	−15	.	45	.
$\delta\hat{T}(0)$		2	.	−30	.	90
$\mu\delta^2\hat{T}(0)$			6	.	−60	.
$\delta^3\hat{T}(0)$				20	.	−168
$\mu\delta^4\hat{T}(0)$					70	.
$\delta^5\hat{T}(0)$						252
$\sum\hat{T}_r^2$	8	168	1512	6600	15400	19656

For example,

$$\mu\delta^3\hat{T}_5(0;7)=-84, \quad \delta^2\hat{T}_4(0;8)=-50, \text{ etc.}$$
$$\hat{T}_4(x;7)=70x^2(x^2-1)/4!-50x^2/2!+30,$$
$$\hat{T}_4(x;8)=70(x^2-\tfrac{1}{4})(x^2-\tfrac{9}{4})/4!-60(x^2-\tfrac{1}{4})/2!+45, \text{ etc.}$$

The computation of the triangular portion of the tables (the values of $\sum_t \mathrm{T}_r{}^2$ being as before, § 3 (16)) was carried out by preparing a fixed triangle from the coefficients in the Legendre polynomials, together with a varying triangle, depending on q, of the numerical values of the quasi-central factorials which appear in the centred Tchebychef polynomials, and multiplying corresponding elements of these. Thus for graduation as far as the quintic case one uses

---

* The expressions in central, not mean central factorials, were found in a different manner by Miss F E. Allan, and more recently by P. Lorenz. References B, 1, 6 (*c*), and 12.

$$\begin{array}{cccccc} 1 & \cdot & -1 & \cdot & \frac{1}{4} & \cdot \\ & 2 & \cdot & -2 & \cdot & \frac{1}{2} \\ & & 6 & \cdot & -5 & \cdot \\ & & & 20 & \cdot & -14 \\ & & & & 70 & \cdot \\ & & & & & 252 \end{array} \quad \text{with} \quad \begin{array}{cccccc} 1 & \cdot & q^2-1 & \cdot & (q^2-1)(q^2-4) & \cdot \\ & 1 & \cdot & q^2-1 & \cdot & (q^2-1)(q^2-4) \\ & & 1 & \cdot & q^2-4 & \cdot \\ & & & 1 & \cdot & q^2-4 \\ & & & & 1 & \cdot \\ & & & & & 1 \end{array}$$

or with

$$\begin{array}{cccccc} 1 & \cdot & q^2-\frac{1}{4} & \cdot & (q^2-\frac{1}{4})(q^2-\frac{9}{4}) & \cdot \\ & 1 & \cdot & q^2-\frac{9}{4} & \cdot & (q^2-\frac{9}{4})(q^2-\frac{25}{4}) \\ & & 1 & \cdot & q^2-\frac{9}{4} & \cdot \\ & & & 1 & \cdot & q^2-\frac{25}{4} \\ & & & & 1 & \cdot \\ & & & & & 1 \end{array}$$

according as $n=2q$ is even or odd.

For any special value of n such tables as these may be computed with ease and rapidity.

$n=4$.

$$\begin{array}{|ccc|} \hline 1 & \cdot & -3 \\ & 2 & \cdot \\ & & 6 \\ \hline \multicolumn{1}{c}{4} & 20 & \multicolumn{1}{c}{36} \end{array}$$

$n=5$.

$$\begin{array}{|cccc|} \hline 1 & \cdot & -6 & \cdot \\ & 2 & \cdot & -8 \\ & & 6 & \cdot \\ & & & 20 \\ \hline \multicolumn{1}{c}{5} & 40 & 126 & \multicolumn{1}{c}{160} \end{array}$$

$n=6$

$$\begin{array}{|ccccc|} \hline 1 & \cdot & -8 & \cdot & 10 \\ & 2 & \cdot & -16 & \cdot \\ & & 6 & \cdot & -25 \\ & & & 20 & \cdot \\ & & & & 70 \\ \hline \multicolumn{1}{c}{6} & 70 & 336 & 720 & \multicolumn{1}{c}{700} \end{array}$$

$n=7$.

$$\begin{array}{|cccccc|} \hline 1 & \cdot & -12 & \cdot & 30 & \cdot \\ & 2 & \cdot & -20 & \cdot & 30 \\ & & 6 & \cdot & -50 & \cdot \\ & & & 20 & \cdot & -84 \\ & & & & 70 & \cdot \\ & & & & & 252 \\ \hline \multicolumn{1}{c}{7} & 112 & 756 & 2400 & 3850 & \multicolumn{1}{c}{3024} \end{array}$$

$n=8$.

$$\begin{array}{|cccccc|} \hline 1 & \cdot & -15 & \cdot & 45 & \cdot \\ & 2 & \cdot & -30 & \cdot & 90 \\ & & 6 & \cdot & -60 & \cdot \\ & & & 20 & \cdot & -168 \\ & & & & 70 & \cdot \\ & & & & & 252 \\ \hline \multicolumn{1}{c}{8} & 168 & 1512 & 6600 & 15400 & \multicolumn{1}{c}{19656} \end{array}$$

$n=9$.

$$\begin{array}{|cccccc|} \hline 1 & \cdot & -20 & \cdot & 90 & \cdot \\ & 2 & \cdot & -36 & \cdot & 126 \\ & & 6 & \cdot & -90 & \cdot \\ & & & 20 & \cdot & -196 \\ & & & & 70 & \cdot \\ & & & & & 252 \\ \hline \multicolumn{1}{c}{9} & 240 & 2772 & 15840 & 50050 & \multicolumn{1}{c}{91728} \end{array}$$

$n=10$

$$\begin{array}{|cccccc|} \hline 1 & \cdot & -24 & \cdot & 126 & \cdot \\ & 2 & \cdot & -48 & \cdot & 252 \\ & & 6 & \cdot & -105 & \cdot \\ & & & 20 & \cdot & -294 \\ & & & & 70 & \cdot \\ & & & & & 252 \\ \hline \multicolumn{1}{c}{10} & 330 & 4752 & 34320 & 140140 & \multicolumn{1}{c}{343980} \end{array}$$

$n=11.$

$$\boxed{\begin{array}{rrrrrr} 1 & . & -30 & . & 210 & . \\ & 2 & . & -56 & . & 336 \\ & & 6 & . & -140 & . \\ & & & 20 & . & -336 \\ & & & & 70 & . \\ & & & & & 252 \end{array}}$$

$11 \quad 440 \quad 7722 \quad 68640 \quad 350350 \quad 110074^{(17}$

$n=12.$

$$\boxed{\begin{array}{rrrrrr} 1 & . & -35 & . & 280 & . \\ & 2 & . & -70 & . & 560 \\ & & 6 & . & -160 & . \\ & & & 20 & . & -448 \\ & & & & 70 & . \\ & & & & & 252 \end{array}}$$

$12 \quad 572 \quad 12012 \quad 128700 \quad 800800 \quad 311875^{(17}$

$n=13.$

$$\boxed{\begin{array}{rrrrrr} 1 & . & -42 & . & 420 & . \\ & 2 & . & -80 & . & 720 \\ & & 6 & . & -200 & . \\ & & & 20 & . & -504 \\ & & & & 70 & . \\ & & & & & 252 \end{array}}$$

$13 \quad 728 \quad 18018 \quad 228800 \quad 170170^{(17} \quad 801965^{(17}$

$n=14.$

$$\boxed{\begin{array}{rrrrrr} 1 & . & -48 & . & 540 & . \\ & 2 & . & -96 & . & 1080 \\ & & 6 & . & -225 & . \\ & & & 20 & . & -630 \\ & & & & 70 & . \\ & & & & & 252 \end{array}}$$

$14 \quad 910 \quad 26208 \quad 388960 \quad 340340^{(17} \quad 190467^{(18}$

$n=15$

$$\boxed{\begin{array}{rrrrrr} 1 & . & -56 & . & 756 & . \\ & 2 & . & -108 & . & 1350 \\ & & 6 & . & -270 & . \\ & & & 20 & . & -700 \\ & & & & 70 & . \\ & & & & & 252 \end{array}}$$

$15 \quad 1120 \quad 37128 \quad 636480 \quad 646646^{(17} \quad 423259^{(18}$

$n=16.$

$$\boxed{\begin{array}{rrrrrr} 1 & . & -63 & . & 945 & . \\ & 2 & . & -126 & . & 1890 \\ & & 6 & . & -300 & . \\ & & & 20 & . & -840 \\ & & & & 70 & . \\ & & & & & 252 \end{array}}$$

$16 \quad 1360 \quad 51408 \quad 100776^{(17} \quad 117572^{(18} \quad 888844^{(18}$

$n=17.$

$$\boxed{\begin{array}{rrrrrr} 1 & . & -72 & . & 1260 & . \\ & 2 & . & -140 & . & 2310 \\ & & 6 & . & -350 & . \\ & & & 20 & . & -924 \\ & & & & 70 & . \\ & & & & & 252 \end{array}}$$

$17 \quad 1632 \quad 69768 \quad 155040^{(17} \quad 205751^{(18} \quad 177769^{(19}$

$n=18.$

$$\boxed{\begin{array}{rrrrrr} 1 & . & -80 & . & 1540 & . \\ & 2 & . & -160 & . & 3080 \\ & & 6 & . & -385 & . \\ & & & 20 & . & -1078 \\ & & & & 70 & . \\ & & & & & 252 \end{array}}$$

$18 \quad 1938 \quad 93024 \quad 232560^{(17} \quad 348194^{(18} \quad 340724^{(19}$

$n=19.$

$$\boxed{\begin{array}{rrrrrr} 1 & . & -90 & . & 1980 & . \\ & 2 & . & -176 & . & 3696 \\ & & 6 & . & -440 & . \\ & & & 20 & . & -1176 \\ & & & & 70 & . \\ & & & & & 252 \end{array}}$$

$19 \quad 2280 \quad 122094 \quad 341088^{(17} \quad 572033^{(18} \quad 629028^{(19}$

$n=20.$

$$\boxed{\begin{array}{rrrrrr} 1 & . & -99 & . & 2376 & . \\ & 2 & . & -198 & . & 4752 \\ & & 6 & . & -480 & . \\ & & & 20 & . & -1344 \\ & & & & 70 & . \\ & & & & & 252 \end{array}}$$

$20 \quad 2660 \quad 158004 \quad 490314^{(17} \quad 915253^{(18} \quad 1123265^{(20}$

$n=21$.

1	.	−110	.	2970	.
	2	.	−216	.	5616
		6	.	−540	.
			20	.	−1456
				70	.
					252
21	3080	201894	$692208^{(17}$	$143008^{(19}$	$194699^{(110}$

$n=22$.

1	.	−120	.	3510	.
	2	.	−240	.	7020
		6	.	−585	.
			20	.	−1638
				70	.
					252
22	3542	255024	$961400^{(17}$	$2187185^{(19}$	$328555^{(110}$

$n=23$.

1	.	−132	.	4290	.
	2	.	−260	.	8190
		6	.	−650	.
			20	.	−1764
				70	.
					252
23	4048	318780	$131560^{(18}$	$328078^{(19}$	$541149^{(110}$

$n=24$

1	.	−143	.	5005	.
	2	.	−286	.	10010
		6	.	−700	.
			20	.	−1960
				70	.
					252
24	4600	394680	$177606^{(18}$	$483483^{(19}$	$871852^{(110}$

$n=25$

1	.	−156	.	6006	.
	2	.	−308	.	11550
		6	.	−770	.
			20	.	−2100
				70	.
					252
25	5200	484380	$236808^{(18}$	$701050^{(19}$	$137661^{(111}$

8. NUMERICAL USE OF CENTRAL FACTORIAL MOMENTS.

The reduced central factorial moments will be defined as follows: If the number of data is even, we multiply each datum u_x by the corresponding value of the reduced central or mean central factorial polynomial of appropriate degree which appears in the Newton-Bessel formula, and sum over all values. Thus the values of x are here $\pm\frac{1}{2}$, $\pm\frac{3}{2}$, $\pm\frac{5}{2}$, . . . , etc. On the other hand, if the number of data is odd, we shall use the values of the polynomials of the Newton-Stirling formula, the values of x then being 0, ± 1, ± 2, ... The resulting reduced central or mean central factorial moments, which we shall denote by $m_{\{r\}}$ and $m_{\{r\}'} \equiv \mu m_{\{r\}}$, are readily obtained by processes of repeated summation from the outer termini towards the centre, in a way which will be clear from the illustrative example, p. 72.

By using § 3 (6) with the central expressions for the polynomials of § 6 (4), we derive

(1)
$$
\begin{aligned}
na_0 &= m_{\{0\}'},\\
a_1\sum \hat{T}_1^2 &= 2m_{\{1\}},\\
a_2\sum \hat{T}_2^2 &= -(q^2-\tfrac{1}{4})m_{\{0\}'} + 6m_{\{2\}'},\\
a_3\sum \hat{T}_3^2 &= -2(q^2-\tfrac{9}{4})m_{\{1\}} + 20m_{\{3\}},\\
a_4\sum \hat{T}_4^2 &= \tfrac{1}{4}(q^2-\tfrac{1}{4})(q^2-\tfrac{9}{4})m_{\{0\}'} - 5(q^2-\tfrac{9}{4})m_{\{2\}'} + 70m_{\{4\}'},
\end{aligned}
$$

and so on, or with the other set of quasi-factorial coefficients in q (*loc. cit.*), central and mean central moments being then interchanged, according as n is odd or even. (The distinction is not very vital in numerical practice, when the tables are used and the summational process is well understood.) Thus, using (1) above, we can compute the coefficients a_i from the central moments by employing the entries in the *columns* of the tables of § 7, and at the same time can estimate S^2 step by step as before, the work taking about half the time * that it did by the earlier method.

This being done, we may next use the *rows* of the table upon the values a_i, obtaining μU_0 or U_0, δU_0 or $\mu\delta U_0$, and so on, as the case may be. To complete the calculation it is very convenient to break up U_x into an *even* function V_x and an *odd* function W_x, with the obvious consequences that the values of U_0, or μU_0, and its central differences of even order are the same as those of V_0, or μV_0; while the central differences of odd order are the same as those of W_0, or μW_0. The symmetry or alternation of even and odd functions and their differences about the central origin then enables us to complete the solution in a very simple manner.

For a first example we shall take the same data as in § 5.

x.	u.	$\hat{\Sigma}$	$\mu\hat{\Sigma}^2$.	$\hat{\Sigma}^3$.	$\mu\hat{\Sigma}^4$.	$\hat{\Sigma}^5$.	u^2.
$-\frac{9}{2}$	17	17	17	17	17	17	289
$-\frac{7}{2}$	40	57	74	91	108	125	1600
$-\frac{5}{2}$	47	104	178	269	377 (677)	502	2209
$-\frac{3}{2}$	49	153	331 (433·5)	600			2401
$-\frac{1}{2}$	52	205					2704
$\frac{1}{2}$	69	545					4761
$\frac{3}{2}$	111	476	(1470·5) 1198	2392			12321
$\frac{5}{2}$	123	365	722	1194	(2977) 1781	2483	15129
$\frac{7}{2}$	127	242	357	472	587	702	16129
$\frac{9}{2}$	115	115	115	115	115	115	13225
	750						70768

Hence,

$$m_{\{0\}} = 750, \quad m_{\{1\}'} = 1470{\cdot}5 - 433{\cdot}5 = 1037, \quad m_{\{2\}} = 2392 + 600 = 2992,$$

$$m_{\{3\}'} = 2977 - 677 = 2300, \quad m_{\{4\}} = 2483 + 502 = 2985.$$

[*Note.*—The summations are from the outer values to the centre. The bracketed entries, obtained by taking only *half* of the last item in the preceding column, are mean central factorial moments; *cf.* § 2 (vii) and (viii).]

---

* Unless one possesses a very efficient machine for ordinary repeated summation. By either mode of summation, ordinary or central, it is easy on a machine with a capacious register to sum two columns at once, and thus save the time and trouble of copying

Bringing into play the table of § 7 for $n = 10$ and building around it as before, we obtain:—

				— a —							
		75.	6·285.	−0·0101.	−0·1100.	−0·07642	μV_0.	δW_0	$\mu\delta^2 V_0$.	$\delta^3 W_0$.	$\mu\delta^4 V_0$.
	750	1	.	−24	.	126	65·61				
|	1037		2	.	−48	.		17·85			
m	2992			6	.	−105			7·96		
|	2300				20	.				−2·20	
	2985					70					−5·35
$\sum \dagger_r^2$		10	330	4752	34320	140140					

The details of the computation are precisely similar to those described earlier in § 5. We obtain, with less trouble, the same values of the a_t, and can estimate S^2 in the same way.

Finally, using properties of odd and even functions, we build the following two half-tables of differences of V and W:—

x	V	δV.	δ^2V.	δ^3V.	δ^4V.	W.	δW.	δ^2W.	δ^3W.
						(−8 93)		(1·10)	
		0		0			17 85		−2 20
$\frac{1}{2}$	65 61		7·96		−5 35	8·93		−1·10	
		7·96		− 5·35			16·75		−2·20
$\frac{3}{2}$	73·57		2·61		−5 35	25·68		−3·30	
		10·57		−10·70			13·45		−2·20
$\frac{5}{2}$	84·14		− 8·09		−5 35	39 13		−5·50	
		2 48		−16 05			7 95		−2·20
$\frac{7}{2}$	86·62		−24·14			47·08		−7·70	
		−21·66					0·25		
	64·96					47 33			

whence, taking finally the sum and the difference of corresponding V's and W's, we derive the graduated values U:—

x.	$-\frac{9}{2}$.	$-\frac{7}{2}$.	$-\frac{5}{2}$.	$-\frac{3}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$.	$\frac{3}{2}$.	$\frac{5}{2}$.	$\frac{7}{2}$.	$\frac{9}{2}$.	Σ.
U	17·6	39·5	45 0	47·9	56 7	74 5	99·3	123·3	133·7	112·3	749·8

in good agreement with the results found in § 5.

By adopting central methods we have renounced the check upon remote terminal values. Other checks, not quite so simple, can easily be devised, however; and we still have the checks upon the sum, and upon S^2.

To illustrate the case of an *odd* number of data, the procedure being slightly different, we shall modify our earlier example. If, using the difference table of the example in § 5, or the two given above, we extrapolate an eleventh value for U, we obtain approximately the value 35. Supposing then that we annex this value to the ungraduated data and proceed to graduate, we ought to reproduce this value 35, while obtaining for the rest the same graduated values as before.

x.	u.	$\mu\hat{\Sigma}$.	$\hat{\Sigma}^2$.	$\mu\hat{\Sigma}^3$.	$\hat{\Sigma}^4$.	$\mu\hat{\Sigma}^5$.	u^2.
−5	17	17	17	17	17	17	289
−4	40	57	74	91	108	125	1600
−3	47	104	178	269	377	502 (990·5)	2209
−2	49	153	331	600 (868)	977		2401
−1	52	205 (239·5)	536				2704
0	69						4761
1	111	(545 5) 511	1373				12321
2	123	400	862	(2230·5) 1544	2481		15129
3	127	277	462	682	937	(2467·5) 1227	16129
4	115	150	185	220	255	290	13225
5	35	35	35	35	35	35	1225
	785						71993

Hence,

$$m_{\{0\}'}=785,\quad m_{\{1\}}=1373-536=837,\quad m_{\{2\}'}=2230{\cdot}5+868=3098{\cdot}5,$$

$$m_{\{3\}}=2481-977=1504,\quad m_{\{4\}'}=2467{\cdot}5+990{\cdot}5=3458.$$

Taking the table for $n=11$ and building around it, we have

	— a —									
	71·36.	3 805.	−0·6422	−0·2446.	−0 07672.	V_0.	$\mu\delta W_0$.	$\delta^2 V_0$.	$\mu\delta^3 W_0$.	$\delta^4 V_0$.
785	1	.	−30	.	210	74 51				
837		2	.	−56	.		21·31			
m 3098·5			6	.	−140			6·89 −		
1504				20	.				−4·89	
3458					70					−5 37
$\sum \hat{T}_r^2$	11	440	7722	68640	350350					

(The values of S^2 will be found to be 15973, 9604, 6419, 2312, and, for the quartic, 250, the last being the same as before.)

The two half-tables of differences for V and W are then as follows:—

x.	V.	δV.	δ^2V.	δ^3V.	δ^4V.	W.	δW.	δ^2W.	δ^3W.
		(− 3·44)		(2·69)					
0	74·51		6·89		−5·37	0		0	
		3·44		− 2·69			21·31		−4·89
1	77·95		4·20		−5·37	21·31		− 4·89	
		7·64		− 8·06			16·42		−4·89
2	85 59		− 3·86		−5·37	37·73		− 9·78	
		3·78		−13·43			6·64		−4·89
3	89·37		−17·29		−5·37	44·37		−14·67	
		−13·51		−18·80			− 8·03		−4·89
4	75·86		−36·09			36·34		−19·56	
		−49·60					−27·59		
5	26·26					8·75			

whence, by taking the sum and the difference of corresponding V's and W's, we derive the U's as before.

x.	−5.	−4.	−3.	−2.	−1	0.	1	2	3.	4.	5.	Σ.
U	17·5	39·5	45·0	47·9	56·6	74·5	99·3	123·3	133·7	112·2	35·0	784·5

It appears from the above example that the difference of procedure between the cases of an odd and an even number of data is in practice of very slight importance.

These methods, both non-symmetrical and central, depending as they do on summations combined with a few multiplications, are simple and well adapted to computation by the aid of calculating machines.

9. CONNEXION WITH THE DETERMINANTAL METHOD

It will be shown in this section that the determinantal approach by way of moments, provided that *factorial* moments and sums are used instead of the ordinary power moments and sums usually employed here, leads directly to the Tchebychef polynomials

Let $T_r(x)$ be assumed to have the form

$$(1) \quad . \quad . \quad . \quad T_r(x) = c_0 + c_1 x + c_2 x^{(2)} + \dots + c_r x^{(r)},$$

and let the orthogonal conditions be taken in a form like that already used in § 3, namely,

$$(2) \quad . \quad . \quad \sum (x+s)^{(s)} T_r(x) = 0, \qquad s = 0, 1, 2, \dots, r-1.$$

Substituting (1) in this and carrying out the summations according to the properties of factorial polynomials, § 2 (xii), we obtain r homogeneous linear equations in the $r+1$ unknowns c_i, the array of coefficients being the r-by-$(r+1)$ array,

$$(3) \quad . \quad M = \begin{bmatrix} n & \frac{n^{(2)}}{2} & \frac{n^{(3)}}{3} & . \quad . \quad . & \frac{n^{(r+1)}}{r+1} \\ \frac{(n+1)^{(2)}}{2} & \frac{(n+1)^{(3)}}{3} & \frac{(n+1)^{(4)}}{4} & . \quad . \quad . & \frac{(n+1)^{(r+2)}}{r+2} \\ \frac{(n+2)^{(3)}}{3} & \frac{(n+2)^{(4)}}{4} & \frac{(n+2)^{(5)}}{5} & . \quad . & \frac{(n+2)^{(r+3)}}{r+3} \\ . \quad . \quad . & . \quad . & . \quad . & . \quad . \quad . & . \quad . \\ \frac{(n+r-1)^{(r)}}{r} & . \quad . & . \quad . & . \quad . & \frac{(n+r-1)^{(2r)}}{2r} \end{bmatrix}$$

(The determinant * formed from the first r columns of this array is a known determinant. The persymmetric determinant which has elements s_i, the sums of powers of natural numbers, can be transformed into it.)

The ratios one to another of the coefficients c_i will therefore be the ratios of the determinants M_i obtained by deleting the proper column, the $(i+1)$th, of the array M. By examining M_i we find that all of the factors involving n in the factorial numerators of the elements may be removed from each row and column, yielding the composite factor

---

* *Annals of Math.*, (2), vol. xxxii, 1931, pp. 461–462.

$$(4) \quad . \quad . \quad \{n^r(n^2-1)^{r-1}(n^2-4)^{r-2} \ldots (n^2-\overline{r-1}^2)\}(n-i-1)^{(r-i)}.$$

The numerical factor which then remains in M_i will be the appropriate determinant P_i of the persymmetric array,

$$(5) \quad . \quad . \quad . \quad \mathrm{P} = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & . & . & . & \frac{1}{r+1} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & . & . & . & \frac{1}{r+2} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & . & & . & \frac{1}{r+3} \\ . & . & . & . & . & . & . \\ \frac{1}{r} & \frac{1}{r+1} & \frac{1}{r+2} & . & . & . & \frac{1}{2r} \end{bmatrix}.$$

The ratios of the determinants P_i can be found simply thus: Place below the array P a final row of elements $1, x, x^2, \ldots, x^r$ and evaluate the resulting determinant We obtain a polynomial $P_r(x)$ of degree r. Now we must have

$$(6) \quad . \quad \int_0^1 x^s P_r(x)dx = 0, \qquad s = 0, 1, 2, \ldots, r-1,$$

since, by carrying out the integrations on the last row of the determinant, we obtain elements which are the same as those of an earlier row, the $(s+1)$th; while at the same time

$$(7) \quad . \quad . \quad . \quad . \quad . \quad . \quad \int_0^1 x^r P_r(x)dx \neq 0,$$

since the persymmetric determinant of elements in harmonic progression happens to be known, and can in any case be shown in various ways * to be positive. Hence, apart from the constant factor, which we may fix by our convention, $P_r(x)$ must be the Legendre polynomial for the range of integration (0, 1)—that is, it must be given by

$$(8) \quad . \quad . \quad P_r(x) = \left(\frac{d}{dx}\right)^r \{x(x-1)\}^r/(r!)^2,$$

and the mutual ratios of the determinants P_i in the array P are the mutual ratios of the coefficients in this polynomial. Combining finally (1) and (4) and (8), we derive the Tchebychef polynomials as given in § 3 (14).

In conclusion, it may be remarked that the order of procedure in § 8 above can be applied to the case of graduation of continuous data by a

---

* That it should not vanish is indeed the condition for the existence of the Legendre polynomials.

polynomial. Suppose that over the range $(-q, q)$ we have to graduate an empirical curve, broken polygon, or the like, by a polynomial, minimising the integral of the squared deviations. We should in the first place compute, in general by methods of numerical integration, the reduced power moments of the data. To these we might then apply a triangular table of the coefficients of $x^r/r!$ in the Legendre polynomials and their derivatives, with values of the integral of $P_r^2(x)$ for column divisors, exactly in the manner of § 8. By this means we should derive the values of the coefficients a_t in a Legendre expansion, from which S^2 could be evaluated as before, together with values, at the origin, of the graduating polynomial and its derivatives, from which a Taylor expansion could be at once written down.

The author wishes finally to express his indebtedness to Professor E. T. Whittaker, who first stimulated his interest in this subject by a course of lectures on Tchebychef's theory, given in 1924

APPENDIX.

A. *References to Finite Differences, Sums, Factorial Moments.*

1. ELDERTON, W. P., 1927. *Frequency Curves and Correlation* (2nd edition, London). Central Summation, pp. 22–23.
2. FREEMAN, H., 1931. *Actuarial Mathematics* (Cambridge). Differences of product, pp. 116–117 and 371, Ex. 10. Sum of a product, p. 132, Ex 9. Summation by parts, pp. 102, 112, Ex. 39; p 117, Ex. 1; p. 118. Central factorials, p. 127.
3. HARDY, G. F., 1909. *Construction of Tables of Mortality* (London) Summation and central summation for moments, pp. 59–60; and Note E, pp. 124–128.
4. SHEPPARD, W. F., 1914. (*a*) "On Factorial Moments in Terms of Sums and Differences," *Proc. Lond Math. Soc.*, (2), vol. xiii, pp. 81–96.
   ——, 1923. (*b*) "On the Reciprocal Correspondence of Differences and Sums," *ibid.*, (2), vol. xxi, pp 291–305.
5. STEFFENSEN, J. F., 1927 *Interpolation* (Baltimore). Various kinds of factorial polynomials, pp. 8–9 Difference of product, pp. 11–12. Summation by parts, p. 89. Repeated summation, pp. 94–103 Central summation, p. 99. Factorial moments, p. 101.

B. *References to Polynomial Graduation by Least Squares.*

1. ALLAN, F. E., 1930. *Proc. Roy. Soc. Edin.*, vol l, pp. 310–320.
2. CHOTIMSKY, V., 1925. *The Smoothing of Statistical Series by Least Squares** (*Tchebychef's Method*) (in Russian), Soviet Press, Moscow and Leningrad, p. 87.

* I am indebted to Professor J. F. Steffensen for the privilege of perusing his copy of this monograph.

3. DAVIS, H. T., and V. V. LATSHAW, 1930. *Annals of Math.*, (2), vol. xxxi, pp. 52–78.
4. ELDERTON, W. P., 1903. *Biometrika*, vol. ii, pp. 474–480.
5. ESSCHER, F., 1920. "Ueber die Sterblichkeit in Schweden, 1886–1914," *Meddel. Lunds Astr. Observ.*, (2), vol. xxiii, pp. 10–21.
6. FISHER, R. A., 1921. (*a*) *Journ. Agr. Sci.*, vol. ix, pp. 107–135.
   ——, 1924–25. (*b*) *Phil. Trans. Roy. Soc. Lond.*, B, vol. ccxiii, pp. 89–142, esp. pp. 99–113.
   ——, 1930. (*c*) *Statistical Methods for Research Workers* (3rd edition, Edinburgh), pp. 124–132, esp. pp. 126–127.
7. GRAM, J. P., 1882. A Thesis, *Om Raekkendviklinger bestemte ved Hjaelp af de mindste Kvadraters Methode* (Copenhagen, 1879). Recast later as a memoir (in German), *J. für Math.*, vol. xciv, pp. 41–73.
8. GREENLEAF, H. E. H., 1932. *Annals of Math. Statistics*, vol. iii, pp. 204–255.
9. IRWIN, J. O., 1931. *Journ. Roy. Stat. Soc.*, vol. xciv, p. 577. (A survey.)
10. ISSERLIS, L., 1927. *Biometrika*, vol. xix, pp. 87–93.
11. JORDAN, CH., 1922. (*a*) *Proc. Lond. Math. Soc.*, (2), vol. xx, pp. 297–325.
    ——, 1927. (*b*) *Statistique Mathématique* (Paris), pp. 25–32, 270–275, 331–334.
    ——, 1929. (*c*) *Journ. Soc. Hongr. Statist.*, p. 35.
12. LORENZ, P., 1932. *J. für Math.*, vol. clxviii, pp. 170–175.
13. PEARSON, K., 1903. (*a*) *Biometrika*, vol. ii, pp. 9–23.
    ——, 1921. (*b*) *Ibid.*, vol. xiii, pp. 296–299.
14. POINCARÉ, H., 1896. *Calcul des Probabilités* (Paris), 21me Leçon, pp. 251–263.
15. ROMANOVSKY, V, 1927. *Biometrika*, vol. xix, pp. 93–99.
16. SHEPPARD, W. F., 1912. (*a*) *Proc. Fifth Intern. Congr. Math.* (Cambridge), vol. ii, p. 348.
    ——, 1914. (*b*) *Proc. Lond. Math. Soc.*, (2), vol. xiii, p. 97.
    ——, 1914, 1915. (*c*) *Journ. Inst. Act.*, vol. xlviii, pp. 171–185, 390–412; vol. xlix, pp. 148–157.
17. SHERRIFF, C. W. M, 1920. *Proc. Roy. Soc. Edin.*, vol. xl, pp. 112–128.
18. TCHEBYCHEF, P. L., 1858. *Œuvres*, vol. i, pp. 203–230, 381–384, 473–498, 541–560, 701–702; vol. ii, pp. 219–242.
19. WHITTAKER and ROBINSON, 1926. *Calculus of Observations* (2nd edition, London and Glasgow), pp. 291–296.

(*Issued separately February* 3, 1933)

VII.—On the Measurement of Spatial Distance in a Curved Space-time. By **H. S. Ruse,** M.A., D.Sc.

(MS. received June 20, 1932 Read January 9, 1933.)

THE problem of defining the concept of spatial distance in a general riemannian space-time has been discussed by E. T. Whittaker,[*] who showed how to translate into the terms of four-dimensional geometry the method adopted by astronomers in determining the distance of a star from the earth by means of a comparison of its absolute and apparent luminosities. The problem was further considered by the present writer,[†] who derived a different formula for spatial distance by a method which was essentially one of partitioning the whole of space-time into space *and* time relative to the given observer. It was suggested that the new spatial distance might be that determined practically by parallax-measurements, though the evidence in support of this suggestion was perhaps hardly sufficient to carry conviction.

It is the purpose of this paper to give a third definition of spatial distance, which, it is claimed, corresponds to that determined by the use of rigid measuring-rods. The formula derived therefrom is the same as that in the writer's earlier paper. Yet a fourth definition, due to Professor Whittaker, is shown in § 5 to yield the same formula.

§ 1. DISTANCE[‡] IN A GALILEAN FIELD.

It is necessary as a preliminary to make a few remarks concerning the definition of spatial distance in a flat space-time.

If an observer O at rest at the point (x_0, y_0, z_0) relative to a given system of rectangular cartesian axes receives at time t_0 a light-pulse emitted by an object Q at the point (x_1, y_1, z_1), then the distance between the object and the observer as measured by the latter is

$$r = \{(x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2\}^{\frac{1}{2}}. \qquad . \qquad . \qquad . \qquad (1.1)$$

This really amounts to a definition of spatial distance in a galilean field, and is of course the ordinary formula for the distance between two points in a three-dimensional euclidean space. It is assumed that the distance is the same whether the object be in motion relative to the observer or not. Moreover, it should be noted that r is the distance between the fixed point O and the point where the object was when the light left it; or, from the observer's point of view, between himself and the point where the object appears to be at time t_0.

---

[*] *Proc. Roy. Soc.*, A, vol. cxxxiii, 1931, p. 93.

[†] *Proc Roy. Soc. Edin.*, vol. lii, 1932, p. 183.

[‡] The word *distance* will be used to mean *spatial distance*. Displacements between point-events in the four-dimensional world will be called *intervals*.

If t_1 denote the time at which the light left the object, and if the velocity of light be taken as unit, then of course

$$t_0 = t_1 + r$$

or

$$r = t_0 - t_1. \qquad . \quad . \quad . \quad . \quad . \quad (1.2)$$

Now the metric of a galilean space-time is given by

$$ds^2 = dt^2 - dx^2 - dy^2 - dz^2. \quad . \quad . \quad . \quad . \quad (1.3)$$

If s_0 is the proper-time of the observer at the world-point (t_0, x_0, y_0, z_0), then since x_0, y_0, z_0 are constants, it follows from (1.3) that $ds_0^2 = dt_0^2$. So

$$\frac{dt_0}{ds_0} = 1, \quad \frac{dx_0}{ds_0} = \frac{dy_0}{ds_0} = \frac{dz_0}{ds_0} = 0.$$

Hence the observer's world-line has in space-time the direction with contravariant components (1, 0, 0, 0). Also the vector $\overrightarrow{QO}$ giving the displacement between the two world-points has covariant components $t_0 - t_1$, $-(x_0 - x_1)$, $-(y_0 - y_1)$, $-(z_0 - z_1)$. Hence $t_0 - t_1$, and therefore r, is equal to the orthogonal projection on the observer's world-line of the displacement $\overrightarrow{QO}$.

§ 2. UNDERLYING ASSUMPTIONS.

The familiar process adopted in everyday life when the distance between two points is measured with a rigid scale cannot for a variety of reasons be readily expressed in the terms of general relativity; for example, except when the distance between the points is less than the length of the rod, the process is one which cannot be performed instantaneously, and in a general space-time different results might be expected according to the time taken. And in any case two points on or near the earth's surface are too close to one another for the refinements of general relativity to be of significance in this connection.

It is therefore true that the formula of this paper represents spatial distance as determined by rigid measuring-rods only in a somewhat conventional sense, but an attempt has been made to introduce no argument not in accord with the basic ideas of the theory of relativity.

The principles underlying the work of the succeeding paragraphs are briefly as follows. It is assumed that:

(i) In a curved space-time there exists in the neighbourhood of any given observer a region which is flat to a high degree of approximation (the observer's *local galilean space-time*); for in any space-time a transformation of coordinates can in general be made such that in the new system the first-order derivatives of the fundamental tensor vanish at any given point.*

(ii) Rigid measuring-rods are, according to the fundamental ideas of relativity, the ultimate instruments with which observers set up spatial mesh-systems in their local galilean space-times.

(iii) If a light-pulse is emitted by an object at the world-point Q *in the local galilean space-time* of an observer and received by him at the world-point O, then the spatial distance between them, as determined by the observer with a rigid scale at the instant of the arrival of the pulse, is equal to the orthogonal projection of the (null) displacement $\overrightarrow{QO}$ on the observer's world-line

It is convenient to call the distance between an observer and an object in his local galilean space-time a *distance-element*. The spatial distance between an observer and *any* object will be defined as a sum (or rather an integral) of a series of distance-elements

Suppose, then, that in a general space-time O and S are respectively any observer and any object with prescribed world-lines, which are not necessarily geodesics. Suppose, further, that a light-pulse leaves S and arrives at O at the instant when he wishes to determine the distance OS, so that the world-points of S and O at the respective instants of the emission and reception of the pulse lie on a null geodesic.

Imagine that along the light-track between S and O is a succession of auxiliary observers $O_1, O_2, \ldots, O_n$, each in possession of a rigid scale which provides in his own locality the unique standard of length. If these observers are numerous enough that S is in the local galilean space-time of O_1, O_1 in that of O_2, O_2 in that of O_3, and so on, O_n being finally in that of O, then the distance-elements $SO_1, O_1O_2, \ldots, O_{n-1}O_n, O_nO$ may be determined respectively by $O_1, O_2, \ldots, O_n$, O. The integral of these distance-elements as $n \to \infty$ is defined to be the spatial distance OS.

To state the same thing more picturesquely, suppose that the observers are near enough to one another that each can touch his neighbours with his measuring-rod, and so determine the distance-element between himself and a neighbour by a single instantaneous observation, and that this measurement is made at the moment when the light-pulse passes him. Then the sum (integral) of these smaller distances is by definition the total distance OS. It is, moreover, to be noticed that, since the light-pulse may be assumed to carry with it an image not only of the object S but also of each of the auxiliary observers, the final observer O will see by its aid not only S but each of the auxiliaries in the act of making his measurement, and will therefore regard the whole operation of determining the distance as instantaneous.

---

* Eddington, *Mathematical Theory of Relativity*, 1924, p. 77.

One important point has been ignored, namely, that a distance-element depends on the direction of the observer's world-line—that is, on his time-direction in the four-dimensional world If each auxiliary observer is allowed to have an arbitrary world-line the distance OS as above defined is indeterminate.

Now the time-direction of each auxiliary observer must be one which is acceptable to the chief observer O, and it is natural therefore to suppose that O requires each of the others to have the same time-direction as himself at the instant when the measurement is made Geometrically this means that the tangents to the world-lines of O_1, O_2, . . , O_n at the points where they are cut by the null geodesic SO must be parallel to the tangent at O to the world-line of the chief observer. But unfortunately the word *parallel* has no unique meaning in a curved riemannian manifold, so that the spatial distance SO will depend on the definition of parallelism chosen Since, however, the parallelism of Levi-Civita is fundamental in general relativity, it seems natural to require that the unit four-vector tangential to the world-line of any auxiliary observer O_r at the point where it is cut by the null geodesic OS shall be that obtained by Levi-Civita parallel transport, along the null geodesic OO_r, of the unit four-vector tangential to the world-line of the chief observer O at the point where it is cut by the same null geodesic.

In short, therefore, the chief assumptions made in the following work are that it is legitimate to introduce into space-time a series of fictitious observers, and that once this has been done their world-directions may be taken as parallel in the sense of Levi-Civita. Concerning the former assumption it may perhaps be remarked that there would seem to be no difference in principle between introducing a series of observers into a general space-time and introducing one, as is commonly done, into a galilean space-time; in each case it is assumed that the prescribed nature of the four-dimensional world is unaffected by their presence.

§ 3. PRELIMINARY FORMULÆ.

In order to find a mathematical expression for spatial distance as defined in § 2 it is necessary to establish a few formulæ belonging to the theory of normal coordinates *

Let

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu \quad . \quad . \quad . \quad . \quad (3.1)$$

define the metric of space-time in terms of general coordinates (x^0, x^1, x^2, x^3). Suppose that the world-point S of the object at the instant when the light-pulse leaves it † is $(x'^\mu) \equiv (x'^0, x'^1, x'^2, x'^3)$, and that the world-point O of the chief observer at the instant when he receives it is $(\bar{x}^\mu)$. Let $(\bar{k}^\mu)$ be the contravariant components of the unit vector at $(\bar{x}^\mu)$ tangential to his world-line, so that, if τ is his proper-time at this point,

$$\bar{k}^\mu = \frac{d\bar{x}^\mu}{d\tau} \quad . \quad . \quad . \quad (3.2)$$

A bar placed above any functional symbol will denote evaluation at the point $(\bar{x}^\mu)$.

By supposition S and O lie on the same null geodesic, the equations of which satisfy, with a proper choice of the parameter λ, the system of differential equations

$$\frac{d^2x^\mu}{d\lambda^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\lambda}\frac{dx^\beta}{d\lambda} = 0, \quad (\mu = 0, 1, 2, 3) \quad . \quad . \quad (3.3)$$

where $\Gamma^\mu_{\alpha\beta}$ is the Christoffel symbol $\{\alpha\ \beta, \mu\}$. If a^μ is the value of $dx^\mu/d\lambda$ at the point $(\bar{x}^\mu)$ for the null geodesic OS, then, as is well known, the parametric equations of this null geodesic may be written

$$x^\mu = \bar{x}^\mu + a^\mu\lambda + \tfrac{1}{2}\bar{\Gamma}^\mu_{\alpha\beta}a^\alpha a^\beta\lambda^2 + \text{higher terms}, \quad (\mu = 0, 1, 2, 3)$$

these equations being in fact solutions of the system (3.3). Writing

$$y^\mu = a^\mu\lambda, \quad (\mu = 0, 1, 2, 3) \quad . \quad . \quad . \quad . \quad (3.4)$$

we get

$$x^\mu = \bar{x}^\mu + y^\mu + \tfrac{1}{2}\bar{\Gamma}^\mu_{\alpha\beta}y^\alpha y^\beta + \ . \ ., \quad . \quad . \quad . \quad (3.5)$$

giving a transformation from the system of coordinates (x^μ) to a system (y^μ) [Riemann-Veblen normal coordinates], in terms of which the null geodesic OS has the parametric equations (3·4), the constants a^μ determining its direction. From (3.5) it follows that $(y^\mu) \equiv (0, 0, 0, 0)$ when

---

* See Veblen, *Invariants of Quadratic Differential Forms* (Camb. Math Tract, No. 24, 1927), chap. vi.

† This notation differs slightly from that of my previous paper, in which the coordinates of *S* were denoted by unaccented letters In this paper x^0, x^1, x^2, x^3 are used exclusively for current coordinates.

$x^\mu = \bar{x}^\mu$, so that O is the origin of the new system of coordinates. Moreover, by (3.5),

$$\left(\frac{\partial x^\mu}{\partial y^\nu}\right)_0 = \delta^\mu_\nu \quad . \quad . \quad . \quad . \quad . \quad (3.6)$$

where the suffix o indicates, like a superposed bar, evaluation at the new origin O. An asterisk will be affixed to the components in the y-system of a tensor or other quantity. So for example

$${}^*g_{\mu\nu} = \frac{\partial x^\alpha}{\partial y^\mu}\frac{\partial x^\beta}{\partial y^\nu} g_{\alpha\beta}.$$

It is to be noticed that, if $T^{\mu\ \ \ \sigma}_{\nu\ \ \ \rho}$ be any tensor-field, then considering the tensor at the point O,

$$T^{\mu\ \ \cdot\,\cdot\,\sigma}_{\alpha\ \ \cdot\,\cdot\,\beta}\left(\frac{\partial x^\alpha}{\partial y^\nu}\right)_0 \qquad \left(\frac{\partial x^\beta}{\partial y^\rho}\right)_0 = \left({}^*T^{\kappa\ \cdot\,\cdot\,\beta}_{\nu\ \dots\ \rho}\right)_0\left(\frac{\partial x^\mu}{\partial y^\kappa}\right)_0 \qquad \left(\frac{\partial x^\sigma}{\partial y^\beta}\right)_0$$

by the ordinary transformation-law for tensors, so by (3 6),

$$T^{\mu\ \ \cdot\,\cdot\,\sigma}_{\nu\ \ \cdot\,\cdot\,\rho} = ({}^*T^{\mu\ \ \ \sigma}_{\nu\ \ \ \rho})_0$$

Hence the components of a tensor *at the point O* are unaltered by the transformation (3.5) In particular, the components $\bar{k}^\mu$ of the vector specifying the direction of the observer's world-line remain unchanged.

Further, transforming (3.3) to the new system, we get

$$\frac{d^2y^\mu}{d\lambda^2} + {}^*\Gamma^\mu_{\alpha\beta}\frac{dy^\alpha}{d\lambda}\frac{dy^\beta}{d\lambda} = 0 \qquad (3.7)$$

Substituting from (3 4), it at once follows that

$${}^*\Gamma^\mu_{\alpha\beta}a^\alpha a^\beta = 0 \qquad (3.8)$$

§ 4. DERIVATION OF THE FORMULA FOR SPATIAL DISTANCE.

In normal coordinates, then, the equations of the null geodesic OS are

$$y^\mu = a^\mu\lambda, \quad (\mu = 0, 1, 2, 3) \quad . \quad . \quad . \quad (4.1)$$

where the a's are constants and λ the parameter. Let (y'^μ) be the coordinates of S in the y-system, and λ' the corresponding value of the parameter, so that $y'^\mu = a^\mu\lambda'$.

Consider an auxiliary observer O_ν at the point (y^μ) on the null geodesic. Let ${}^*k_\mu$ be the covariant components of the unit vector in the direction of his world-line, this being parallel in the sense explained in § 2 to the direction $\bar{k}_\mu (= \bar{g}_{\mu\alpha}\bar{k}^\alpha)$ at O. Suppose that the auxiliary observer $O_{\nu+d\nu}$ next to O_ν on the side of him nearer S has coordinates $(y^\mu + dy^\mu)$, with parameter $\lambda + d\lambda$, so that

$$dy^\mu = a^\mu d\lambda \quad . \quad . \quad . \quad . \quad . \quad . \quad (4.2)$$

Next consider any other pair of observers O_s and O_{s+ds} between O and O_y having coordinates.

$$s^\mu = a^\mu \lambda_s \quad \text{and} \quad s^\mu = a^\mu(\lambda_s + d\lambda_s) \quad . \quad . \quad . \quad (4.3)$$

respectively, λ_s being the value of λ at O_s. Then, if ${}^*k_\mu(s)$ and ${}^*k_\mu(s) + d{}^*k_\mu(s)$ are the direction-vectors of the world-lines of O_s and O_{s+ds} respectively, the ordinary formula for Levi-Civita parallel transport gives

$$d{}^*k_\mu(s) = {}^*\Gamma^\alpha_{\mu\beta}(s) \, . \, {}^*k_\alpha(s) \, . \, ds^\beta,$$

${}^*\Gamma^\alpha_{\mu\beta}(s)$ being the value at (s^μ) of the Christoffel symbol ${}^*\{\mu\beta, \alpha\}$. Integrating along OO_y and using (4 3), we obtain the world-direction of the observer O_y, namely,

$${}^*k_\mu = \bar{k}_\mu + \int_0^\lambda {}^*\Gamma^\alpha_{\mu\beta}(s) \quad {}^*k_\alpha(s) \quad a^\beta d\lambda_s \quad . \quad . \quad . \quad (4.4)$$

Now the spatial distance between O_y and O_{y+dy} as determined by O_y's measuring-rod is, in accordance with § 2, equal to the projection on O_y's world-line of the interval $\overrightarrow{O_{y+dy}O_y}$, it being assumed that O_{y+dy} is in O_y's local galilean space-time. But this projection is

$$-{}^*k_\mu dy^\mu,$$

by the ordinary formula of riemannian geometry.

Using (4 2), and integrating along the null geodesic OS (*i.e.* from $\lambda = 0$ to $\lambda = \lambda'$), we get the total spatial distance OS, namely,

$$\delta = -\int_0^{\lambda'} {}^*k_\mu a^\mu d\lambda.$$

Substituting from (4.4),

$$\begin{aligned} \delta &= -\int_0^{\lambda'} \bar{k}_\mu a^\mu d\lambda - \int_0^{\lambda'} d\lambda \int_0^{\lambda} {}^*\Gamma^\alpha_{\mu\beta}(s) \, . \, a^\mu a^\beta \quad {}^*k_\alpha(s) \, . \, d\lambda_s \\ &= -\int_0^{\lambda'} \bar{k}_\mu a^\mu d\lambda \end{aligned}$$

by (3.8). Hence

$$\begin{aligned} \delta &= -\bar{k}_\mu a^\mu \lambda' \\ &= -\bar{k}_\mu y'^\mu \quad . \quad . \quad . \quad . \quad (4.5) \end{aligned}$$

If now s be the function representing the geodesic distance between the world-points $(\bar{x}^\mu)$ and (x'^μ) of the observer and the star, then *

$$y'^\mu = -\bar{g}^{\mu\nu} \frac{\partial \Omega}{\partial \bar{x}^\nu}, \quad . \quad . \quad . \quad (4.6)$$

where $\Omega = \frac{1}{2}s^2$. Since the two world-points lie on the same null geodesic, it is of course true that $s = 0$. Nevertheless, the function s has an analytical existence, so that equation (4.6) has a definite meaning, the equation

---

* Ruse, *Proc. London Math. Soc.*, vol. xxxii, 1931, p 90.

$s=0$ (or $\Omega=0$) being introduced after the performance of the differentiation. Substituting from (4.6) in (4.5),

$$
\begin{aligned}
\delta &= k^{\nu}\frac{\partial\Omega}{\partial x^{\nu}} \\
&= \frac{d\bar{x}^{\nu}}{d\tau}\frac{\partial\Omega}{\partial\bar{x}^{\nu}}, \quad \text{by} \quad (3.2) \\
&= \frac{\partial\Omega}{\partial\tau}
\end{aligned}
$$

This is the formula for spatial distance obtained in the earlier paper already referred to

§ 5. ANOTHER DEFINITION OF SPATIAL DISTANCE.

The differential equations of the null geodesics are

$$\frac{d^2x^{\mu}}{d\lambda^2}+\Gamma^{\mu}_{\alpha\beta}\frac{dx^{\alpha}}{d\lambda}\frac{dx^{\beta}}{d\lambda}=0, \quad . \quad . \quad . \qquad (5.1)$$

combined with the first integral

$$g_{\mu\nu}\frac{dx^{\mu}}{d\lambda}\frac{dx^{\nu}}{d\lambda}=0, \quad . \quad . \quad . \qquad (5.2)$$

and it is to be noticed that these equations are unaltered if the parameter λ is replaced by $a\lambda$, where a is any constant

Professor E. T. Whittaker has defined * the spatial distance between an observer O and an object S, these being on the same null geodesic, to be equal to $a\int_{S}^{O}d\lambda$ taken along the null geodesic, the constant a being so determined that, when S is very near to O, $a d\lambda$ is equal to the spatial distance in the instantaneous three-dimensional space of the observer (the projection of $\overrightarrow{SO}$ on his world-line).

Adhering to the notation of the previous sections, let $(\bar{x}^{\mu})$, (x'^{μ}) be the coordinates of the observer and object respectively, and (k_{μ}) the observer's covariant direction-vector Now (5.1) can be solved to give, *inter alia*,

$$\frac{dx^0}{d\lambda}=a\mathrm{Y}(x^{\mu}), \quad . \quad . \quad . \quad . \qquad (5.3)$$

a being the unknown constant and $\mathrm{Y}(x^{\mu})$ a known function of x^0, x^1, x^2, x^3. If a is chosen according to the requirement of the definition, it follows easily that an explicit formula for the new spatial distance D in any riemannian space-time is

$$\mathrm{D}=\mathrm{Y}(\bar{x}^{\mu})\left\{k_0+\sum_{p=1}^{3}k_p\left(\frac{dx^p}{dx^0}\right)_0\right\}\int_{x'}^{\bar{x}}\frac{dx^0}{\mathrm{Y}(x^{\mu})}, \quad . \quad . \qquad (5.4)$$

---

* Kermack, M'Crea, and Whittaker, *Proc. R.S.E.*, vol. liii, 1933, p. 35.

where $\left(\frac{dx^\nu}{dx^0}\right)_0$ relate to the direction of the null geodesic at $(\bar{x}^\mu)$, and the integral is taken along the null geodesic.

It is a simple matter to show that the spatial distance D is precisely the same as the δ of § 4. For, if the coordinates are transformed to the normal system (y^μ) defined in § 3, the equations of the null geodesic OS have the form

$$y^\mu = \alpha a^\mu \lambda, \qquad (\mu = 0, 1, 2, 3), \qquad . \quad . \quad . \quad (5.5)$$

where the constant α is inserted to indicate the arbitrariness of the parametrisation already referred to. Therefore

$$\frac{dy^0}{d\lambda} = \alpha a^0,$$

so that, if $^*\mathrm{Y}(y^\mu)$ is the function in the y-system corresponding to the function $\mathrm{Y}(x^\mu)$ of (5.3), we have

$$^*\mathrm{Y}(y^\mu) = a^0 \quad \text{(constant)}$$

The invariant formula (5.4) at once gives

$$\mathrm{D} = a^0\left\{\bar{k}_0 + \sum_{p=1}^{3}\bar{k}_\nu\left(\frac{dy^\nu}{dy^0}\right)_0\right\}\int_{y^1}^{0}\frac{dy^0}{a_0}, \qquad . \quad . \quad (5.6)$$

where, as before, (y'^μ) are the normal world-coordinates of S, (0, 0, 0, 0) being those of O. By (5.5),

$$\begin{aligned}\frac{dy^\nu}{dy^0} &= \frac{a^\nu}{a^0}\\ &= \frac{\alpha a^\nu \lambda'}{\alpha a^0 \lambda'},\end{aligned}$$

where λ' is the value of the parameter at S, hence

$$\frac{dy^\nu}{dy^0} = \frac{y'^\nu}{y'^0}$$

everywhere along the null geodesic. Moreover, the integral in (5.6) clearly has the value $-y'^0/a^0$, so that

$$\begin{aligned}\mathrm{D} &= -a^0\left\{\bar{k}_0 + \sum_{p=1}^{3}\bar{k}_\nu\frac{y'^\nu}{y'^0}\right\}\frac{y'^0}{a^0}\\ &= -\bar{k}_\mu y'^\mu\end{aligned}$$

or

$$\mathrm{D} = \delta, \quad \text{by} \quad (4.5).$$

§ 6. SPATIAL DISTANCE IN EINSTEIN'S CYLINDRICAL WORLD.

In conclusion, it may be of interest to give a formula for the distance δ in the Einstein world, which is specified by *

$$ds^2 = dt^2 - \left\{\frac{dr^2}{1 - r^2/R^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2\right\}, \quad . \quad . \quad (6.1)$$

the observer being at the "space"-origin $r=0$ and the object at the world-point (t, r, θ, ϕ). If $\bar{t}$ be the observer's "time"-coordinate and τ his proper-time, then by (6.1),

$$d\tau^2 = d\bar{t}^2,$$

so that we may take $\tau = \bar{t}$.

Assuming from the spherical symmetry of the spatial part of the quadratic form (6.1) that the geodesic distance from the observer's world-point to that of the object is independent of θ and ϕ, it is easily shown by solving the equations of the geodesics that in this case

$$2\Omega = (t - \tau)^2 - R^2\{\text{arc sin } (r/R)\}^2 \quad . \quad . \quad (6.2)$$

Hence

$$\delta = \partial\Omega/\partial\tau = -(t - \tau),$$

or, since we may now put $\Omega = 0$ (expressing the fact that the observer and object are on the same null geodesic),

$$\delta = R \text{ arc sin } (r/R) \quad . \quad . \quad . \quad (6.3)$$

It may be noticed that, as $R \to \infty$ and (6.1) to galilean-polar form, $\delta \to r$, the radial distance from the origin in a galilean field.

Writing ρ for δ, that is, putting $\rho = R \text{ arc sin } (r/R)$, (6.1) becomes

$$ds^2 = dt^2 - d\rho^2 - R^2 \sin^2 (\rho/R)\{d\theta^2 + \sin^2 \theta d\phi^2\}, \quad . \quad (6.4)$$

where, accepting the definition of spatial distance given in this paper, ρ is the true radius-vector for the observer at the origin

---

* Eddington, *op. cit.*, p. 156, equation (67.12) with $R \sin \chi = r$

(*Issued separately March* 30. 1933.)

VIII.—A Case of Non-disjunction in the Fowl. By Professor F. A. E. Crew, M.D., D.Sc., Institute of Animal Genetics, University of Edinburgh. (With Five Plates.)

(MS received December 6, 1932. Read February 6, 1933.)

THE application of the facts of criss-cross inheritance to commercial poultry breeding is widespread, and every year witnesses the hatching of thousands of chickens, the plumage-colour character of whose mothers was genetically silver, whilst that of their sires was genetically gold. Dominant silver and its allelomorph, recessive gold, form a pair of sex-linked characters, the genes corresponding to them being placed in the X-chromosome. Such evidence as exists points directly to the conclusion that in the fowl the male is the homogametic sex (XX), the female the heterogametic (XY or XO). The mating silver hen, (SX)Y, and gold cock, (sX)(sX), gives, in more or less equal numbers, gold daughters, (sX)Y, and silver sons, (SX)(sX), and, since these colours can readily be distinguished in the newly hatched chick, it is possible to identify males and females amongst day-olds by reference to their down colour. The diagnostic value of a sex-linked character depends upon the precision of the distributive mechanism of the sex-chromosomes.

It is reasonable, in view of what is known of *Drosophila* and other forms, to expect that, amongst the thousands of "sex-linked" chicks that are hatched each year, there will be some which will exhibit the outward manifestations of faulty distribution of the sex-chromosomes. I have therefore endeavoured to maintain contact with a number of breeders known by me to be interested in the scientific aspects of poultry breeding, and have asked them to inform me of any unexpected happening. A number of birds which have broken the rules of sex-linked inheritance have been given to me and are under observation. One of these, which, through the kindness of Professor Punnett, was sent to me by Mr Marcus Slade, of Stokenchurch, High Wycombe, Bucks, is responsible for this communication.

The bird was out of the mating Light Sussex hen × Rhode Island Red cock. In colour it was precisely that gold that one would expect in the gold offspring of this mating, but it was a typical male. Though I did not breed this bird myself, I had no hesitation in accepting the statements made by Mr Marcus Slade concerning it—in the first place, because I know the man; and, secondly, because the bird, in its characterisation, exhibited its own pedigree Mr Slade knew of several other gold cocks that had been hatched out of eggs supplied by him from his pen of Light Sussex × Rhode Island Red. It will be noted that a gold chick is much more likely to be kept than is a silver, for ordinarily the silvers are males and therefore are commonly discarded.

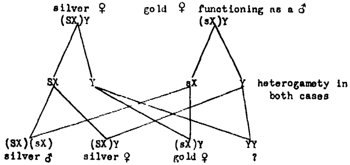
This particular bird could be a case of sex-reversal in early life; that is, a hen of the correct colour (gold) which had become, during the first seven months of its life, a cock—a far-fetched notion maybe, but nevertheless one that had to be entertained. Or it might have been that, through mutation, the gene for silver in a heterozygous male (SX) (sX) became transformed into that for the recessive gold. Or it might have been a case in which the plumage colour that was exhibited was not the sex-linked gold, but an autosomal bay, chestnut, or mahogany. Or, again, it might be a case of secondary non-disjunction of the sex-chromosomes Breeding tests associated with cytological examination alone could decide this matter.

After having performed a laparotomy on this bird, satisfied myself that it possessed a typical male sex-equipment, and removed a small portion of one testis for cytological examination, I mated him to Light Sussex hens, known from past breeding to behave as silvers in a sex-linked mating. One hundred and seventeen chicks were produced, and on examination, either as dead-in-shell or alive, 68 were found to be silvers and 49 golds When the living chicks were 6–12 weeks old, laparotomy was performed on each bird, the gonads were examined, and a small piece of gonadic tissue was removed for histological and cytological examination Amongst the golds was 1 male, amongst the silvers were 2 females, whilst the rest were either silver males or gold females, giving a total of 67 males and 50 females.

These results can best be appreciated when given in tabular form:

Silvers		Golds.	
68		49	
♂♂	♀♀	♂♂	♀♀
66	2	1	48

Thus far, the results favoured the view that this exceptional bird was a case of secondary non-disjunction of the sex-chromosomes. He was a male, and therefore presumably possessed two X-chromosomes. Since his mother was genetically a silver, and, possessing only one X-chromosome, could not carry the gene for gold, it follows that both of his X-chromosomes must have been derived from his father. The spermatozoon which his father provided must have included two X-chromosomes following upon non-disjunction during meiosis. But, since the bird originated in an egg, this could have been a Y-chromosome-bearing egg, so that, in sex-chromosome constitution, this bird would be (sX) (sX)Y. Since there were other gold cocks produced by the pen from which he was derived, it follows that non-disjunction was not rare during spermatogenesis in his father The fact that this exceptional bird produced exceptional offspring eliminates the possibility that he was a bird in which mutation had replaced the gene for silver in a (SX) (sX) male with that for gold. It also shows that he was not a case of sex-reversal, for, had he been, then, mated with silver hens, he would have been expected to produce silver daughters and gold daughters in equal numbers, but only silver sons, and the sex-ratio would presumably be 50 100 since the YY birds would not appear. This will be seen from the ensuing diagram.



Furthermore, the facts that among the offspring only golds and silvers appeared, and that in the great majority of cases the golds were females and the silvers were males, make it certain that the plumage characters involved in the experiment were the sex-linked silver and gold and not autosomal characters such as were present in the birds studied by Agar (1924) and Hays (1926). Moreover, it was evident that the autosomal inhibitor of sex-linked gold described by Taylor (1932) was not complicating the experiment

It became possible, therefore, to assume that indeed this exceptional bird was (sX) (sX)Y in sex-chromosome constitution, and to predict the probable cytological peculiarities of his offspring. The following genetic and cytological types were to be expected:—

	Non-disjunctional Gold ♂.		Normal Silver ♀	
P_1	(sX)(sX)Y	×	(SX)Y	The (sX) and (sX)Y classes of sperm will be far more common than the other two classes for the reason that the two X's disjoin far more often than do the X and the Y. (Bridges, 1916; Muller, 1918)
Gametes	sX (sX)Y ——— (sX)(sX) Y		SX Y	

	Genotype.	Phenotype	Cytologically.	Genetically.	Reason for Genetical Abnormality.
F_1	1. (SX)(sX)	silver ♂	normal	normal	.
	2 (SX)(sX)Y .	do.	abnormal	do.	.
	3. (SX)(sX)(sX)	do.	do.	do.	
	4. (SX)Y . .	silver ♀	normal	abnormal	Y from father. X from mother.
	5 (sX)Y .	gold ♀	do.	normal	
	6 (sX)YY	do	abnormal	do.	
	7. (sX)(sX)Y .	gold ♂	do.	abnormal	Both X's from father.
	8. YY . .			.	.

Disregarding for the time being the genetically exceptional birds, the gold male and the silver female, since these will be rare, it is seen that, among the genetically unexceptional birds, two out of every three silver males, and one out of every two gold females, might be expected to be cytologically abnormal. Cytological investigation was therefore directed toward the identification of the XXX and XXY males and the XYY females. But, in the light of previous studies on the chromosomes of the fowl, it was assumed that the small Y would not easily be distinguished from the small autosomes, and that most probably only the XXX males would be recognised. Embryos, from normal Brown Leghorns and also from the pen of Light Sussex females and the exceptional gold male, ranging in age from 3–6–17 days, were taken to supply further material for cytological examination.

Strong Flemming, medium Flemming, Allen, 2 BD (La Cour), or 2 BE (La Cour) were used as fixatives, and to each fixative solution 1 per cent. of urea solution was added. The sections, 5–15 μ, were stained with iron-hæmatoxylin, gential violet, and new magenta crystal. The drawings were made with an H.I. 90., 1·30 apochromatic oil-immersion objective, and × 15, × 18, and × 30 compensating eyepieces, using a Zeiss camera lucida at an initial magnification of about 2300, 3500, and 4500. I am indebted for the microphotographs to Mr H. C. Osterstock, of the staff of the John Innes Horticultural Institution.

I wish to acknowledge my indebtedness to my colleague Dr Koller for the help he gave to me in the cytological investigation of this material.

The difficulty which attends the study of the chromosome complex in the fowl arises in the facts that the chromosomes are numerous, differing greatly in size amongst themselves, and that there are many relatively small ones. It has been shown in both plants and animals that the number of such small chromosomes is variable, that they are non-essential to the species, and that, most probably, they result from fragmentation (Darlington, 1932) The diploid number of chromosomes of the fowl has been stated by different investigators in recent years to be between 32 and 36. Stevens (1923), Hance (1924, 1926), Shiwago (1924, 1926), and Akkeringa (1927) have all given very clear figures of the male and the female chromosome complexes. Akkeringa fixed the number at 32 in both sexes, but found many cells with a greater number than this. Hance regarded 35–36 as the probable number, although in many cells he found 32–33. Drawings by Miss Stevens, which appear in a paper by Miss Boring, show 34. In the present study the tissues from the control birds showed chromosome complexes with 32–35 chromosomes Plate I, figs 1–3, illustrate the female, and fig 4 the male complexes of 17-day-old embryos. Plate I, figs 5 and 6, 7 and 8, 9 respectively, illustrate male and female complexes as given by Shiwago, Hance, and Stevens, and in these the number of the chromosomes is 32, 38, 33, and 31 respectively.

In an attempt to determine the exact number of chromosomes, both spermatogenesis and oogenesis were examined In the secondary spermatocyte and in the oocyte the haploid complex is present, and therefore the number of chromosomes can be determined more easily The number in the haploid complex of the control birds was in some cases 15, in others 16. The chromosomes, as are seen in Plate III, figs. 1, 2, 3, are thicker and shorter than in the mitotic division (Plate III, figs 4, 5, 6) The figure of Miss Stevens (Plate III, fig. 1) shows that the number of chromosomes in the first spermatocyte is 21, but it would seem that her figure represents metaphase of the secondary spermatocytes where the short chromosomes are already separated and scattered in the metaphase plate. This precocious separation of the small chromosomes is quite common in species in which great size differences between the chromosomes exist.

In oogenesis the prophase stage was found most suitable for study. The ovary of a 3-months-old hen showed developing eggs of different sizes, and it was found possible to follow the pairing of homologous chromosome threads, and also to identify the chiasmata and loops. Serial sections made it possible to follow the whole chromosome complex during diplotene. Plate IV, figs *2a*, *3a*, and *4a* depict three eggs with chromosomes in the diplotene stage, and figs. *2b*, *3b*, and *4b* show schematically the chromosome arrangement. The number of paired bivalents was found to be 15 in most instances. In some cells a very long chromosome thread was found which was looked upon as the long X-chromosome with a small Y-chromosome associated with one of its ends. The chromosomes remain in association at diplotene for a much longer period of time than at any other stage. Plate V represents a cross-section of an ovary showing eggs of different sizes. They are in the growth stage, the yolk around the ovum is multiplying, and the volume of the eggs is increasing rapidly. Nevertheless, their nuclei all show the same diplotene stage, which appears to indicate that the prophase of meiosis proceeds relatively slowly during oogenesis This observation is in close agreement with that of others who have shown that the maturation division in eggs which contain large quantities of yolk proceeds very slowly as compared with spermatogenesis in the same species.

However, it was found that the number of prophase bivalents during oogenesis, and the haploid number of chromosomes in the secondary spermatocytes showed exactly the same variation as was encountered in a study of mitosis It would seem that the exact number of the chromosomes of the fowl cannot be determined, using methods so far available, because of the number and the behaviour of the small chromosomes in the complex. These are far too variable to make exactness in counting possible. This being so, it was hopeless to expect that a count of the chromosomes in the tissues of the exceptional bird and of his offspring could explain the reason for his disobeying the rules of sex-linked inheritance.

This polisomy or aneuploid chromosome number, commonly encountered in the complexes of the control fowls, has previously been remarked upon by other investigators. It is probably the result of fragmentation and/or fusion of the small chromosomes. In the present study several instances were found in which there was indication of fusion on the part of these small chromosomes. They were found to be arranged either in a line or else grouped so closely together that, as a result of fixation, they assumed the appearance of being attached and stained as a single chromosome. This phenomenon was observed by Akkeringa (1927) Hance (1926) suggests that fragmentation rather than fusion is responsible, but no evidence of fragmentation was encountered in the present study. Another likely cause of this polisomy is non-disjunction: a phenomenon which would give rise, in equal numbers, to cells with higher and cells with lower chromosome numbers. This irregularity in respect of the small chromosomes is probably very common, and can be responsible for the variation in chromosome numbers recorded by different investigators. If, as is thought, these small chromosomes are unessential to the species, if they do not carry the genes for characters being studied in an experiment, it is unlikely that irregularities in their distribution will have any detectable effects genetically.

It was necessary, therefore, to study the morphology of the chromosomes rather than their number. The chromosomes of the fowl can be grouped into three size classes—long, medium, and small. This variation in size is very well revealed in sections from the embryo, and is illustrated in Plate II, fig. 1. The long and the medium-sized chromosomes exhibit a tendency to arrange themselves at the periphery of the metaphase plate, whilst the small ones are situated centrally (Plate I, figs. 1–4). Such an arrangement is characteristic of complexes in which great size differences between the individual chromosomes exist It has been shown recently in many instances that it is due to mechanical forces which operate at the time of prophase, and also to the arrangement of the spindle at metaphase.

The relative length of the long and small chromosomes has been given by Shiwago (1926) and Akkeringa (1927), according to whom the long chromosomes are about 3 μ, whilst the small members of the complex are about 5–6 times less than this. In the present study the difference between the long and the small would seem to be much greater, the small ones being 10, 25, or even 30 times less than the longest. The difference between the present findings and those of Shiwago and Akkeringa is probably due to differences in the age of the embryo, for older embryos have thicker and somewhat shorter chromosomes than the very young (see Plate I, figs 3–4, and Plate II, figs. 1–9), whilst different fixative and staining techniques may also be partially responsible.

The chromosomes of the fowl exhibit a very great capacity for extreme condensation and contraction. Hance (1926*b*), who investigated chromosome behaviour in tissue culture, found that the chromosomes at prophase in mitosis were attenuated and more twisted than was the case in tissues derived from growing embryos. It is probable that this difference is a reflection of the differences in environment, for cells *in vitro* lose their characteristic properties and dedifferentiation occurs. A comparison between cells cultivated *in vitro* and cells taken from the living organism might therefore be expected to reveal different features.

The form of the long chromosomes is variable. The attachment constriction, which is one of the few definite and permanent characters of the individual chromosome, determines the form of the chromosome; but the avian chromosomes do not show the *loci* of the spindle-fibre attachment until the very end of metaphase (Plate III, figs. 4 and 5); and only when the chromosomes are arranged at the metaphase plate, and at the time of the separation of the two daughter halves, can the attachment constriction be located. It becomes necessary, therefore, when describing the form of the chromosomes, to state which particular stage is being examined, for even at metaphase there is still great variation. It is probably because the tissues were fixed at different stages of prophase and metaphase that the figures and descriptions of the chromosomes given by different investigators differ. The two longest chromosomes in the diploid complex show very clearly a submedian attachment constriction (Plate I, fig. 4), whilst others of the longer chromosomes have subterminal, terminal, and, in the case of one pair, a median attachment constriction.

Cytological differences between the male and the female complexes have been described by Guyer (1916), Hance (1924), and Shiwago (1924) The presence of two relatively very long chromosomes in the male and of one of the same length and form in the female, in association with a very small chromosome, is taken to indicate that the male is XX, the female XY. In the present study there were found in the males of the control birds two chromosomes which were the longest in the whole complex (Plate I, fig. 4) Their precocious condensation and their permanent structure with short and long limbs, the permanent submedian *locus* of the attachment constriction and the fact that they exhibited the phenomenon of succession at division (Plate III, fig. 6) were accepted as evidence that they were qualitatively different from the remaining chromosomes in the complex. In the female only one chromosome possessing the same properties was observed (Plate I, figs. 1–3). In oogenesis, as has been stated, only one long thread was found during prophase These are therefore regarded as the X-chromosomes. In the male they lie frequently side by side, although in certain instances they are arranged in the metaphase plate as mirror images. Gonomeric groupings, in which the parental chromosomes exhibit a tendency to remain apart, as described by Shiwago, were not identified in the present study.

In the tissues derived from the offspring of the exceptional bird, in certain metaphase plates, an unusually long chromosome was found. Its length, being calculated by means of camera lucida drawings, was found to be twice as long as the longest chromosome in the complex of the control birds. It is illustrated in Plate II, figs 4, 5, and 7, and Plate IV, fig. 1*b*. The most probable explanation of this long chromosome is that it consists of two, these being attached at the shorter limb or at the attachment constriction. In other cells, three instead of two members of the complex showed the characteristic submedian attachment constriction (Plate II, figs. 2, 3, 6, and 8), and the size of the extra chromosome exactly resembled the two longest ones, which were identified as the X-chromosomes in the male complex. These chromosomes, as did the extremely long ones, exhibited stronger condensation, they were thicker and they stained more deeply than the rest For these reasons they were regarded as the X-chromosomes

Among the control birds two types were identified: males with a variable number of small autosomes, but with two large X-chromosomes; and females with a variable number of small autosomes, but with one large X-chromosome. The Y-chromosome could not be recognised as such with any degree of certainty for the reasons that in size it is very similar to the small autosomes and that the number of these was variable.

Among the progeny of the exceptional male three types could be identified cytologically (in all the number of small autosomes was variable): XX males, X females, and XXX males. For the reasons given, the XXY male and the XYY female could not be recognised. The exceptional bird himself appeared to have the XX sex-chromosome constitution. His exceptional gold son also appeared to be XX, whilst in his exceptional daughters only one X was present. The XXX individuals were silver males.

The presence of trisomic diploids, 3X : 2A, in which the X was represented thrice instead of twice, among the progeny of this exceptional bird can be accepted as proof that non-disjunction was concerned in their origin, and that their sire, being exceptional in colour and behaving as a genetic gold in mating, was himself a non-disjunctional individual with the XXY sex-chromosome constitution.

Non-disjunction of the small autosomes would seem to be followed by no disadvantageous or recognisable repercussions, but non-disjunction of the large X-chromosomes, because of their relation to the sex-determining mechanism and to the distribution of the genes for the sex-linked characters, can be recognised by genetical peculiarities.

SUMMARY

An exceptional gold cock, *ex* Light Sussex × Rhode Island Red, when mated to Light Sussex females, produced 66 silver males, 2 silver females, 48 gold females, and 1 gold male. Cytological examination detected trisomic diploid silver males amongst the progeny. For the reasons that the number of the small chromosomes in the complex is variable, and that the Y cannot readily be distinguished from the small autosomes, it was not possible to identify any other cytologically exceptional types (XXY and XYY). The combined genetic and cytological evidence,

however, is sufficient to warrant the conclusion that this exceptional cock was an instance of secondary non-disjunction of the sex-chromosomes, having the constitution (sX) (sX)Y.

After the MSS. had gone to Press there appeared in the *Journal of Genetics* (vol. xxvi, No. 3, pp. 345–350, December 1932) a paper by Mr M. J. D. White on "The Chromosomes of the Domestic Chicken," in which the diploid number is given as 66±2. No Y-chromosome was identified· the male is XX, the female presumably XO. The largest autosomes seem very similar to the X-chromosome in size and shape The illustrations are impressive. However, it remains impossible to identify more than 32–35 chromosomes in my preparations, and differences in our counts must be referred to differences in our techniques. Further, it is the case that I did not pay very much attention to the small fragments of chromatin material. I have considered the possibility that I had mistaken a large autosome for an extra X-chromosome, and am satisfied that by reference to the site of the attachment constriction I have distinguished the one from the other.

BIBLIOGRAPHY.

AGAR, W., 1924. "Experiments with Certain Plumage Colour and Pattern Factors in Poultry," *Journ. Genet.*, vol. xiv, pp. 265–272.

AKKERINGA, L. J., 1927. "Die Chromosomen bei einigen Hühnerassen," *Zeitschr. f. mikr. anat. Forschung.*, vol. viii, pp. 325–342.

BORING, A M., 1923. "Notes by N. M Stevens on the Chromosomes of the Domestic Chicken," *Science*, vol. lviii, No 1491, pp. 73–74.

BRIDGES, C. B., 1916. "Non-disjunction as Proof of the Chromosome Theory of Heredity," *Genetics*, vol. i, pp. 1–163.

DARLINGTON, C. D., 1932. *Recent Advances in Cytology.* London Churchill.

GUYER, M. F., 1916. "Studies on the Chromosomes of the Common Fowl as seen in Testes and Embryos," *Biol. Bull*, vol. xxxi, pp. 221–225.

HANCE, R. T., 1924. "The Somatic Chromosomes of the Chick and their possible Sex Relations," *Science*, vol. lix, No 1532, pp. 424–425.

HANCE, R. T., 1926*a*. "Sex and the Chromosomes in the Domestic Fowl (*Gallus domesticus*)," *Journ Morph. and Phys.*, vol. xlii, pp. 119–145.

HANCE, R. T., 1926*b*. "A Comparison of Mitosis in Chick Tissue-culture and in Sectioned Embryos," *Biol. Bull*, vol. l, pp. 155–159

HAYS, F. A., 1926. "Inheritance of Plumage Colour in the Rhode Island Red Breed of Domestic Fowl," *Genetics*, vol xi, pp. 355–371.

MULLER, H. J., 1918. "Genetic Variability, Twin Hybrids, and Constant Hybrids in a Case of Balanced Lethal Factors," *Genetics*, vol. iii, pp. 422–499.

SHIWAGO, P. J, 1924. "The Chromosome Complexes in the Somatic Cells of the Male and Female of the Domestic Chicken," *Science*, vol. lx, No. 1541, pp. 45–46.

SHIWAGO, P. J, 1926. "The Chromosome Complexes in the Somatic Cells of the Male and Female of Domestic Chicken. Genetics of Domestic Fowl," *Mem. Anikowa Gen. Stn.*, pp. 75–76 (original Russian, p. 129; English summary).

TAYLOR, L. W., 1932. "An Inhibitor of Gold Colour in Chickens," *Proc. 6th Intern. Cong. Genetics*, pp. 259–260.

DESCRIPTION OF PLATES

PLATE I.

Figs 1, 2, and 3. Somatic chromosome complexes from the 17-day-old female embryo. (× 4·500.)

Fig. 4. Somatic chromosome complex from a 17-day-old male embryo. (× 4·500)

Fig. 5. Diploid complex of cock, after Shiwago. (× 3·500.)

Fig. 6. Diploid complex of hen, after Shiwago. The small Y-chromosome is depicted (× 3·500.)

Fig. 7. Diploid complex of cock, after Hance. (× 4·200.)

Fig. 8. Diploid complex of hen, after Hance (× 4·200)

Fig 9. Diploid complex of cock, after Stevens. (× 3·200.)

PLATE II.

Fig. 1. Somatic chromosome complex of a 5-day-old embryo, showing size differences. Two X-chromosomes can be identified (× 4·500)

Figs. 2 and 3. Somatic chromosome complex of a silver son of the exceptional bird. Three X-chromosomes are present. (× 4·500)

Figs. 4 and 5. Somatic chromosome complex from the same The two X-chromosomes are attached (× 4·500.)

Fig. 6. Somatic chromosome complex from embryo derived from exceptional bird. Three free X-chromosomes are to be found (× 4·500.)

Figs. 7, 8, and 9 Chromosome complex of embryo with three X-chromosomes. In fig. 7 two of the X's probably attached. (× 4·500.)

PLATE III.

Fig. 1. Haploid complex of a secondary spermatocyte, after Stevens The number of chromosomes is 21. (× 3·200.)

Figs. 2 and 3. Haploid chromosome complex of the secondary spermatocytes. The number is 16 and 15. (× 4·500.)

Fig. 4. Mitotic metaphase with long chromosomes and centrally arranged small chromosomes. (× 4·500.)

Fig. 5. Mitotic metaphase to illustrate the onset of metaphase. (× 4·500)

Fig. 6. Mitotic anaphase. The long chromosomes are delayed in their movements towards the poles (succession). They are regarded as the X-chromosomes. (× 4·500.)

Fig. 7. Diplotene bivalents representing the most frequent types during oogenesis. On the right, the single X-chromosome is depicted. (× 2·300)

PLATE IV.

Fig. *1a*. Microphotographs of somatic metaphase from a silver son of the exceptional bird. The three X-chromosomes can be seen. (× 2·300.)

Fig *1b* Microphotographs of somatic metaphase from the same. Two attached X-chromosomes are depicted. (× 2·300.)

Figs. *2a* and *3a*. Eggs with diplotene bivalents during oogenesis. The chiasmata are shown in figs. *2b* and *3b*. (× 1·500.)

Fig. *4a*. Diplotene stage, from the female. *4b* depicts the chiasma formation. (× 2·300)

PLATE V

Section of an ovary from a 3½-months-old bird The developing eggs differ in volume, but the diplotene stage is present in all Stained with new magenta crystal.

(*Issued separately March* 30, 1933)

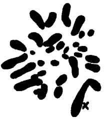


FIG. 1.

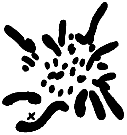


FIG. 2.

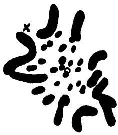


FIG. 3.

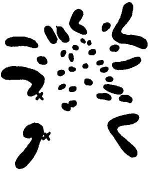


FIG. 4

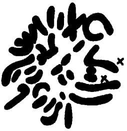


FIG. 5.

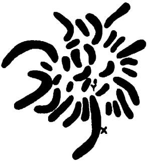


FIG. 6.

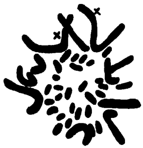


FIG. 7.

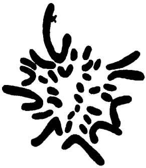


FIG 8.

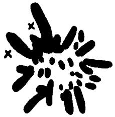


FIG 9.

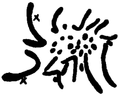


FIG. 1.

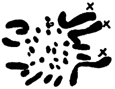


FIG. 2.

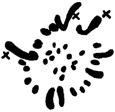


FIG. 3

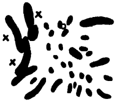


FIG. 4.

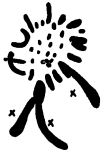


FIG. 5.

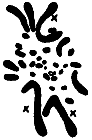


FIG. 6.

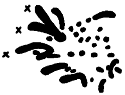


FIG 7

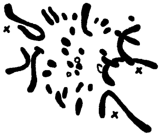


FIG. 8.

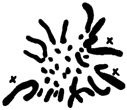


FIG. 9.

F. A. E. CREW.

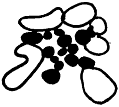


Fig. 1.

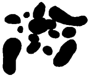


Fig. 2

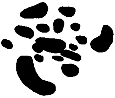


Fig. 3.

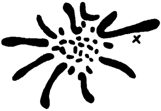


Fig. 4

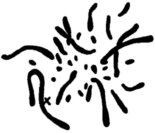


Fig. 5.

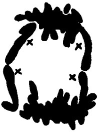


Fig 6.

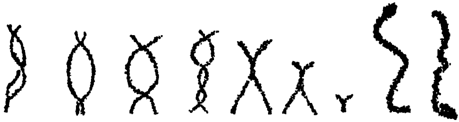


Fig. 7

Fig. 1*a*. Fig 1*b*.

Fig 2*a*

Fig. 3*a*

Fig 4*a*.

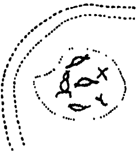


Fig. 2*b*.

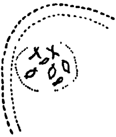


Fig. 3*b*.

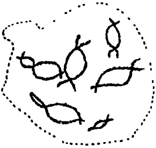


Fig. 4*b*.

F A E Crew. [Plate IV.

F A E CREW [PLATE V

IX.—The Intellectual Resemblance of Twins.

By **Louis Herrman**, M.A. (Research Assistant in the Department of Social Biology), and **Lancelot Hogben**, M A , D.Sc. (Research Professor of Social Biology in the University of London).

(MS. received December 8, 1932 Read February 6, 1933)

1. STATEMENT OF THE PROBLEM AND PREVIOUS WORK.

THE characteristics of social behaviour in man are conditioned by previous experience What is observed is the product on the one hand of a certain genetic constitution and on the other of an intricate, prolonged, and at present largely obscure, process of training and physical environment, including both the environment of the fœtus and family influences, social and physical. The experimental methods for detecting differences due to single gene substitutions cannot be applied directly Indeed, we can see no immediate prospect of applying to social behaviour methods of genetic analysis such as have led to the mapping of the chromosomes in animals and in plants. With methods available at present, genetic inquiry can undertake to detect whether any gene differences are associated with observed differences, and whether such gene differences are recognisable throughout a comparatively wide or narrow range of social and physical environment.

The first prerequisite for such an inquiry is a quantitative method for describing some particular aspect of social behaviour Ideally such a method should possess two characteristics These are, first, high reliability—that is to say, a close measure of correspondence between repeated tests on the same individual examined successively within a short interval of time—and second, ability to yield an index characteristic for the individual measured over a comparatively long period of time. The only descriptive methods applicable to the social behaviour of human beings at present capable of fulfilling both requirements are tests of the type originally devised by Binet, and subsequently modified by a large school of workers, including Spearman, Terman and Burt among others. Given a quantitative method of high reliability for describing any characteristic or characteristics of social behaviour, the problem stated above suggests inquiry along several different lines.

In surveying these several lines of inquiry it is first necessary to distinguish between gene differences which are recognisable throughout the range of environment to which members of the same fraternity are exposed and gene differences which are recognisable throughout the whole range of environment to which members of different families are exposed. Within the first group it is necessary to draw a further distinction. Members of the same family may have the same birth-rank (when plural births occur), or, as is more usually the case, the birth-rank of two members of the same family may be different. Presumably such differences of environment operating upon different individuals within the same family unit are less in the first case than in the second. Different methods of investigation are appropriate to each type of problem.

1. To detect gene differences associated with observed differences between individuals of the same birth-rank brought up together in the same family, a comparison of the resemblance of identical and non-identical twins may be made

2. A comparison of the resemblance between ordinary sibs * and fraternal twins may be used to determine how far environmental differences associated with different birth-rank limit the exhibition of gene differences between individuals belonging to and brought up in the same family

3. For elucidating gene differences which manifest themselves as observed differences between individuals belonging to different families at different social levels, it is much more difficult to obtain the requisite data. Two methods assist us to arrive at a conclusion. One is to standardise the environment of individuals who are not genetically related. The other is to vary the environment of individuals who are. The first is that employed by Freeman, Holzinger and Mitchell (4), who have placed on record correlations between adopted children and sibs of the families into which they have been adopted. The second is to compare twins or sibs reared apart. No adequate sample of twins reared apart has yet been studied in this connection Freeman, Holzinger and Mitchell (*op cit.*) have recorded correlations for Intelligence Quotients of sibs adopted into different homes

The present inquiry is concerned with the first two, and more especially the first of the three problems stated in the preceding paragraph There have been several previous investigations of this kind. The results in the form of correlation coefficients are summarised in Table I. From this it will be seen that the numbers hitherto examined have not been so large as to dismiss the need for fuller investigation. Two points upon

---

* Here and throughout sib is used to denote brothers and sisters of different birth-rank. Increase in crossing over as the age of the parent advances must also be taken into consideration in discussing birth rank

TABLE I

Author	Test	Like-sex.		Like-sex			Mixed Pairs	All Fraternal (Like and Mixed)
		Identical	Fraternal	Boys	Girls	All Like-sex.		
Merriman (1924)	Stanford-Binet 5–9 years			0·80 ± 0·08 (10)	0·91 ± 0·03 (19)	0·88 ± 0·03 (29)	0·77 ± 0·06 (18)	
	Stanford-Binet 10–16 years			0·89 ± 0·03 (17)	0·81 ± 0·05 (21)	0·86 ± 0·03 (38)	0·30 ± 0·14 (20)	
	Beta 5–9 years			0·93 ± 0·05 (7)	0·71 ± 0·10 (9)	0·92 ± 0·03 (16)	0·52 ± 0·15 (12)	
	Beta 10–16 years			0·75 ± 0·08 (13)	0·89 ± 0·03 (16)	0·84 ± 0·04 (29)	0·64 ± 0·09 (19)	
	National Intelligence 5–9 years			0·92 ± 0·04 (7)	0·96 ± 0·01 (24)	0·95 ± 0·01 (31)	0·75 ± 0·07 (23)	
	National Intelligence 10–16 years			0·89 ± 0·03 (24)	0·92 ± 0·02 (37)	0·86 ± 0·02 (61)	0·83 ± 0·04 (28)	
Lauterbach (1925)	Terman			0·81 ± 0·03 (71)	0·73 ± 0·04 (63)		0·56 ± 0·06 (78)	
Wingfield (1928)	Average of various	0·90 ± 0·02 (45)				0·82 ± 0·03 (76)	0·59 ± 0·08 (26)	0·70 ± 0·04 (57)
Holzinger (1929)	Binet	0·88 ± 0·02 (50)	0·63 ± 0·06 (52)					
	Otis	0·92 ± 0·01 (50)	0·62 ± 0·06 (52)					
Herrman and Hogben (1932)	Otis (Corrected by Spearman formula for attenuation)	0·86 ± 0·04 (65)	0·48 ± 0·08 (96)	0·69 ± 0·04 (126)	0·63 ± 0·05 (141)	0·66 ± 0·02 (267)	0·53 ± 0·06 (138)	

(The figures in brackets denote the number of pairs tested.)

which previous researches suggest more careful inquiry will be evident from the facts summarised therein. The first is that no previous publication has recorded data bearing upon the resemblance of both twins and ordinary sibs tested with the same test, at the same time, under the same conditions, and by the same observer. The second is that no previous investigation has simultaneously compared the resemblance of fraternal twins of unlike sex and of dizygotic twins of the same sex. The significance of the first issue has been stated already That of the second demands further elucidation

Differences in social behaviour between fraternal twins of like sex and fraternal twins of unlike sex may arise either from differences in environment or from differences in genetic constitution hitherto overlooked As regards the first we have no reason to suppose that the social environment of twins of opposite sex will be quite as similar as the social environment of boy pairs or girl pairs. Therefore it is important to ascertain whether in fact fraternal twins of unlike sex are as much alike as fraternal twins of the same sex. Clearly if they are, environmental differences of this category do not significantly affect observed differences in the behaviour of twins The second issue has recently been pointed out by one of us (Hogben, 6) The fraternal correlation arising when transmission is sex-linked differs according to whether the sibs compared are boy pairs, girl pairs, or mixed pairs. If an appreciable amount of variability arises from sex-linked genes the observed correlation for mixed pairs will be lower than for either boy pairs or girl pairs, the last-named being highest of all The theoretical values are briefly summarised in Table II, in which the figures refer to the case of a single gene substitution and the values for complete dominance refer to equal frequencies of the allelomorphic genes.

TABLE II —CORRELATION COEFFICIENTS FOR FRATERNAL RESEMBLANCES.

	Dominance Complete		Heterozygote Intermediate.	
	Autosomal	Sex-linked	Autosomal.	Sex-linked
Brother brother	0 42	0 50	0 50	0 50
Brother sister	0 42	0 29	0·50	0 35
Sister sister	0 42	0 67	0·50	0·75

In the present inquiry special attention has been directed to the following points:—

(*a*) To secure a sample of dimensions adequate to separate from the entire group of like-sex twins a large group of individuals exclusively belonging to the dizygotic type for comparison with twins of mixed sex.

(*b*) To test a comparable group of sibs under the same conditions, all individuals of the same pair of sibs or twins being tested simultaneously by group tests

(*c*) To preserve the strictest scrutiny of resemblance with respect to the sexes of the pairs studied, and, if sibs, of birth-rank also

(*d*) To utilise Stocks' finger-print criterion for the diagnosis of fraternal and identical twins.

2. DIFFERENTIAL DIAGNOSIS OF MONOVULAR AND DIOVULAR TWINS.

Organised co-operation between a number of large hospitals willing to undertake an autopsy for the detection of corpora lutea in the ovaries of mothers of twins dying in childbirth might supply the requisite data to establish the genesis of twins In the meantime there is no direct evidence for the view that twins of the same sex are of two kinds, the individuals of one type being formed in the usual way from separate ova, and the separate individuals of the other type from one and the same ovum. Of late this belief has been placed upon a more satisfactory foundation than it formerly enjoyed by two types of statistical inquiry. One is embodied in Fisher's analysis of Lauterbach's data (3) It will be referred to under section 5 The other is embodied in the extremely important contribution of the recent work of Dr Stocks (12). Stocks approached the issue in the following way Two main types of data have strengthened the belief in the monovular and diovular origin of identical twins One is the displacement of the ratio of like-sex to mixed-sex pairs from what would be expected if all twins were of diovular origin. The other is the existence, within the like-sex group, of twins who are much more alike than ordinary brothers or sisters Evidently the hypothesis stated above is greatly strengthened if both considerations can be harmonised on a quantitative basis. This is what the work of Stocks has succeeded in doing.

If the hypothesis that identical twins are of monovular origin is correct, the statistics for the respective frequencies of like and mixed-sexed pairs permit us to predict what proportion—roughly a half in European countries —of all like-sexed pairs belong to the identical type It should also be possible to divide all twins of like sex into two groups, one of which yields a higher correlation than the other for any measurable characteristic whatsoever. The former group must form the same fraction of the whole as the proportion of identical twins in the like-sex group predicted on the basis of the sex ratio. Clearly it is a matter of convenience what characteristic we select to separate the twins of like sex into two such groups. Stocks found that finger-print resemblance of mixed pairs never exceeded a certain figure which permitted him to separate the like-sex twins into two classes. One of these yielded correlations in the neighbourhood of unity and the other correlations of the fraternal order of magnitude for a wide variety of anatomical and physiological measurements. The magnitude of the two classes in his inquiry corresponds with remarkable fidelity to what would be predicted from the sex ratio.

The results of Stocks' inquiry show that in his own hands the finger-print method of diagnosis is a most valuable instrument For that reason the present writers decided at the outset to take finger-prints of all the children tested in this investigation. The criterion Stocks prescribes is stated thus in his monograph: "Compare the corresponding finger-patterns on the same-sided hands of the two sets and mark off those which are similar, giving a total of α similar pairs.

" Examine the remainder one by one for similarity with the corresponding finger of the opposite-sided hand of the other child, provided that that finger has not already been marked, and add the number of similar pairs of fingers so found to α, giving the total β, in looking for crossed similarity one pattern must, of course, be the mirror image of the other.

"Compare the corresponding finger-patterns of each child separately, left with right, and count the total pairs γ, of similar pattern, allowing for reversal of pattern.

"If $\beta = 7$ or more, or alternatively if β is greater than γ, the twins may be regarded as monozygotic; if $\beta = 6$ or less *and* β is equal to or less than γ, they may be regarded as dizygotic."

The standard of similarity which Stocks adopted is stated thus: Similar finger-prints "are not only of the same class (arch, whorl, loop), but are so much alike as regards general configuration, inclination of axes, position of deltas, and number of ridges as to make *them appear the same to a casual examination, without counting the ridges or looking for minutiæ.*"

Whether the finger-print method of diagnosis can yield the same clear-cut results in the hands of another investigator depends upon how far the specification of similarity given by Stocks is sufficiently definite to eliminate personal idiosyncrasies in making a "casual examination" which does not involve actually counting the ridges or looking for minutiæ. Stocks examined 52 mixed-sex pairs of whom only 3 had more than 5 fingers alike on corresponding hands, and none of them had more than 6 similar patterns on corresponding fingers of corresponding hands. Roughly, half of the like-sex pair twins had more than 5 pairs of similar finger-prints.

The present writers have examined a somewhat larger sample and have included in their survey a control group of ordinary sibs. Table III gives the mean number of finger-print resemblances on corresponding hands, together with the number of individuals in each class examined. The criterion adopted was, as nearly as the specification permits, that which Stocks prescribes. It will be seen that there is a significantly greater resemblance in the finger-print patterns of twins of like sex as compared with twins of unlike sex, and that there is both among twins and sibs closer similarity between girls than boys, possibly indicative of the contribution of sex-linked genes to finger-print differences. The frequency distributions are given in percentages in Table IV. From this it is seen that the dispersion for mixed-sex pairs of twins or for ordinary sibs is much more pronounced than it was in the sample examined by Stocks. This table refers only to resemblance in fingers of corresponding

TABLE III.—FINGER-PRINT RESEMBLANCES (SAME HAND).

		Mean	Standard Error of Mean	No. Examined.
Twins	Boy pairs	5·66	±0·19	133
	Girl pairs	6·14	±0·20	148
Sibs:	Boy pairs	4·05	±0·25	66
	Girl pairs	4·54	±0·29	57
Twins	Like-sex pairs	5·91	±0·04	281
Sibs:	Like-sex pairs	4·27	±0·16	123
Twins.	Mixed-sex pairs	3·97	±0·18	143
Sibs	Mixed-sex pairs	4·73	±0·21	97

TABLE IV.—FREQUENCY DISTRIBUTION OF FINGER-PRINT RESEMBLANCES (IN PERCENTAGES)

		0	1	2	3	4	5	6	7	8	9	10
Twins:	Boy pairs .	0	3·8	5·3	9·0	10·5	15·0	21·0	14·3	8·3	9·0	3·0
	Girl pairs.	0	5·4	3·4	6·8	12·8	10·8	9·4	16·9	17·6	8·1	8·8
	Like-sex pairs .	0	4·6	4·3	7·8	11·3	13·2	14·9	15·7	13·2	8·5	5·1
	Unlike-sex pairs	4·2	8·4	13·3	17·5	17·5	15·4	10·5	8·4	3·5	1·4	0
	All twin pairs .	1·4	5·9	7·3	11·9	13·7	13·9	13·4	13·2	9·9	6·1	4·0
Sibs:	Boy pairs. .	4·5	7·6	10·6	18·2	19·7	12·1	13·6	9·1	4·5	0	0
	Girl pairs. .	3·5	5·3	10·5	10·5	19·3	21·1	8·8	10·5	7·0	3·5	0
	Like-sex pairs .	4·1	6·5	10·6	14·6	19·5	16·3	11·4	9·7	5·7	1·6	0
	Unlike-sex pairs	2·1	4·2	7·2	14·4	15·4	23·7	16·5	5·1	7·2	3·1	1·0
	All sibling pairs	3·2	5·5	9·1	14·5	17·7	19·5	13·6	7·7	6·4	2·3	0·5

hands. The difficulty of separating twins into two classes by finger-print diagnosis in this investigation is brought into prominence in Table V.

TABLE V.—PERCENTAGE FINGER-PRINT RESEMBLANCES IN TWINS AND SIBLINGS.

No. of Fingers Similar.		Under Seven	Six	Over Six
Twins	Boy pairs	64·6	21·0	34·6
	Girl pairs	48·6	9·4	51·4
	Mixed pairs	86·8	10·5	13·3
Sibs	Boy pairs	86·3	13·6	13·6
	Girl pairs	79·0	8·8	21·0
	Mixed pairs	83·5	16·5	16·4
Twins	Like-sex pairs	56·6	14·9	43·0
	Unlike-sex pairs	86·8	10·5	13·3
Sibs	Like-sex pairs	82·7	11·2	17·3
	Unlike-sex pairs	83·5	16·5	16·4

No criticism of the work of Stocks, whose investigation merits the highest commendation, is implied in the statement that the present writers could not succeed in using the finger-print diagnosis to effect such a clear-cut separation as he was able to do. *Prima facie*, there is no reason why any single criterion should be able to distinguish all identical twins of the same sex. On the other hand, a single criterion may be used to separate from the entire class of like-sex twins a group of pairs all of whom are dizygotic. In taking finger-prints of children who were tested in the present inquiry, other physical similarities, height, eye and hair colour, ear and dental configuration were carefully noted, and the entire group of twins of like sex were subdivided into four classes showing different grades of similarity independent of finger-print pattern. The finger-print distribution of the individuals in each of these classes is given in Table VI. This table is divided into four sections. Henceforth individuals belonging to the bottom section on the left hand of the table will be referred to as the "D" group. Individuals belonging to the top section on the right-hand side will be referred to as the "M" group of like sex-twins. The "D" group consists of individuals with less than six similar finger-patterns on the same hand and a complex of striking physical dissimilarities in other particulars. The "M" group consists of individuals all of whom display more than six finger-print similarities and somewhat less uniformity in other criteria of physical similarity. On the basis of the generally accepted hypothesis it seems justifiable to suppose that the "D" group so defined will be exclusively composed of dizygotic pairs, and that the "M" group will contain a very high proportion of monozygotics with an appreciable dilution of the dizygotic type. In fact, both these suppositions are found to be consistent with the results obtained from the application of Fisher's method to the analysis of the intelligence test data.

TABLE VI.—RELATION OF FINGER-PRINT SIMILARITY TO GENERAL RESEMBLANCE.

	No. of Finger-prints Similar.										
	0	1.	2	3.	4	5	6	7	8	9	10.
I. "Identical" .	0	0	0	1	0	3	10	17	18	19	13
II. "Probably" identical	0	0	0	0	0	3	1	4	4	1	3
III. "Possibly" identical	0	1	1	1	2	3	2	3	2	2	1
									"M"		
			"D"								
IV. Non-identical . .	0	12	11	20	31	28	29	20	11	3	0

TABLE VII.—AGE DISTRIBUTION OF TWINS

		Years									
Like-sex Pairs		7–8	8–9	9–10	10–11	11–12	12–13	13–14	14–15.	15–16.	Totals
"M"	Boys	..	8	5	8	4	9	5	...	...	39
	Girls .	2	7	9	18	14	4	5	1	1	61
"D"	Boys .	1	5	13	6	15	8	5	1	.	54
	Girls .	1	6	10	7	10	10	5	2	...	51
"X"	Boys .	1	5	4	8	12	6	4	..	..	40
	Girls .	1	7	9	5	9	5	2	1	1	40
All boys	.	2	18	22	22	31	23	14	1	.	133
All girls .	.	4	20	28	30	33	19	12	4	2	152
All like sex.	.	6	38	50	52	64	42	26	5	2	285
All unlike sex	.	..	14	23	17	40	29	14	2	..	139
Totals	.	6	52	73	69	104	71	40	7	2	424

3. SAMPLE OF CHILDREN EXAMINED

The children examined were elementary school pupils in the London County Council area. After a preliminary survey of the twin population of the L.C.C. schools, the schools were selected if they contained six or more pairs of twins of the requisite ages. The Intelligence Group Tests employed were suitable for normal children of eight years of age or over. The bulk of the children examined were between 8 and 14. The age distribution of the twins is given in Table VII. The like-sex pairs are classified as boy or girl pairs in three groups. "M" and "D" have been previously defined. "X" represents the remainder of the like-sex pairs that do not come within these two classes.

The distribution of ages is of especial importance in connection with siblings As it is very difficult to attain a standard of perfection represented by zero correlation between any individual index, such as Intelligence Quotient, and Age, it is highly desirable when comparing pairs of individuals to minimise age differences as far as it is practicable From Table VII, which gives the mean ages of each pair of sibs examined, together with the interval between the birth of each pair, it will be seen that 90 per cent of the sibs examined did not differ by more than three years, and the mean age of rather more than half of the pairs lies between nine and twelve years of age Roughly, two-thirds of the sib pairs differed by less than two years in age.

4. THE TESTS EMPLOYED.

The Group Test selected was the Otis Advanced (Form A). This includes ten categories of performances: 1. Directions. 2. Word Opposites. 3. Pied Sentences. 4. Proverbs. 5. Numerical Problems. 6. Geometrical Figures. 7. Analogies. 8. Formal Similarities. 9. Narrative Completion 10 Verbal Memory. The Otis Group Test was selected for two principal reasons One is its high coefficient of reliability, which is given by Kelley as 0·97. The other is the very extensive tests on which the published norms are based. The age distribution of the scores for the Otis tests have been published for a sample of 25,226 unselected pupils from 200 cities of the United States. The Intelligence Quotient (I.Q) referred to in what follows is the ratio of the Mental Age to the Chronological Age expressed as a percentage, Mental Age being defined as the age corresponding to the score recorded in the published norms. The justification for using this index resides in the extent to which it yields a value independent of age for one and the same individual. The first prerequisite of such an inquiry as this is therefore to ascertain the correlation between the I.Q calculated on the basis of the American norms and the ages of the L.C.C. school children to whom the test was administered. In this investigation each pair of sibs was entered upon a card, one member on the left-hand side, the other on the right. Since the test scores of some individuals were below the lowest figure for which an age norm is given, there is a slight discrepancy between the number of individuals respectively included in the contingency tables for age and I.Q. for the right-hand and left-hand members of a pair. The correlations obtained are given in Table IX.

TABLE VIII —SIBLINGS—AGE DISTRIBUTION

Age Groups.	8 to 10.	10–1 to 12	12–1 to 14	14–1 to 16.	Tot	8 to 10.	10–1 to 12.	12–1 to 14	14–1 to 16	Tot.	8 to 10.	10–1 to 12.	12–1 to 14.	14–1 to 16	Tot	8 to 10.	10–1 to 12	12–1 to 14.	14–1 to 16.	Tot.	Totals
Interval.	Unlike Sex.					Boy Pairs.					Girl Pairs					Like Sex.					All Sibs.
0–1	1	2	...	..	3	1	...	1	1	3		...	2		2	1	...	3	1	5	8
1·1–2	16	25	22	1	64	15	22	13	2	52	9	19	12	1	41	24	41	25	3	93	157
2 1–3	5	9	8	2	24	8	4	5	1	18	4	5	9	1	19	12	9	14	2	37	61
3·1–4	...	7	4	...	11		4	2	.	6	.	3	1	..	4		7	3	. .	10	21
4·1–5	.	..	...	. .	0	...	3	...		3	..	1	1	.	2		4	1	.	5	5
5 1–6	. .	1	..	..	1	...	...		.			.	.	.			.	...		. .	1
	22	44	34	3	103	24	33	21	4	82	13	28	25	2	68	37	61	46	6	150	253

Percentage under 3 years' interval {
Unlike sex, 88 3
Like sex, 90·0
All sibs, 89 3

TABLE IX.—CORRELATION OF AGE AND I.Q.

Group	*r*	*σ*	N
A. All children tested (twins or sibs)—			
(i) Right-hand members	-0·16	±0 04	570
(ii) Left-hand "	-0 12	±0 04	566
B Sibs—			
(i) Younger members	-0·08	±0 07	213
(ii) Elder "	-0 35	±0 06	211

For the children as a whole the correlation between age and I.Q. does not differ from zero so much as to introduce a serious error in treating the I.Q. as constant for age in the subsequent analysis. It will be noted, on the other hand, that the correlation for the group of elder sibs is significantly high. The reason for this is not far to seek. The method of selection does not yield a strictly random sample of the population for two reasons. Brighter children leave the elementary schools for Central or Secondary schools at about twelve years of age, so that the children between twelve and fourteen may be expected to be a relatively backward group A second selective feature in the inquiry arises from the fact that the published norms for the test begin at eight years of age. Hence very few subnormal children can be included in the eight-year old class. Both these selective factors are brought clearly into prominence in the graph (fig. 1) of Age and mean I Q. for each age group in half-yearly intervals This shows more clearly than the correlation coefficients given in the preceding table the independence of I.Q. and Age within the range of nine to twelve years The importance of this consideration will be recognised later in dealing with sib correlation for I.Q. A second issue which demands some attention is the magnitude of the error arising out of the reliability of the test, *i.e.* the correlation between successive tests on the same individual carried out within a short interval of time. In this investigation at each school visited a corresponding number of pairs of twins and pairs of siblings were examined simultaneously. Thus in all cases both individuals of a pair of either type were examined by the same test on the same occasion under the same conditions of illumination,

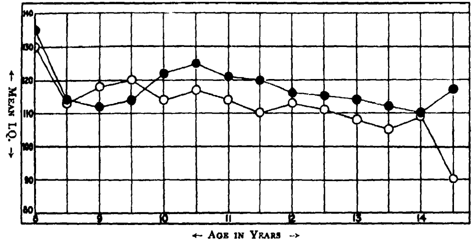


FIG. 1.—Two samples respectively of 576 (white circles) and 570 (black circles) unrelated children. I Q. plotted against mean age in six monthly class intervals

ventilation, temperature, humidity, etc. The same examiner carried out the testing throughout the investigation In these circumstances it is doubtful whether it is appropriate to apply Spearman's correction for attenuation. Actually the point is not of very great importance because of the very high reliability of the tests. In Table I the coefficients given have been adjusted by the use of Spearman's formula Elsewhere in this publication no such correction has been made.

5. HOMOGENEITY OF THE DATA.

If the monovular hypothesis concerning the origin of identical twins is correct, like-sex twins constitute a heterogeneous group. Therefore, in so far as the observed differences with respect to a given measurement made on both members of a mixed pair of twins or a pair of sibs of like sex approximate to a normal distribution, a standard relation between the mean deviation and the mean square deviation for the Gaussian curve may be used to demonstrate the heterogeneous character of a group of like-sex pair twins. This fruitful suggestion was first made by Fisher (3), and used by him in the analysis of Lauterbach's data.

The relation which exists between the mean difference and the mean square difference in a normal population is given by

$$\frac{\Sigma\delta^2}{n} = \frac{\pi}{2}\left[\frac{\Sigma\delta}{n}\right]^2,$$

where δ is the difference of any one pair found by subtracting the smaller measurement from the greater. If the population is homogeneous the difference between the quantities on the left- and right-hand side of the equation should not differ significantly from zero. For testing the significance the standard deviation is given by:

$$\sigma = \frac{\overline{d^2}}{\sqrt{n}} \cdot \sqrt{2\pi - 6}, \qquad \text{where } \overline{d^2} = \frac{\Sigma\delta^2}{n}.$$

In Table X the mean square differences, etc. for the I Q. measurements of each class of twins and sibs are severally given. As far as they go the results are consonant with Fisher's analysis of Lauterbach's physical measurements. That is to say, the difference between the left- and right-hand expressions in the relation given above differs from zero by less than one and a half times the standard deviation for the group of twins of unlike sex and for the groups of sibs of like and of unlike sex. For the group of like-sex twins it is nearly two and a half times its standard deviation. When the classes are still further subdivided according to sex, the contrast is not so striking; but the ratio of the difference to its standard deviation for both boy pairs of twins and girl pairs of twins is still greater than for mixed twins or for boy, girl, or mixed pairs of sibs of different ages. One striking fact brought out by the table is that the "D" pairs of like-sex twins as previously defined are relatively more homogeneous than the "M" pairs.

TABLE X.—MEAN DIFFERENCES IN I.Q.

	$\frac{\Sigma\delta}{n}$	s.d of $\frac{\Sigma\delta}{n}$.	$\frac{\Sigma\delta^2}{n}$	$\left[\frac{\Sigma\delta}{n}\right]^2$	$\Delta = \frac{\Sigma\delta^2}{n} - \frac{\pi}{2}\left[\frac{\Sigma\delta}{n}\right]^2$.	$\frac{\Delta}{\sigma}$	σ	n.
A Twins								
(*a*) Boy pairs	16·8	±1·2	467·1	282·2	+24·1	1·1	22·1	126
(*b*) Girl pairs	13·3	±0·9	295·8	176·9	+18·1	1·4	13·2	141
(*c*) Mixed pairs	17·9	±1·2	522·3	320·4	+19·3	0·9	23·5	138
B. Sibs								
(*a*) Boy pairs	23·3	±2·1	895·5	542·9	+43·2	0·8	52·6	82
(*b*) Girl pairs	18·4	±1·7	529·9	338·6	− 1·7	0·0	33·1	68
(*c*) Mixed pairs	21·2	±1·7	721·5	449·4	+16·0	0·4	37·8	103
C Twins.								
(*a*) "M" pairs	10·3	±0·9	192·0	106·1	+25·4	2·4	10·5	95
(*b*) "D" pairs	17·7	±1·5	523·3	313·3	+31·4	1·1	28·4	96
D Twins and Sibs								
(*a*) Like-sex twins	14·9	±0·8	376·7	222·0	+28·2	2·3	12·2	267
(*b*) Like-sex sibs	21·1	±1·4	729·8	445·2	+20·9	0·7	31·7	150
E Twins and Sibs								
(*a*) Unlike-sex twins	17·9	±1·2	522·3	320·4	+19·3	0·9	23·5	138
(*b*) Unlike-sex sibs	21·2	±1·7	721·5	449·4	+16·0	0·4	37·8	103

Two issues suggested by the data in Table X demand closer scrutiny. The first is how far the mean difference for the "M" pairs differs from what would be the mean difference of a completely homogeneous group of monozygotic twins. The second is to what extent the higher values of the sibling pairs are due to imperfect correlation between I.Q. and Age at the limits of the age scale. For this reason it was thought desirable

1. To separate from the "M" group the sixty-five pairs which were judged to be most certainly identical. These sixty-five pairs are taken from sixty-seven individuals having more than six finger-print resemblances in the top row of Table VI, two being rejected because they did not yield an I.Q.
2. To separate from the sibs a mixed group of which both members of a given pair were between the ages of eight and a half and twelve years. The total number of pairs in this group was 103.

The final result of this selection is given in Table XI. The data embodied in it is reserved for subsequent discussion.

TABLE XI.—MEAN DIFFERENCES IN THE I Q. OF SELECTED GROUPS.

Monozygotic twins (65)	9·2 ± 1·0
Dizygotic twins of like sex (96)	17·7 ± 1·5
Twins of mixed sex (138)	17·9 ± 1·5
All pairs of sibs between 8½ and 12 years (103)	16·8 ± 2·3

6. CORRELATION FOR THE INTELLIGENCE QUOTIENT OF TWINS.

In making correlation tables for the I.Q.'s of twins and siblings care was taken to eliminate a possible source of error by alternating the position in the rows and columns of the higher and lower scores. To facilitate the utilisation of data here presented by subsequent workers it has been thought desirable to tabulate the coefficients of correlation both for the raw scores and I.Q. as defined elsewhere. The sibling coefficients will be dealt with in a subsequent section after consideration of order of birth.

TABLE XII.—CORRELATION COEFFICIENTS FOR TEST SCORES AND I.Q OF TWINS

	Raw Score	No Tested	I Q	No Tested
A. Like sex—				
" Monozygotic " pairs	.	65	0·84 ±0·04	65
" M " pairs	0·87 ±0·02	96	0·81 ±0·03	95
" D " pairs	0·75 ±0·04	100	0·47 +0·08	96
All like-sex pairs	.	..	0·64 ±0·02	267
Boy pairs	0·79 +0·03	133	0·67 ±0·04	126
Girl pairs	0·80 ±0·03	141	0·61 ±0·05	141
B. Mixed pairs	0·63 ±0·05	139	0·51 ±0·06	138

In Table XII there are two features that merit a word of explanation. The group referred to as "monozygotic" as distinct from the "M" pairs are the same individuals as were judged to be monozygotic for the purpose of assigning a minimum estimate for the mean difference in I.Q. of identical twins in the preceding section. The numbers of individuals included in the computation of the coefficients for raw scores and for I.Q. do not tally in every case. This is because the contingency table for Raw Scores in some cases included pairs one member of which scored less than the requisite total corresponding to the minimal age in the scale of norms used to evaluate the I Q. The final result of this part of the inquiry is summarised in Table XIII, which also includes the correlation for Intelligence Quotient of siblings found in this investigation.

TABLE XIII.—CORRELATION COEFFICIENT FOR I.Q OF TWINS

Monozygotic twins (65)	0·84 ±0·04
Dizygotic twins of like sex (96)	0·47 ±0·08
Dizygotic twins of unlike sex (138)	0·51 ±0·06
Sibs (like or mixed pairs) (103)	0·32 ±0·09

7. INFLUENCE OF ORDER OF BIRTH.

The correlation between siblings has been reserved for separate treatment for reasons already stated. Siblings differ from fraternal twins in that the constituent individuals of a pair differ in birth rank and are separated by an interval of years. Hence any contribution to the observed variance in virtue of differences in birth-rank or errors arising through imperfect correlation between the Intelligence Quotient and Age call for special attention. Owing to the relatively low fertility of the social group from which these children were selected, it transpired that, with one exception, one member of each pair examined was a first-born offspring. Hence any effect due to the handicapping or otherwise of the first-born must influence the results obtained considerably. The birth-rank of the individuals included in the sibling group examined is given in Table XIV.

TABLE XIV.—SIBLINGS—ORDER IN FAMILY OF PAIRS IN THE DIFFERENT GROUPS

	First and Second	First and Third	First and Later	Second and Third	Second and Later	Third or Later and Fourth or Later.	Totals
Boys	74	3	5	0	0	0	82
Girls	64	4	0	0	0	0	68
All like-sex pairs	138	7	5	0	0	0	150
Unlike-sex pairs	94	7	1	0	0	1	103
All sib pairs .	232	14	6	0	0	1	253

Hitherto the influence of birth-rank upon the I.Q has been studied very little. Reference to the importance of recent work by Thurstone and Jenkins (13) will be made at a later stage The influence of birth-rank is sufficiently well established in connection with characteristics which are closely dependent upon or associated with the Intelligence Quotient to emphasise the need for very special attention to the problem. The observations which emerge from the present inquiry are summarised in Table XV.

As would be expected, the means here exhibited represent a group of children above the average This is not surprising in view of the fact that no children of decidedly retarded type can be included in the lower-age groups. The results show a significant falling off of the second-

TABLE XV.—MEAN I.Q. FOR CHILDREN OF DIFFERENT BIRTH-RANK IN THE SIBLING GROUP

First-born.		Second-born.		Third or Later.	
Boys. Mean s.d.	Girls. Mean s d.	Boys. Mean s d.	Girls. Mean s.d.	Boys Mean s d.	Girls. Mean s.d
119·3 ± 2·2	115·6 ± 2·0	114·9 ± 2·1	108·3 ± 1·9	114·6 ± 9·0	96·3 ± 15·0
Combined value for boys and girls					
117·5 ± 1·5		111·8 ± 1·4		108·5 ± 8·0	

born as compared with the first-born. Of the third-born nothing can be stated, because the number on which the mean value is based is small and hence the standard error of the mean is large. The difference between the mean values for first-born and second-born children is approximately 2·7 times its standard deviation. If the odds in favour of significance are not overwhelming on the figures as they stand, there are other good reasons for regarding the difference recorded as an underestimate rather than an exaggeration of what would be recorded for a more representative sample. The selection of data would tend to favour a higher I.Q. for the second-born than for the first, because the second-born constitute a younger-age group It has already been pointed out that the elder group would be expected to contain relatively more retarded and relatively less bright children than the younger group. This is evident from consideration of the way in which the material was selected, from the negative correlation for age and I.Q., and from the graphical presentation of the data given in fig. 1. Hence a more representative sample might be expected to yield a rather higher figure for the first-born and a rather lower figure for the second-born.

8. CORRELATION OF I Q. FOR SIBLINGS.

The correlations obtained for the Intelligence Quotients of siblings by various authors vary considerably. Dr Kate Gordon's data for orphan children, analysed by Elderton (2), yielded a value of 0·53 This was based upon orphanage children, and it may well be that any social factors involved in birth-rank are eliminated in an institutional upbringing. Pintner (10) gives a value of 0·33. The values hitherto recorded are mostly based upon the Binet test, and range between the figure given by Pintner and so high a value as 0·68 by Hildreth (5). Apart from differences arising from the mode of upbringing of the children studied and error due to a significant age variation of the I.Q., a further reason for disagreement may be sought in positive correlation of unrelated children brought up in the same school when the material is selected in such a way as in the present investigation, which includes children taken from 88 different schools. This would tend to raise rather than to lower the observed correlation.

An analysis of all the data derived from the sibling pairs examined in the present inquiry yields coefficients of correlation for the Otis Advanced examination as set forth in Table XVI Since the figures recorded for various types of pairs classified according to sex were low, it was thought desirable to take a group of sibs, excluding the youngest and the eldest. It has already been shown that variations of the I.Q. with age may be treated as negligible within the range of eight and a half to twelve years. For pairs of sibs of which both members fall within these limits the figure obtained agrees with that given by Pintner and is significantly lower than Elderton's value for a group of children brought up under institutional conditions

TABLE XVI.—CORRELATION COEFFICIENT FOR THE I.Q. OF SIBLINGS

All like-sex pairs (150)	0·27 ± 0·07
Girl pairs (68)	0·26 ± 0·11
Boy pairs (82)	0·22 ± 0·10
Mixed pairs (103)	0·39 ± 0·08
All pairs between 8½ and 12 years (103)	0·32 ± 0·09

9 GENERAL DISCUSSION.

The results of this investigation will now be subjected to more general discussion, with special reference to (1) agreement and difference between observations of different investigators on different samples; (2) an attempt to elucidate the contribution of genetic differences to the observed variance in the I Q of offspring of the same parents brought up together in the same family environment, whether (*a*) of the same birth-rank or (*b*) separated by an interval in age.

(1) *Comparison of the Results of different Investigators.*

Concerning the following issues there is now complete unanimity between the testimony of different workers:—

(i) The resemblance of like-sex twins is greater than that of mixed pairs, and the resemblance of twins of unlike sex is somewhat greater than that between siblings of different ages.

(ii) The resemblance of twins judged to be monozygotic is greater than that of twins judged to be dizygotic, the correlation for I.Q. in the case of identical twins being of the order 0·85.

Whether dizygotic twins of like sex are more alike than dizygotic twins of unlike sex cannot be stated with any confidence. No previous investigator has assigned figures for both In the present investigation neither differs significantly from the theoretical correlation predicted on the theory of autosomal transmission in a uniform environment. It may be noted, however, that the correlation coefficient and the mean difference recorded for dizygotic twins of like sex refers to a group deliberately selected for dissimilarity in physique. This may prejudice the result somewhat for a reason which will be set forth more fully below A possible indication of the significance of sex as affecting the environment to which two members of a twin pair are exposed is given by the fact that boy pairs appear to be more alike than girl pairs according to the data of Lauterbach and of the present writers All that can be legitimately stated on the basis of this investigation is that the subjects examined provide no conclusive evidence for the belief that difference of environment within the family group arising from sex difference do not appreciably manifest themselves in the method of analysis that has been adopted

(2) *The Contribution of Nature and Nurture to Observed Differences in the I Q of Twins*

To assign a value for the relative " contribution " of genetic differences and environmental influences tending to differentiate offspring of the same birth-rank brought up in the same families, two formulæ have been proposed by Holzinger. These will be referred to as the T and the t^2 formulæ, and they will now be applied to an analysis of the present data and those given by other investigators In doing so, we wish to make clear that we do not interpret them to mean that the contributions of nature and nurture to variability in a mixed population of different families are truly additive. All we are entitled to ask is how much the variability, measured either as mean differences or mean square differences, within the family is reduced, when we have eliminated all gene differences such as distinguish one member of a fraternity from another member of the same fraternity.

(*a*) The first formula may be written

$$T=\frac{\Delta_f-\Delta_i}{\Delta_f}.$$

Here Δ_f is the mean difference between dizygotic twins of like sex and Δ_i is the mean difference between identical twins. The values for T given by different observers using different tests are shown in Table XVII.

TABLE XVII.—VALUE OF T GIVEN FOR THE I.Q. BY DIFFERENT INVESTIGATORS.

Author.	T.	Test.
Holzinger . .	. 0·5–1·0	Binet and Otis
Tallman .	0·61–0·62	Stanford Revision
Herrman and Hogben	. 0·92–0·62	Otis

The two figures of Holzinger himself refer to different age-groups. Those attributed to Tallman are calculated by the present writers from her published data. Tallman herself, whose work is not referred to in Table I, which includes only that of authors who have employed the method of correlation in the analysis of their data, gives the value of Δ_i and the mean difference Δ_m for twins of unlike sex. She examined a comparatively large sample of which 84 were mixed pairs, and 158 like-sex pairs of which 63 were judged to be identical on the basis of facial resemblance. Two estimates of the value of T are given for her data, one on the assumption that dizygotic twins of like sex are not more alike than dizygotic twins of unlike sex, and the other on the assumption that her identical group was homogeneous and that the remaining like-sex pairs were all of the dizygotic type. Of the two figures given for the data of the present inquiry, the first is based upon a literal application of the formula, the second is calculated on two assumptions, both justified by experience in the present investigation. It is assumed that 45 per cent. of like-sex twins are monozygotic and that dizygotic twins of like sex are not more alike than mixed pairs. Needless to say, these assumptions tend to minimise the contribution of Nurture.

(*b*) The second formula proposed by Holzinger is

$$t^2 = \frac{r_i - r_f}{1 - r_i}.$$

Two different estimates of t^2 given by Holzinger and those derived from the data of this investigation are as follows:—

Holzinger . .	2·0–3·8
Herrman and Hogben . .	2·3–2·6

As for physical characteristics, there is a considerable discrepancy between the estimates derived from these two formulæ. There is remarkably satisfactory agreement between the figures obtained by applying one and the same formula to the data of different authors, as shown in Table XVIII, which is partly based upon figures given by Stocks. For the I.Q. an approximate mean is given. One thing at least is clear from this table. Whichever criterion of the relative "potency" of Nature and Nurture is

TABLE XVIII

	This Investigation.		Stocks.		Holzinger		Von Verschuer	Lauterbach
	T.	t^2	T.	t^2	T	t^2.	T	t^2
Height .			1·3	10·4	1·7	4·0	1·7	6·6
Weight	..		1·4	7·0		3·6	0·8	7·5
I Q	·75	2·4	.		·75	3·0		

adopted, genetic differences make less difference to variation in I Q than to observed variation in such physical characteristics as weight and height among individuals brought up in the same family environment

Holzinger's two formulæ do not interpret the relative "potency" of nature and nurture in precisely the same sense The ratio T represents the relative efficacy of genetic and environmental factors in terms of the average difference between members of the same family—that is to say, the *mean* differences. The second formula t^2 measures the relative importance of genetic and environmental differences with respect to the *variance* within the family It thus involves *mean square* differences. We can interpret the discrepancy between the ratio of T and t^2 in the following way: The fact that T is less than unity indicates that genetic influences account for less than half the mean difference between individuals of the same birth-rank and family if produced from separate ova The fact that t^2 is greater than unity might signify that genetic differences are more important than differences of environment in producing striking discrepancies between members of the same family and the same birth-rank. However, there is another interpretation for the fact that t^2 is greater than T. This is that striking differences of environment are more likely to occur as between two non-identical twins than between two identical twins.

With the notable exception of Stocks, few investigators have hitherto recognised that it is more easy to give a minimal estimate of the contribution of genetic differences to observed variation in physique or social behaviour than to give a maximal estimate of the contribution of environmental influences. Sometimes estimates have been given, as in an investigation into home influences by Burks (1), when the method employed could not possibly yield more than an arbitrary minimum for the total effect of nurture. One point mentioned by Stocks is specially relevant to the method adopted in this inquiry. In a certain sense an organism selects its own environment. A gene, which would only be manifest as some simple anatomical or physiological peculiarity in a carefully standardised environment under laboratory conditions, may in nature bring the organism into situations which reveal genetic differences that would not be manifest if the environment were perfectly uniform. In an insect a single gene substitution may result in reversing the sign of a tropism. In nature a negatively phototropic animal will tend to move into more humid surroundings than a positively phototropic individual. Thus the presence of a gene whose effect is only manifest under conditions of humidity such as that which determines the fruit-fly mutant referred to as "vestigial abdomen" would perhaps be more readily recognised in nature among negatively phototropic than among positively phototropic flies In the Biblical narrative Jacob and Esau were apparently dizygotic twins. It is stated that "Esau was a cunning hunter, a man of the field, and Jacob was a plain man living in tents." It is conceivable that the disposition to hunt is genetically determined It is also easy to see that such a disposition equips the individual with a different social and physical environment from that in which he would find himself if he did not possess it

Such formulæ as those used above involve the assumption that the environment of two dizygotic twins does not differ more than the environment of two identical twins. This assumption is certainly false How significantly so in the present instance we have no means of deciding. The difficulty of interpretation arising on this account is increased by the method of separating from the like-sex group an exclusively identical and exclusively fraternal class. To secure a perfectly homogeneous group of dizygotic twins of like sex, individuals showing very striking physical differences were chosen Holzinger found that dizygotic twins of like sex were more alike than mixed pairs appear to be from the data presented by Lauterbach and by the present writers. By the method of selection employed, the "D" group of this investigation are rather less alike than twins of unlike sex If Holzinger's figure is more correct, sex is a significant factor in determining environmental differences which influence the I Q at some stage in the development of the individual. One explanation of the discrepancy is that Holzinger's group of supposedly dizygotic twins may have included some identical pairs. The other is that the "D" group of this investigation excluded all dizygotic individuals so much alike as to defy confident diagnosis. Hence, if physical similarity is a factor in deciding differences of environment, a too exacting method of diagnosis may lead to an underestimate of the resemblance between dizygotic twins of like sex.

Taking all these considerations into account, the present authors are entirely in sympathy with the main conclusion which Holzinger draws from his own study. Contrary to a widely spread belief which attributes differences in I.Q. predominantly to genetic differences, it is extremely unlikely that genetic differences account for more than one-half the mean differences among offspring of the same parent having the same birth-rank and brought up together in the same family environment. Hence genetic conclusions derived from existing studies upon widely different racial and occupational groups are devoid of any scientific validity

(3) *Influence of Birth-rank.*

It has already been made clear that comparisons of siblings and twins such as are embodied in the present investigation can only yield information concerning how far differences of nurture limit the manifestation of gene differences among offspring of the same parents brought up together in the same family environment They throw no light whatsoever on the wider problem of variation as between individuals of one family at one social level or physical environment and individuals of another family at another social level or in a different family environment Whether such differences of environment as siblings encounter in virtue of a disparity of age or different position in the family group significantly influence variation in the I Q cannot be decisively answered from the data presented here The mean differences for ordinary sibs are not greater than for twins. On the other hand, evidence for the significance of birth-rank is contributed by the low correlation for ordinary siblings and the mean differences of first- and second-born children. From one point of view the material of this investigation was chosen in such a way as to minimise environmental differences arising from different position in the family group; since the mean age difference of the sibs examined was very small. Any difference between first-born children and children born subsequently appeared as a systematic deviation throughout the inquiry In a recent study, Thurstone and Jenkins (13) found the I.Q tended to increase with successive births in a group of retarded and delinquent children; but they also note the existence of other evidence pointing to the fact that genius appears to occur among first-born children with disproportionate frequency. In the group of relatively bright children studied by the present writers, the first-born children proved to be brighter than those born later. Probably, owing to paucity of material, the twin pairs were a more strictly random selection of individuals than the sibs examined. Any selection involved in taking pairs of sibs would minimise the difference in their intellectual status. The balance of evidence is in favour of the view that the intellectual resemblance of fraternal twins is greater than that of ordinary sibs brought up together under ordinary conditions of family life; but that fraternal twins are not more alike than ordinary siblings reared in institutions where the social environment is more uniform than that of a normal family. From the standpoint of animal genetics it would not be surprising if such a difference existed. Sewall-Wright (11) has found a significantly higher correlation, with respect to certain physical characteristics in the guinea-pig, for littermates as compared with offspring of the same parents but belonging to different litters.

(4) *Indications of Sex-linkage.*

Indications of the contribution of sex-linked genes to variability in finger-print pattern has already been mentioned. The present investigation did not yield evidence of the contribution of sex-linked genes to differences in I Q. If sex-linked genes appreciably affect variability in I.Q. male pairs of offspring of the same parent should show less resemblance than female pairs, and mixed pairs should be least alike. Actually boy twins appear to be more alike than girl twins. In the sib-group girl pairs were slightly more similar than boy pairs, but any genetic significance which might be attached to the small difference observed must be ruled out on account of the greater resemblance of unlike-sex pairs It would be necessary to investigate a much larger sample before conclusions could be drawn concerning the contribution of sex-linked genes

10. SUMMARY.

Comparison of the response of twins and siblings to a group intelligence test shows that the mean difference between the intelligence quotients of individuals belonging to the same fraternity and having the same birth-rank is reduced by somewhat less than a half, when no gene differences directly or indirectly influence the difference between individuals belonging to one and the same pair

ACKNOWLEDGMENTS.

Acknowledgment is made to the Education Officer of the London County Council, to Professor C. L. Burt, and to the School Principals who kindly extended facilities for this investigation; to Professor J. B. S. Haldane, F.R.S., and Dr R. A. Fisher, F.R.S., who discussed with the writers some of the points raised in the investigation; to Chief Inspector Battley of the Finger-print Department of Scotland Yard for technical advice; and also to the Rockefeller Research Fund of the London School of Economics for a grant in aid of the research.

REFERENCES.

(1) Burks, 1928. "The Relative Influence of Nature and Nurture upon Mental Development," *27th Year-Book, N.S.S.E.*, part 1, chap. x.

(2) Elderton, 1923. "A Summary of the Present Position with regard to the Inheritance of Intelligence," *Biometrika*, vol. xiv.

(3) Fisher, 1925. "The Resemblance between Twins: a Statistical Examination of Lauterbach's Measurements," *Genetics*, vol. x, No. 6.

(4) Freeman, Holzinger, and Mitchell. "The Influence of Environment on the Intelligence, School Achievement, and Conduct of Foster-children," *27th Year-Book, N.S.S.E.*, part 1, chap. ix.

(5) Hildreth, 1925. "The Resemblance of Siblings in Intelligence and Achievement," *T.C. Contribs. to Educ.*, No. 186.

(6) Hogben, 1932. "Filial and Fraternal Correlations in Sex-linked Inheritance," *Proc. Roy. Soc. Edin.*, vol. lii, pp 331–336.

(7) Holzinger, 1929. "The Relative Effect of Nature and Nurture Influences on Twin Differences," *Journ. Educ. Psych.*, vol. xx, No. 6.

(8) Lauterbach, 1925. "Studies in Twin Resemblance," *Genetics*, vol. x, No. 6.

(9) Merriman, 1924. "The Intellectual Resemblance of Twins," *Psych. Monographs*, 33, Princeton, U.S.A.

(10) Pintner, 1930. *Intelligence Testing* (2nd ed.). New York.

(11) Sewall-Wright, 1926. *American Naturalist*, vol. lx, p. 552.

(12) Stocks, 1930. "A Biometric Investigation of Twins and their Brothers and Sisters," *Annals of Eugenics*, vol. iv, parts 1 and 2.

(13) Thurstone and Jenkins, 1931. *Birth-order, Parent—Age and Intelligence*, University of Chicago, U.S.A.

(14) Tallman, 1928. "A Comparative Study of Identical and Non-identical Twins with respect to Intelligence Resemblances," *27th Year-Book, N.S.S.E.*, part 1, chap. vii.

(15) Wingfield, 1928. *Twins and Orphans* (Dent).

(*Issued separately April* 12, 1933.)

X.—Spermatogenesis in *Drosophila obscura* Fallen. I. The Cytological Basis of Suppression of Crossing-over.

By **P. Ch. Koller,** Ph.D., and **Thelma Townson,** M.A., Institute of Animal Genetics, University of Edinburgh. *Communicated by* Professor F. A. E. Crew. (With three Plates.)

(MS. received December 9, 1932. Read February 6, 1933.)

INTRODUCTION.

During recent years it has become increasingly possible to interpret unusual genetic phenomena by reference to peculiarities in chromosome behaviour. Examples of such cytological explanation are found in the studies of triploidy by Bridges and Anderson (1924) and Redfield (1930, 1932), and of translocations by Dobzhansky and Sturtevant (1930), Rhoades (1931), and Dobzhansky (1932*a*). But, thus far, the phenomenon of complete linkage in the male of *Drosophila* has not been explained satisfactorily by direct observation of spermatogenesis. Doubtless this has been due partly to technical difficulties, and partly to the fact that chromosome behaviour is only now beginning to be understood. However, the first difficulties have been eliminated by Metz (1926), Guyenot and Naville (1929), and Huettner (1930); and the second by Belar (1928), and more recently by Darlington (1932*b*).

The present study deals with spermatogenesis in *D. obscura*, and attempts to offer a theoretical explanation, based on the observations obtained, for the absence of crossing-over in the male.

MATERIAL AND TECHNIQUE.

The flies were obtained in 1928 from America through the kindness of Professor Lancefield, and have since been bred in the Institute of Animal Genetics, Edinburgh.

Newly hatched males were used. The maturation of *D. obscura* takes longer than in *D. melanogaster*, and the testes of young flies nearly always exhibit every stage of spermatogenesis. The only disadvantage is that the spermatogonial cells and chromosomes are smaller than in other species of *Drosophila*. However, this was counterbalanced by the fact that large numbers of cells exhibit different stages of spermatogenesis, and the chromosomes are arranged in one plane at metaphase, as described by Metz (1926).

The testes were dissected out from the abdomen in a 2 per cent. urea-solution and fixed in strong Flemming to which urea was added. After thirty to sixty minutes' fixation, the testes were transferred to 1 per cent. chromic acid for three to twelve hours. The stain used was Newton's gentian violet; it was absorbed deeply by the chromosomes and the nucleolus, but only in a slight degree by what Metz (1926) calls the "pseudo-chromatin" materials.

DIVISION OF THE SPERMATOGONIA.

The chromosomes at prophase are somewhat indistinct and, owing to the shortness of the period between the onset of division and metaphase, their behaviour cannot be studied satisfactorily. They appear as more or less thick threads deeply stained with gentian violet They absorb the stain more rapidly than do the spermatocyte chromosomes, so that in many cases the spermatogonial cells can be easily identified by the strong staining property of their chromosomes.

At metaphase the chromosomes are arranged in the equatorial plate, the homologous chromosomes lying side by side. The chromosomes in the spermatogonia of *D. obscura* are always arranged in one plane during metaphase, which appears not to be the case in any other *Drosophila* species. Plate I, figs. 1, 2 and 3, show spermatogonial chromosomes at metaphase. Only fig. 1 illustrates the whole chromosome complex. The heteromorphic sex-chromosomes can be identified. The X-chromosome is V-shaped and has a median or nearly median attachment constriction, while the Y-chromosome is rod-shaped with a submedian attachment constriction, which sometimes divides the chromosome into a long and short limb as illustrated in fig. 3. Three out of the four autosomes are rod-shaped, all about the same length, and slightly shorter than the Y-chromosome. Frolowa and Astaurow (1929) illustrated the chromosomes in the Russian species of *D. obscura*, and in their figures the autosomes are somewhat V-shaped. Metz's depiction (1916, 1926) of the complex of *D. obscura* resembles ours, showing rod-shaped autosomes. In a few spermatogonial metaphases (fig. 2) the different size of the autosomes is due to the different angles from which they are viewed. The fourth pair of autosomes is the smallest in the complex, its length being about ⅛, or even less, of the other autosomes, so that in many cases they are not readily observed (figs. 2 and 3). The spindle fibre attachment constriction of the autosomes is terminal or nearly so.

At metaphase of the spermatogonial division the chromosomes show a tendency to associate in homologous pairs. This association of the homologous chromosomes becomes more intimate at anaphase when the equationally divided chromosomes pass into opposite poles. Plate I, figs. 4, 5 and 6, illustrate the anaphase of the spermatogonial division. Figs. 4 and 5 give the polar view, and fig. 6 the side view of the anaphase stage. This association of homologous chromosomes resembles meiotic pairing, and it is easy to mistake the anaphase configuration of the associated spermatogonial chromosomes for the anaphase of the first spermatocytes, as has been already pointed out by Metz (1926) and by Huettner (1930). In some cells the chromosomes in the polar view are represented as small, round bodies, lying side by side, and this arrangement is very similar to the secondary pairing at meiotic division described and discussed by Lawrence (1931).

Each member of the associated homologous chromosomes has a single structure; no sign of splitting could be observed at anaphase of the spermatogonial chromosomes. The double structure and the apparent "haploid" number are due to the association of the homologous chromosomes during their segregation to the opposite poles. The association of the homologous chromosomes during the spermatogonial division in several *Drosophila* species was observed by Stevens (1908), Metz (1916), Guyenot and Naville (1929), and Huettner (1930). In another species of Diptera, Nonidez (1921) followed in detail the whole process of anaphase association of spermatogonial chromosomes. Metz (1926), in his masterly paper on spermatogenesis of the Drosophilidæ, even described cases where the association of homologous chromosomes begins as early as the prophase of the spermatogonial division. However, in the present study only the anaphase pairing could be followed clearly.

The association of homologous chromosomes in pairs is characteristic of the Diptera, but this phenomenon must not be confused with true chromosome pairing, a term which indicates intimate chromosome conjugation, or synapsis, during the prophase of the first meiotic division. In some cases the association between homologous chromosomes results in complete fusion at the attachment constriction. This process takes place before the chromosomes of the whole complex fuse together to form the nucleus of the first spermatocyte.

At the end of anaphase the chromosomes fuse together and give rise to a deeply stained chromatin mass, from which the ends of the chromosomes sometimes still protrude (Plate I, figs. 7 and 8). Simultaneously, a wall develops round the nuclear material, and the daughter nuclei are formed. However, the space in the nucleus which is occupied by the chromosome complex is only about half that of the total volume, the rest being taken ıp by other nuclear material which has less staining capacity than the chromosomes. In a few cases cells were found in which this nuclear material had migrated from the nucleus, and was dispersed in the cytoplasm. The true nature of this nuclear material was discussed, its behaviour followed with the aid of the Feuglen-reaction and described in detail by Huettner (1930).

MEIOSIS.

The division of the first spermatocyte is preceded by the "growth period," the volume of the nucleus increasing rapidly. The nuclear membrane of the first spermatocyte nucleus during this period is somewhat thicker and coarser in structure than it was in the previous stage. In the nucleus qualitative and quantitative changes take place. At the beginning of the growth period, the nuclear mass disperses and is more or less evenly stained with gentian violet. Soon, however, a small part of this nuclear material condenses and forms a nucleolus or plasmosome, which breaks up into small, slightly stained bodies before the prophase of the meiotic division. Plate I, figs. 9 and 10, illustrate the growth period of the first spermatocyte. In the nucleus the nucleolus and the nuclear material can be seen. Plate I, fig. 11, depicts an early spermatocyte nucleus with a nucleolus and small stained bodies which are derived from the nucleolus; Jeffrey and Hicks (1925) most probably mistook these fragments of the nucleolus for chromosomes. They survive the prophase of the first spermatocyte, sometimes even the meiotic anaphase, and can easily be interpreted as chromosomes, as was pointed out by League (1930).

The rate of growth is very rapid, and the cells in successive layers differ greatly in size. Those spermatocytes which are in a more advanced stage show a nucleus, which is increased in volume, as well as other changes in the nuclear material. The small nuclear aggregates, which are derived from the break-up of the nucleolus or plasmosome, are present in various numbers. Plate II, fig. 1, illustrates a nucleus, showing the condensed nucleolus and four large and four small chromatin aggregates. The staining capacity of the nucleolus is exaggerated in this figure to illustrate more clearly the smaller aggregates. They are sometimes seen to be connected by thin threads with each other and with the nucleolus.

The break-up of the nuclear chromatin material into smaller aggregates is followed by their diffusion. The aggregates loosen, become thread-like, and stain less deeply than in the condensed stage. Huettner and Guyenot and Naville found that the spermatocytes at this stage, which is most probably the early prophase, contain fine single threads which were identified as the leptotene chromosome threads. However, in our investigation, as also that of Metz, single threads like the leptotene chromosomes were not found in the early prophase of the first spermatocyte division. This is most probably due to the different technique and material used by Huettner and Guyenot and Naville.

The diffusion of the chromatin aggregates into threads is illustrated in Plate II, fig. 2. The threads have a double structure, showing a V-shape, and are associated at one end. This arrangement was the most common in the first spermatocyte. Fig. 2 does not show the supernumerary and "pseudo-nuclear" aggregates, but illustrates only the prophase chromosomes as they develop from the chromatin material The chromosomes are thicker than the early prophase chromosomes in normal meiosis. Plate II, fig. 3, illustrates the same stage in the first spermatocyte. One chromosome pair shows close submedian association or pairing. Plate II, figs. 4, 5 and 6, show the nucleus of the first spermatocyte with paired chromosomes where the association is terminal, subterminal, or median.

Plate II, figs. 7, 8 and 9, represent more advanced stages of prophase. The chromosomes remain in pairs, but condensation takes place and the loose granulated chromosomes become more deeply stained. In some first spermatocytes during this stage the X- and Y-chromosomes are seen to pair. The inequality of the members of the pair and the structure of the associated chromosomes, suggested that they are the sex-chromosomes. In *D. obscura* a very characteristic feature in the prophase of the first meiotic division is the association of short segments only of the chromosomes. This association most probably represents a partial synapsis or conjugation of homologous portions of chromosomes.

The condensation of the paired chromosome threads is rapidly progressing and they decrease in length. In this stage—the diakinesis—the chromosomes can be followed very easily, as the staining capacity is much better, and they appear as deeply stained double elements. Plate II, figs. 11 and 12, illustrate the diakinesis stage in the first spermatocyte. The X- and Y-chromosomes can be identified in the complex. This stage is closely similar to the stage illustrated in Plate I, figs. 4 and 5, which represent the anaphase of the spermatogonial division.

The prophase stage, as described above, was in many cases obscured by the nucleolus and other chromatin material. The nucleolus usually breaks up into small round bodies or globules, which scatter throughout the nucleus. In *D. obscura*, however, these secondary phenomena, which are very common in other *Drosophila* species, such as *melanogaster* and *virilis*, do not extend to such a degree as to make the observation of the chromosomes impossible during prophase.

Diakinesis is followed by metaphase. The chromosomes, which at diakinesis lie side by side but scattered in the nucleus, are, during metaphase, arranged in the equatorial plate. The members of each paired chromosome bivalent are separated, and are held together only by terminal or subterminal association. The nuclear membrane is still present at the beginning of metaphase, and only when the chromosomes are arranged in the equatorial plate does it gradually disappear and the nuclear spindle develop. The paired chromosomes remain in their loose association at the beginning of metaphase. Owing to the delayed development of the nuclear spindle, the bivalents of *D. obscura* differ from those characteristic of most meiotic metaphases. Plate III, fig 1, illustrates the onset of metaphase with the movement of the chromosomes to the equatorial plate, as seen in polar view. Figs. 2 and 3 illustrate metaphase stages in side view; the nuclear membrane is still present and the spindle has not developed. A more advanced metaphase is seen in fig. 4. Most probably the partial synapsis proceeds in such a way as to leave only a very small part of the two homologous chromosomes remaining in association, but at different levels. When the spindle develops, the associated chromosomes are pulled apart from the point of attachment constriction to the point of association This stage is depicted in Plate III, fig 4, showing the bivalent chromosomes. The unequal sex-chromosomes, as are the homologous autosomes, are associated and paired. The synapsis is always restricted to one end of this chromosome pair, and does not change during prophase. The association of X and Y at metaphase remains terminal or nearly so. If pairing is determined by homology, then the associated segments of X and Y must be homologous. A similar conclusion was drawn by Metz from his cytological studies on other *Drosophila* species. Plate II, fig. 9, and Plate III, figs. 1 and 2, illustrate the partial conjugation of sex-chromosomes.

Anaphase is very short. The chromosomes separate reductionally to the opposite poles, and owing to the slight association, their separation is very rapid. The fibre of the nuclear spindle pulls them apart, and in side view this separation can be followed very clearly. Plate III, fig. 5, shows the anaphase separation in side view. The chromosomes move to the poles, but the rate of movement may be different for different chromosomes. Some tendency to lag could be detected in anaphase, but this phenomenon may be explained by the unusual synapsis during the previous stages. The separating chromosomes sometimes exhibit a double structure, which indicates that they most probably have split in the earlier prophase stages, but the exact time and place of the splitting could not be ascertained. Plate III, fig. 6, shows anaphase stage, some chromosomes showing the double structure.

At the end of anaphase, when the chromosomes have arrived at the poles, they fuse together and form a small round nucleus. This formation of the nucleus represents the end of the first spermatocyte division, which gives rise to two secondary spermatocytes with a reduced chromosome number, and the separation of the cytoplasm into two cells usually proceeds simultaneously, though sometimes the division of the nuclear material is not followed immediately by the division of the cytoplasm. Plate III, fig. 7, illustrates secondary spermatocytes with a comparatively small nucleus.

The division of the secondary spermatocyte into two spermatids is preceded by a rapid increase in the volume of the nucleus. The length and bulk of the chromosomes are comparatively greater than are those of the chromosomes in the first spermatocyte. The individual chromosomes can be identified and two types of haploid chromosome groups—one with the V-shaped X, and the other with the rod-shaped Y—can be seen in many secondary spermatocytes during metaphase. Plate III, figs. 8 and 9, illustrate metaphase plates of secondary spermatocytes. This stage is the one which was found most frequently in the testes of *D. obscura.* The chromosomes are arranged in the metaphase plate, mostly in one plane. Their double structure, which resulted from splitting, cannot be detected in the very short prophase, but in a few cases it was followed from the anaphase of the previous division. The segregation of the chromosomes at anaphase to the opposite poles shows that the rate of separation is sometimes different and causes some chromosomes to lag; most probably the sex-chromosomes remain behind the rod-shaped autosomes in the movement from the equatorial plate to the pole. Plate III, fig. 10, illustrates the anaphase of the secondary spermatocyte.

The formation of spermatids and the development of sperm are similar to these processes already described in other Diptera.

DISCUSSION.

The behaviour of the chromosomes during the maturation division in the male of *Drosophila* has been studied by Metz (1926), Guyenot and Naville (1929), and Huettner (1930). Their observations all substantiate the genetic evidence that there is no crossing-over in the male, but their interpretation of its cause differs. The present study of gametogenesis in *D. obscura* presents additional data, which have been reviewed in the light of recent observations of meiotic chromosome behaviour in plants and animals, and a more probable hypothesis of the cause of this suppression of crossing-over is offered.

Two important hypotheses (Sax's, 1930 and Belling's, 1931, and

Darlington's, 1931) deal with the relation between pairing of homologous chromosomes and crossing-over. Both of them are based on Janssens' (1924) partial chiasmatype hypothesis, but they differ regarding the time relations of chiasma formation and crossing-over.

If the homologous chromosomes do not pair during meiotic prophase, then crossing-over cannot take place. Furthermore, the pairing of homologous chromosomes is the basis for normal chromosome disjunction and segregation. Anderson (1929), studying non-disjunction in *D. melanogaster*, proved that the paired homologous chromosomes pass to the opposite poles more regularly than do chromosomes which failed to pair with their homologues. Similar results were obtained by Dobzhansky (1932*a*), who studied the different translocations in *D. melanogaster* and demonstrated that pairing, crossing-over, and chromosome disjunction are closely interrelated. If there is complete chromosome disjunction which can be followed genetically, then one may infer that the pairing and post-diplotene association of homologous chromosomes must have occurred previously, and that these phenomena represent the cytological basis of the complete genetical segregation. The post-diplotene association of chromosomes, however, is dependent on chiasma formation, which in turn is related to crossing-over, as shown by Beadle (1932). Gowen (1928, 1931) demonstrated in *D. melanogaster* that absence of crossing-over in the female is associated with frequent non-disjunction of all chromosomes, which shows that crossing-over and segregation are interrelated.

In the male of the different *Drosophila* species there is normal distribution of homologous chromosomes to the opposite poles at the anaphase of the first spermatocyte, which indicates that they must have been associated in pairs during the meiotic division. This association must have survived even the post-diplotene stage, and the chromosomes are pulled apart by the spindle fibre after metaphase.

The question arises, therefore, as to why genetical crossing-over does not take place during this meiotic pairing. The cause of complete suppression of crossing-over may be either genetical or cytological.

The pairing of homologous chromosomes, and the formation and distribution of chiasmata (genetical crossing-over) are controlled by factors located in the chromosomes themselves. This problem was discussed by Darlington (1932). Blakeslee (1928) in *Datura stramonium*, and Beadle (1930) in *Zea Mays*, described plants where there is much lack of pairing between homologous chromosomes during meiosis. These plants contain one recessive gene, which is responsible for the absence of crossing-over.

The cytological observations of Metz on several *Drosophila* species, of Guyenot and Naville and Huettner on *D. melanogaster*, and our own on the gametogenesis of *D. obscura*, demonstrate the association and partial synapsis of the homologous chromosomes during prophase of the first spermatocyte division. Such chromosomes associated in pairs are seen in Plate II, figs. 2–10. These observations show that the complete disjunction of the chromosomes in the male of *Drosophila* is preceded by pairing between homologous chromosomes, and it may therefore be concluded that there exists no factor which suppresses crossing-over through the failure of pairing.

In *D. melanogaster* Gowen and Gowen (1922) found a recessive gene in the third chromosome, which, although not preventing the pairing of the homologous chromosomes, suppressed entirely the crossing-over in all chromosomes in the female. Eloff (1932) made a very extensive study of variation in crossing-over, and discussed the genetical basis of complete suppression. If, in the male of *Drosophila*, a genetic factor or factor complex, similar to the factor described by the Gowens, were present, responsible for the direct suppression of crossing-over, then it must be located in the Y-chromosome. Haldane's (1922) law states that if crossing-over is reduced or entirely absent in one sex, that sex is always the heterogametic. Haldane's law favours the assumption that in the male of *Drosophila* the Y-chromosome itself, or factors borne on the Y, may be responsible for complete suppression of crossing-over.

However, several genetical observations made by different investigators, prove that suppression of crossing-over in the male of *Drosophila* is not correlated with the presence of the Y, and consequently cannot be due to a gene or gene complex located in the Y-chromosome. Anderson (1929), studying non-disjunction in *D. melanogaster*, demonstrated that crossing-over in the XXY female had just the same value as in XX flies. Stern (1929) obtained similar results in a study of a line of *D melanogaster* in which the Y had become attached to the X.

The data obtained by genetical investigations indicate that there exists no special factor or factors present only in the male of *Drosophila* and borne on the Y-chromosome, which directly suppresses crossing-over without eliminating the pairing of homologous chromosomes.

During the present study it was found that the chromosomes held together during prophase of the first meiotic division. The association of homologous chromosomes takes place always over a short section of their length. If the locus of association represents a chiasma or chiasmata, then we can apply this fact as a test for the two hypotheses concerned with crossing-over.

Sax (1930) assumes that chiasmata arise by the chromatids opening out in different planes at diplotene and cross-overs occur by the breaking of chiasmata, *i.e.* the crossing-over is conditioned by chiasma formation. If the short distance where the chromosomes associate are chiasmata which did not break, then the absence of crossing-over in the male of *Drosophila* would be easily explained. However, the change of the locus of association at the post-diplotene stages observed in the present study, as well as many other recent observations on the behaviour of chromosome pairing, chiasma formation and crossing-over in many plants and animals, repudiate the assumption that crossing-over arises only through the break of chiasma (*cf.* Sansome and Philp, 1932; Darlington, 1932*b*).

On the second hypothesis, which was elaborated by Darlington, chiasma and post-diplotene association are conditioned by genetical crossing-over; if this is so, then the partial synapsis and association of homologous chromosomes at prophase of the first spermatocyte in *Drosophila* must be the results of crossing-over. This crossing-over, however, cannot be detected in genetical observations. Darlington assumes that two reciprocal chiasmata are formed, and are localised near the attachment constriction, in a segment, which is genetically "inert" in that it does not contain any mutated genes. The existence of such an inert region has been demonstrated both genetically and cytologically in *D melanogaster*, where this region lies in the X-chromosome beyond *forked*, and its length has been calculated by translocation (Dobzhansky, 1932*b*) Metz and Huettner have observed that this region pairs with the Y, from which they conclude that the two are homologous. Darlington assumes that the pairing of chromosomes and the localisation of reciprocal chiasmata only at this genetically inert region are responsible for the absence of crossing-over in the male of *Drosophila*.

Our observations on spermatogenesis of *D. obscura* show that the homologous chromosomes associate before the metaphase of the first meiotic division. The loci of this association are concentrated at the end of the chromosome at the beginning of the first spermatocyte division (see Plate II, figs. 2 and 3). Later, the loci of the partial synapsis alter, and chromosomes with subterminal, submedian, and median association are found (see Plate II, figs. 4–9). The change of the place of association proceeds towards the distal end of the chromosome, and it can be paralleled by the terminalisation of chiasmata in other organisms. Owing to the delayed development of the spindle, however, this movement is disturbed, and the interstitial chiasmata at metaphase are probably due to this absence of the nuclear spindle.

Localisation of chiasmata at the neighbourhood of the attachment constriction was found in *Fritillaria Meleagris* by Darlington (1930), and in *Mecostethus* by McClung (1927). There is genetic evidence provided by Bridges (1929) that in aged females of *Drosophila* there is a reduction of crossing-over at one or both ends of the chromosome (*i.e.* in the X or autosome respectively), and crossing-over is concentrated in the region of the spindle fibre attachment. Sansome and Philp discussed the fact that reciprocal chiasmata (double crossing-over) are more frequent near the spindle fibre attachment than elsewhere.

During the present study it was likewise found that the association of homologous chromosomes at the beginning of the prophase of the first spermatocyte division is concentrated at the end of the chromosomes. If the association of homologous chromosomes is conditioned by chiasmata, *i.e.* by exchanges between the partner chromatids, then our observations support Darlington's view. The concentration of chiasmata at the attachment constriction is probably a necessary consequence of the association between homologous chromosomes in the previous spermatogonial anaphase. The homologous chromosomes are associated in pairs at that stage, and sometimes even fuse together at the end where the spindle fibre attachment is situated. When they reappear at the following prophase of the first spermatocyte division this terminal partial association can again be observed (see Plate II, figs. 1, 2 and 3). Metz argues that the association of homologous chromosomes is so intimate that it even persists through the short resting stage. The homologous chromosomes enter into the prophase of the first spermatocyte with their ends already associated; it is natural, therefore, that the synapsis of the homologous chromosome at the attachment constriction favours the formation of chiasmata The change of loci of chiasmata or association in later stages of prophase is the result of the gradual movement of the associated segments from the proximal to the distal ends. Finally, at the end of the metaphase of the first spermatocyte division (see Plate III, fig. 4), when the nuclear spindle develops, the homologues are arranged with their proximal ends pointing towards the opposite poles, but the distal ends are still in association. Such an arrangement can result only through pairing by chiasmata combined with terminalisation.

It has been shown previously that normal pairing and normal chromosome disjunction occur in the male of *Drosophila*. The present study offers cytological evidence to show that crossing-over—the second step in the mechanism of reduction division—also takes place, though it cannot be demonstrated genetically.

SUMMARY

1. The chromosome complex in the male of *D. obscura* is described. The diploid complex contains 10 chromosomes. The X-chromosome is

V-shaped, the Y and the three pairs of autosomes are rod-shaped. The fourth pair of autosomes is very small.

2. The homologous chromosomes associate in pairs during metaphase and anaphase of the spermatogonial division. The association at anaphase sometimes results in a fusion of the ends of homologous chromosomes, where the attachment constriction is situated.

3. The homologous chromosomes enter into the prophase of the first meiotic division with their ends already associated. The loci of association gradually move towards the distal ends.

4. The X- and Y-chromosomes associate at one of their ends; this association persists through the prophase of the first meiotic division. The conclusion is, therefore, that the two segments are homologous.

5. In discussing the genetical and cytological basis of suppression of crossing-over we find that in the male of *Drosophila* the complete linkage is most probably due to unusual chromosome behaviour.

6. Sax's and Darlington's hypotheses on crossing-over and chiasma formation are discussed. The cytological observations obtained during the present study, tend to support Darlington's view that in the male of *Drosophila* there is localised chiasma formation at the attachment constriction, but crossing-over cannot be detected genetically, the segment being "inert," that is, not containing any mutated genes.

REFERENCES TO LITERATURE

ANDERSON, E. G., 1929. "Studies on a Case of high Non-Disjunction in *Drosophila melanogaster*," *Zeits. f. ind. Abst. u Vererb.*, vol. li, pp. 397–441.

BEADLE, G. W., 1930. "Genetical and Cytological Studies of Mendelian Asynapsis in *Zea Mays*," *Cornell Univ. Agr. Exp. Stn. Memoir*, vol. cxxix, pp. 1–23.

BEADLE, G. W., 1932. "The Relation of Crossing-over to Chromosome Association in *Zea-Euchlena* Hybrids," *Genetics*, vol. xvii, pp. 481–501.

BELAR, K., 1928. *Die Cytologische Grundlagen der Vererbung*, Gebrüder Borntraeger, Berlin.

BELLING, J., 1931. "Chiasmas in Flowering Plants," *Univ. Calif. Publ. Bot.*, vol. xvi, pp. 311–338.

BLAKESLEE, A. F., 1928. (No title.) Quoted in *Carnegie Inst. Washington Year Book*, vol. xxvii, p. 42.

BRIDGES, C. B., and E. G. ANDERSON, 1925. "Crossing-over in the X-chromosomes of Triploid Females of *Drosophila melanogaster*," *Genetics*, vol. x, pp. 418–441.

BRIDGES, C. B., 1929. "Variation in Crossing-over in Relation to Age of Female in *Drosophila melanogaster*," *Carnegie Inst. Washington Publ. No.* 399, pp. 63–89.

DARLINGTON, C. D., 1930. "Chromosome Studies in Fritillaria. III," *Cytologia*, vol. ii, pp. 37–55.

DARLINGTON, C. D., 1931. "Meiosis," *Biol. Rev.*, vol. vi, pp. 221–264.

DARLINGTON, C. D., 1932*a*. "The Control of the Chromosomes by the Genotype and its Bearing on some Evolutionary Problems," *Amer. Nat.*, vol. lxvi, pp. 25–51.

DARLINGTON, C. D., 1932*b*. *Recent Advances in Cytology*, London, Churchill.

DOBZHANSKY, Th., 1932*a*. "Studies on Chromosome Conjugation. I. Translocations involving the Second and Y-chromosomes of *Drosophila melanogaster*," *Zeits. f ind. Abst. u. Vererb.*, vol. lx, pp. 235–286.

DOBZHANSKY, Th., 1932*b*. "Cytological Map of the X-chromosome of *Drosophila melanogaster*," *Biol. Zentr.*, vol. lii, pp. 493–509.

ELOFF, G., 1932. "A Theoretical and Experimental Study on the Changes in the Crossing-over Value, their Causes and Meaning," *Genetica*, vol. xiv, pp. 1–116

FROLOWA, S. L, and B. L. ASTAUROW, 1929. "Die Chromosomengarnitur als systematisches Merkmal," *Zeits. Zellfors.*, vol x, pp. 201–213.

GOWEN, M. S., and J. W. GOWEN, 1922. "Complete Linkage in *Drosophila melanogaster*," *Amer. Nat.*, vol. lvi, pp 449–460.

GOWEN, J. W, 1928 "On the Mechanism of the Chromosome Behaviour in the Male and Female *Drosophila*," *Proc. Nat. Acad. Sci. U S.A.*, vol. xiv, pp. 475–477

GOWEN, J. W., 1931. "Genetic Non-Disjunctional Forms in *Drosophila*," *Amer. Nat.*, vol lxv, pp. 193–213.

GUYENOT, E., and A. Naville, 1929. "Les chromosomes et la réduction chromatique chez *Drosophila melanogaster*," *Cellule*, vol xxxix, pp. 135–168.

HALDANE, J B. S., 1922. "Sex Ratio and Unisexual Sterility in Hybrid Animals," *Jour Genet.*, vol. xii, pp. 101–109.

HUETTNER, A. F., 1930. "The Spermatogenesis of *Drosophila melanogaster*," *Zeits. Zellfors.*, vol. xi, pp. 614–637.

JANSSENS, F. A., 1924. "La Chiasmatypie dans les Insectes," *Cellule*, vol. xxxiv, pp. 135–139.

JEFFREY, E. C., and G. C. HICKS, 1925. "The Reduction Division in Relation to Mutation in Plants and Animals," *Amer. Nat.*, vol. lix, pp. 410–426.

LAWRENCE, W. J. C., 1931. "The Secondary Association of Chromosomes," *Cytologia*, vol. ii, pp. 352–384.

LEAGUE, B. B., 1930. "The Normality of the Maturation Divisions in the Male *Drosophila melanogaster*," *Science*, vol. lxxi, p. 99.

MCCLUNG, C. E, 1927. "Synapsis and Related Phenomena in *Mecostethus*, and *Leptysma* (Orthoptera)," *Jour. Morph. and Phys.*, vol. xliii, pp. 181–264.

METZ, C. W., 1916. "Chromosome Studies on the Diptera. II. The Paired Association of Chromosomes in the Diptera and its Significance," *Jour. Exp. Zool.*, vol. xxi, pp. 213–262.

METZ, G. W., 1926. "Observations on Spermatogenesis in *Drosophila*," *Zeits. Zellfors.*, vol. iv, pp. 1–28.

NONIDEZ, J. F., 1921. "Spermatogenesis in the Fly *Asilus sericeus* Say," *Jour. Exp. Zool.*, vol. xxxii, pp. 165–185.

REDFIELD, H., 1930. "Crossing-over in the Third Chromosome of Triploids of *Drosophila melanogaster*," *Genetics*, vol. xv, pp. 205–252.

REDFIELD, H., 1932. "A comparison of Triploid and Diploid Crossing-over for Chromosome II. of *Drosophila melanogaster*," *Genetics*, vol. xvii, pp. 137–152.

RHOADES, M. M., 1931. "A new Type of Translocation in *Drosophila melanogaster*," *Genetics*, vol. xvi, pp. 490–504.

SANSOME, F. W., and J. PHILP, 1932. *Recent Advances in Plant Genetics*, London, Churchill.

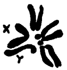
SAX, K., 1930. "Chromosome Structure and the Mechanism of Crossing-over," *Jour. Arnold Arb*, vol. ii, pp. 193–220.

STERN, C., 1929. "Untersuchungen über Aberrationen des Y-chromosoms von *Drosophila melanogaster*," *Zeits. f. ind. Abst. u. Vererb*, vol li, pp 253-353.

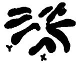
STEVENS, N. M., 1908. "A Study of the Germ Cells of certain Diptera with Reference to the Heterochromosomes and the Phenomena of Synapsis," *Jour. Exp. Zool.*, vol. v, pp. 359–374.

STURTEVANT, A. H, and TH. DOBZHANSKY, 1930. "Reciprocal Translocations in *Drosophila* and their Bearing on Œnothera Cytology and Genetics," *Proc Nat. Acad. Sci. U.S.A*, vol. xvi, pp. 533–536.

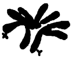
(*Issued separately April* 22, 1933)



1

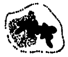


2

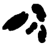


3

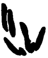
4



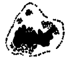
7



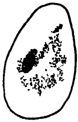
5



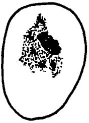
6



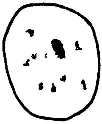
8



9



10

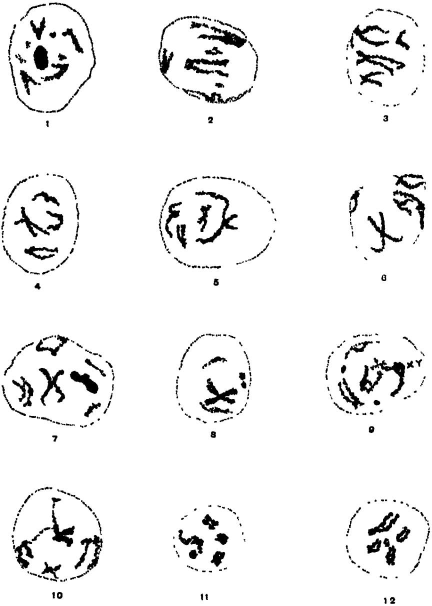


11

P. CH. KOLLER and THELMA TOWNSON. [PLATE I.

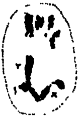
FIGS. 1, 2, and 3.—Metaphase of spermatogonia. The homologous chromosomes arranged in pairs.
FIGS. 4 and 5.—Anaphase of spermatogonia. Polar view. The association of chromosomes is more advanced
FIG. 6.—Anaphase of spermatogonial division. Side view.
FIGS. 7 and 8.—Telophase of spermatogonia.
FIGS. 9 and 10.—The growth period of the first spermatocytes.
FIG. 11.—The growth period of the first spermatocyte with aggregates derived from the nucleus.

The drawings were made with the aid of a Zeiss camera lucida, an H.I. 90, 1·30 apochromate oil-immersion objective and ×30 compensating eyepiece. The magnification of the figures is about 4500.

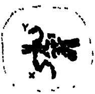


P. CH. KOLLER and THELMA TOWNSON. [PLATE II.

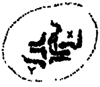
FIG. 1.—Early prophase of the first spermatocyte.
FIGS. 2–10.—Prophase of the first spermatocytes. The chromosomes associated with partial synapsis.
FIGS. 11 and 12.—Diakinesis of the first spermatocytes.



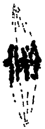
1



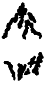
2



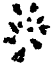
3



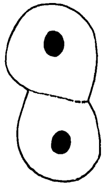
4



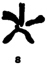
5



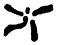
6



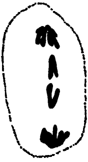
7



8



9



10

P. CH. KOLLER and THELMA TOWNSON. [PLATE III.

FIG. 1.—The onset of metaphase in the first spermatocyte Polar view.
FIGS. 2 and 3—Metaphase plates Side view The nuclear spindle is not yet formed.
FIG. 4.—Late metaphase with bivalents
FIG. 5.—Anaphase of the first spermatocyte.
FIG. 6.—Anaphase Some chromosomes exhibit double structure.
FIG. 7.—Secondary spermatocytes
FIG. 8.—Metaphase of the secondary spermatocyte. The X-chromosome is illustrated in the haploid complex.
FIG. 9.—Metaphase plate with Y-chromosome.
FIG. 10.—Anaphase of the secondary spermatocyte.

XI.—On the Pituitary in Lepidosiren and its Development.

By **T. Kerr**, M.A., Assistant in the Zoology Department, Queen's University of Belfast. *Communicated by* Professor J. GRAHAM KERR, F.R.S. (With Five Text-figs.)

(MS. received February 3, 1933. Read March 6, 1933.)

THE Dipnoi form an archaic group of Fish with so many unusual features that it is of importance to work out the development of all their organs in detail, and this investigation was therefore undertaken. The early stages were studied in the collection of slides in the Zoology Department of Glasgow University, and the adult organs were obtained from the material brought back from the Gran Chaco by Dr. G. S. Carter in 1927. The work was done in Glasgow under Professor J. Graham Kerr, F.R.S., to whom I am deeply indebted. The larval stages are numbered according to his system in Keibel's *Normentafeln*, Heft x (3).

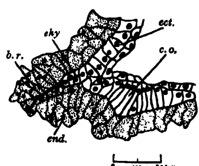


FIG. 1.—Sagittal section of head of larval Lepidosiren, Stage 23

a.l, anterior lobe, *b r*, brain rudiment, *c o*, cement organ, *c.t*, connective tissue, *ect*, ectoderm, *end*, endoderm, *ehy*, ectophysis, *i.l*, intermediate lobe, *inf*, infundibulum, *n.l*, nervous lobe, *o c*, optic chiasma.

The ectodermal rudiment of the pituitary is usually referred to as the "hypophysis," but as this word was originally applied to the entire pituitary of the adult, and still frequently is, it seems worth while to use a quite new word, and "ectophysis" is suggested.

At Stage 23 appears a wedge of cells, formed from the deep layer of the ectoderm, lying between the cement-organ and the brain rudiment (fig. 1). By a late Stage 25 this wedge has become elongated and its tip is deeply buried in the embryonic connective tissue under the primitive fore-brain. This tip, which develops into the ectophysis proper, has separated off by Stage 27, and the wedge of ectoderm is represented merely by a broken chain of cells between the gut and the ventral brain-wall. At Stage 29 the ectophysis lies with one end below the infundibulum and the other below the thickening of the optic chiasma (fig. 2). A lumen has now made its appearance, and the cells have orientated themselves round it in a semi-columnar manner with their nuclei towards the periphery. In transverse section the cavity is oval, with its long axis dorsoventral.

At Stage 30 the ectophysis is curved round the end of the developing infundibulum, and by Stage 32 (fig. 3) it is in the position which it occupies in the adult. The walls separating the cells of ectophysis and infundibulum break down and a proliferation commences among the cells of the distal side of the ectophysial cavity. By Stage 36 this distal part, which will form the anterior lobe in the adult, is noticeably enlarged.

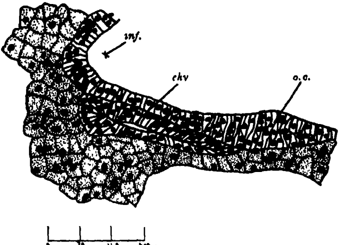


FIG 2.—Sagittal section, Stage 29 Head end to the right

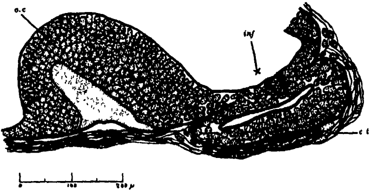


FIG. 3.—Sagittal section, Stage 32 Head end to the left.

In the pituitary of a six-inch Lepidosiren, which was the only stage between larva and adult examined, the part proximal to the ectophysial cavity has enlarged and a slight ingrowth of nerve-fibres has commenced, but no sinuses have formed; the anterior lobe has continued its enlargement.

In the adult the pituitary is a wedge-shaped organ attached to the tip of the infundibulum, with its narrow end directed backwards, sunk into a depression in the cranial floor As may be seen from a sagittal section, shown in fig. 4, its detailed anatomy is unlike that of the pituitary of any other class. The part most closely associated with the infundibulum, the so-called infundibular gland, corresponds to the nervous lobe of the other Vertebrates It consists of bundles of nerve-fibres with groups of cells and irregular sinuses embedded in them. The sinuses, which represent outgrowths from the infundibulum, do not contain blood, and communicate with each other and with the infundibular cavity. There is a good blood supply, as shown in fig. 5, from a few large blood-vessels with intercommunicating small ones running between the bundles of nerve-fibres. The intermediate lobe is represented by the layer, a few cells thick, between the nervous lobe and the gland cavity; it presumably makes use of the vessels on its border

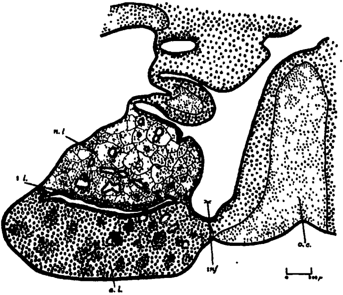


FIG 4 —Sagittal section of the pituitary of the adult, head end to the right

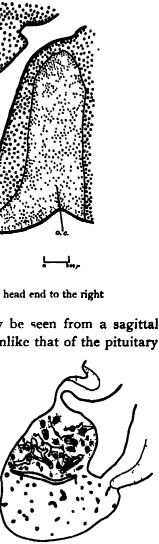


FIG. 5.—Diagram to show the relative vascularity of the various parts

as a blood supply. The rest of the organ is anterior lobe; there is no tuberous lobe. The gland cavity is now flattened in shape and stretches about three-quarters of the way through the organ; its walls throughout are formed of closely-set cells, and it shows no connection with the exterior in these sections.

The anterior lobe is composed of irregular masses of cells separated by thin sheets of connective tissue. There is a good blood supply. The special methods recommended by Bailey (1) and de Beer (2) for differential staining of the cell granules proved with this material inconsistent, possibly owing to its long fixation in formalin; however, after treatment with corrosive sublimate, Mallory's connective tissue stain gave quite good differentiation The basophil cells tended to be arranged in groups, and they were quite frequently orientated about a common centre with their nuclei towards the periphery. The eosinophils were more scattered, and no definite indications of secretory activity were noticed. The cells of the intermediate lobe did not give any reaction.

The early development of the ectophysis as a solid ingrowth recalls that of the Amphibians, Teleosts, and Cyclostomes. The resemblance to the first two ceases at an early stage when a cavity develops, and though the resemblance to the last persists longer, the final stages of development are quite unlike. The resemblance to Reptiles and Selachians is less than to the above, so that the comparative morphology of the gland is obscure.

BIBLIOGRAPHY.

(1) BAILEY, P , 1921. *Journ. Med. Res.*, vol. xlii.

(2) DE BEER, G. R., 1926. *The Comparative Anatomy, Histology, and Development of the Pituitary Body.*

(3) KERR, J. GRAHAM, 1909. Keibel's *Normentafeln zur Entwicklungsgeschichte der Wirbeltiere*, Heft. x.

(4) KERR, J. GRAHAM, 1902. "The Development of Lepidosiren paradoxa —III," *Quart. Journ. Micr. Sci.*, vol. xlvi.

(5) STENDELL, W., 1913 "Die Hypophysis Cerebri" (Oppel's *Lehrbuch der vergleichenden mikroskopischen Anatomie*, VIII).

(*Issued separately June* 13, 1933.)

XII.—Matrices and Continued Fractions. By Professor H. W. Turnbull, F R.S

(MS. received February 21, 1933 Revised MS received April 20, 1933
Read May 15, 1933)

INTRODUCTION.

IN a communication to this Society in 1916 Professor E T. Whittaker[2] gave an illuminating exposition of the use of matrices in dealing with continued fractions. In particular he discussed the problem of differentiating the following function of x:—

$$f(x)=\frac{1}{b+x-}\,\frac{a_1}{b_1+x-}\,\frac{a_2}{b_2+x-}\quad\cdot\cdot,$$

the value of $f'(x)$ being obtained in the first instance from the leading element in the square of a certain reciprocal matrix $\{M(x)\}^{-1}$ (*cf* formula (9), § 3, below) by a method which I have recapitulated at the opening of § 4 The expression for $f'(x)$ was then given as the quotient of two determinants obtained in a simple and specific manner from the matrix $M(x)$ itself. It was remarked that this treatment of continued fractions takes its rise in a work by Sylvester,[1] who incidentally showed that the fraction $f(x)$ could be expressed as the quotient of two determinants. It was clear that this quotient expressed the leading element in the reciprocal of the matrix $M(x)$. and the question naturally arose, what were the values of the other elements of this reciprocal? An answer to this question will be found in § 3 below, which in turn leads to an expression for the derivative of the function $f(x)$ as a series (5), § 4, whose general term is of the form

$$(q_i(x)f(x)-p_i(x))^2/a_1a_2\ .\ .\ .\ a_{i+1},$$

where $p_i(x)/q_i(x)$ is the ith convergent to $f(x)$. It is thought that this is a new type of infinite series such as has not yet received attention, but may well deserve closer scrutiny. I have merely given a preliminary condition, which is sufficient to ensure convergence and which may be satisfactorily applied to certain examples.

Next another well-known type is considered,

$$\frac{1}{1+}\,\frac{a_1x}{1+}\,\frac{a_2x}{1+}\cdot\cdot\cdot,$$

and an analogous result is reached, after which the difficulties inherent in more general representations of $f(x)$ are indicated.

§ 1. CONTINUANT MATRICES.

Ordinary continued fractions depend upon a matrix

$$M = \begin{vmatrix} b_0 & c_1 & & . & .. & . & \\ d_1 & b_1 & c_2 & . & . & . & . \\ . & d_2 & b_2 & c_3 & .. & . & . \\ . & & & & & . & . \\ . & . & & . & .. & d_n & b_n \end{vmatrix} \quad . \quad . \quad (1)$$

which may usefully be called a *continuant*. It is evidently capable of resolution into three diagonal matrices, so that

$$M = B + C + D \quad . \quad . \quad (2)$$

where

$$\left.\begin{aligned} B &= \operatorname{diag}\ (b_0, b_1, \ldots, b_n), \\ C &= \operatorname{diag}_1\ (c_1, c_2, \ldots, c_n), \\ D &= \operatorname{diag}_{-1}(d_1, d_2, \ldots, d_n). \end{aligned}\right\} \quad . \quad . \quad (3)$$

Each of these four matrices consists of $(n+1)$ rows and columns, where the elements are zeros except for those of the diagonals indicated, which are arbitrary. The suffixes ± 1 show that C and D are respectively the first over, and under, diagonal: and these matrices satisfy a product law, which is readily verified:

$$C^2 = \operatorname{diag}_2(c_1c_2, c_2c_3, \ldots, c_{n-1}c_n),$$
$$C^3 = \operatorname{diag}_3(c_1c_2c_3, c_2c_3c_4, \ldots, c_{n-2}c_{n-1}c_n),$$

and so on, while

$$D^2 = \operatorname{diag}_{-2}(d_1d_2, d_2d_3, \ldots, d_{n-1}d_n).$$

Both C and D are essentially singular matrices, and such that

$$C^{n+k} = 0, \quad D^{n+k} = 0, \quad k > 0.$$

On the other hand, the principal diagonal B satisfies the relation

$$B^r = \operatorname{diag}\ (b_0{}^r, b_1{}^r, \ldots, b_n{}^r)$$

for all values of r.

This resolution of a matrix into diagonals is of general application, where instead of three there would be $(2n+1)$ such diagonal components. Products of diagonal matrices are then formed according to the rule

$$\begin{aligned} &\operatorname{diag}_r(x_1, x_2, x_3, \ldots)\ \operatorname{diag}_s(y_1, y_2, y_3, \ldots) \\ &\quad = \operatorname{diag}_{r+s}(x_1y_{r+1}, x_2y_{r+2}, \ldots), \end{aligned}$$

where $r > 0$, $s > 0$, and its analogues when r or s is negative.

§ 2. APPLICATION TO CONTINUED FRACTIONS.

According to the general theory initiated by H. J. S. Smith[3] any non-singular matrix M, whether continuant or not, can be reduced to a simpler form Q by means of an identity

$$HMK=Q=\operatorname{diag}\left(q_0, \frac{q_1}{q_0}, \frac{q_2}{q_1}, \ldots, \frac{q_n}{q_{n-1}}\right), \quad . \quad . \quad (1)$$

where

$$|M|=|Q|=q_n\neq 0, \qquad |H|=|K|=1, \qquad q_i\neq 0.$$

Thus the determinants of M and Q are each equal to q_n, which is assumed to be non-zero. Reciprocally it follows that

$$H^{-1}QK^{-1}=M \quad . \quad . \quad . \quad (2)$$

Among the many rational ways of performing such a reduction the following is of interest, because of its connexion with the theory of continued fractions. The product on the left of (2) is taken to be

$$\begin{bmatrix} 1 & \cdot & \cdot & \ldots \\ \frac{d_1}{q_0} & 1 & \cdot & \ldots \\ \cdot & \frac{d_2q_0}{q_1} & 1 & \ldots \\ \cdot & \cdot & \cdot & \ldots \end{bmatrix} \begin{bmatrix} q_0 & \cdot & \cdot & \ldots \\ \cdot & \frac{q_1}{q_0} & & \\ & & \frac{q_2}{q_1} & \cdot \\ & \cdot & & \end{bmatrix} \begin{bmatrix} 1 & \frac{c_1}{q_0} & \cdot & \ldots \\ \cdot & 1 & \frac{c_2q_0}{q_1} & \\ & & 1 & \cdot \\ & & & \cdot \end{bmatrix}; \quad (3)$$

and when this is multiplied out it is identically equal to the original value of M, provided that the following recurrence relations hold:—

$$q_0=b_0, \quad q_1=b_1q_0-c_1d_1, \quad q_r=b_rq_{r-1}-c_rd_rq_{r-2}, \quad r=2, 3, \ldots \quad . \quad (4)$$

Manifestly the determinants of the first and third matrices (3) are equal to unity, so that all the requirements of the identity (1) are satisfied. It further follows that

$$Q^{-1}=\operatorname{diag}\left(\frac{1}{q_0}, \frac{q_0}{q_1}, \frac{q_1}{q_2}, \ldots, \frac{q_{n-1}}{q_n}\right) \quad . \quad . \quad . \quad (5)$$

and thereafter it is seen that, in (3),

$$H^{-1}=I+D\frac{1}{Q}, \qquad K^{-1}=I+\frac{1}{Q}C,$$

where I denotes the unit matrix. Reciprocally,

$$\left.\begin{aligned} H&=I-D\frac{1}{Q}+\left(D\frac{1}{Q}\right)^2-\left(D\frac{1}{Q}\right)^3+\ldots+(-)^n\left(D\frac{1}{Q}\right)^n, \\ K&=I-\frac{1}{Q}C+\left(\frac{1}{Q}C\right)^2-\left(\frac{1}{Q}C\right)^3+\ldots+(-)^n\left(\frac{1}{Q}C\right)^n, \end{aligned}\right\} \quad . \quad . \quad (6)$$

where each term consists of a diagonal matrix, on or below the principal diagonal for H; on or above for K. If for brevity x_i is written for $d_i q_{i-2}/q_{i-1}$, then

$$H^{-1}=\begin{bmatrix} 1 & . & . & . & \cdots \\ x_1 & 1 & . & . & \cdots \\ . & x_2 & 1 & . & \cdots \\ . & . & x_3 & 1 & \cdots \\ . & . & . & . & . \end{bmatrix}, \qquad H=\begin{bmatrix} 1 & . & . & . & \cdots \\ -x_1 & 1 & . & . & \cdots \\ x_1x_2 & -x_2 & 1 & . & . \\ -x_1x_2x_3 & x_2x_3 & -x_3 & 1 & .. \\ . & . & . & . & . \end{bmatrix} \qquad (7)$$

where the diagonal matrices, given by the terms of (6), are explicitly indicated. Similarly for K and its reciprocal.

Starting with different initial conditions, let a further set of quantities p_r satisfy the same recurrence relations as before:

$$p_0=1, \quad p_1=b_1, \quad p_r=b_r p_{r-1}-c_r d_r p_{r-2}, \quad r=2, 3, \ldots \qquad (8)$$

Also let

$$a_1=c_1d_1, \quad a_2=c_2d_2, \quad \ldots, \quad a_n=c_nd_n. \qquad (9)$$

Then the p_i and q_i serve to define the successive convergents of the continued fraction

$$f_r=\frac{1}{b_0-}\,\frac{a_1}{b_1-}\,\frac{a_2}{b_2-}\cdots\frac{a_r}{b_r}=\frac{p_r}{q_r} \qquad (10)$$

$$(r=0, 1, \ldots, n).$$

As Sylvester[1] has shown, the continued fraction is intimately connected with the matrix M, being in fact the leading element in its reciprocal. This follows at once from (3), and its analogue with p substituted throughout for q. In fact, by taking determinants of (2) and (3) we have

$$q_n=|\,Q\,|=|\,M\,|;$$

and next, by putting $c_1=0$, $b_0=1$, relations (4) become (8) with p replacing q: and hence

$$p_n=|\,P\,|=|\,M_{11}\,|,$$

where $P=\operatorname{diag}(p_0, p_1/p_0, p_2/p_1, \ldots)$, and M_{11} is the new form of M. But if $c_1=0$, $b_0=1$, then the determinant $|\,M\,|$ is precisely the cofactor of its leading element m_{11}. This proves the theorem of Sylvester, since now

$$f_n=\frac{p_n}{q_n}=\frac{|\,M_{11}\,|}{|\,M\,|}.$$

Hence the continued fraction is equal to the leading element in the reciprocal matrix M^{-1}. But it is interesting to go further and to calculate *all* the elements of this matrix, which is possible to do by reciprocating the relation (2).

§ 3. THE RECIPROCAL MATRIX M^{-1}.

By § 2 (7) the product which gives the reciprocal of M in the form

$$M^{-1} = KQ^{-1}H \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

can be exhibited explicitly:

$$\begin{bmatrix} 1 & -\frac{c_1}{q_0} & \frac{c_1c_2}{q_1} & -\frac{c_1c_2c_3}{q_2} & \cdots \\ . & 1 & -\frac{c_2q_0}{q_1} & \frac{c_2c_3q_0}{q_2} & \cdots \\ . & . & 1 & -\frac{c_3q_1}{q_2} & \cdots \\ . & . & . & . & . \end{bmatrix} \begin{bmatrix} \frac{1}{q_0} & . & . & \cdots \\ . & \frac{q_0}{q_1} & . & \cdots \\ . & . & \frac{q_1}{q_2} & \cdots \\ . & . & . & . \end{bmatrix} \begin{bmatrix} 1 & . & . & \cdots \\ -\frac{d_1}{q_0} & 1 & . & \cdots \\ \frac{d_1d_2}{q_1} & -\frac{d_2q_0}{q_1} & 1 & \cdots \\ . & . & . & . \end{bmatrix} \qquad (2)$$

Further, let b_{ij} denote the element in the ith row and the jth column of this product, so that $M^{-1} = B = [b_{ij}]$. Also let

$$\begin{aligned} c_{ij} &= c_{i+1}c_{i+2} \ldots c_j, & (i<j) \qquad d_{ij} &= 0, \\ &= 1, & (i=j) \qquad &= 1 \quad . \quad . \quad . \quad . \qquad (3) \\ &= 0, & (i>j) \qquad &= d_{j+1}d_{j+2} \ldots d_i, \end{aligned}$$

and $\delta_{ij} = 0$ $(i \neq j)$, $\delta_{ii} = 1$, where each suffix i and j takes the values 0, 1, 2, . . ., n. Then the above results may be epitomized by the relation

$$M^{-1} = [b_{ij}] = \left[(-)^{i+j}\frac{c_{ij}q_{i-1}}{q_{j-1}}\right]\left[\frac{q_{i-1}}{q_i}\delta_{ij}\right]\left[(-)^{i+j}\frac{d_{ij}q_{j-1}}{q_{i-1}}\right] \quad . \quad . \qquad (4)$$

where, in each of the square brackets, the ijth element of the matrix concerned is explicitly stated. Here it must be understood that $q_{-1} = 1$.

By matrix multiplication we have

$$\begin{aligned} b_{ij} &= \sum_{k=0}^{n}\sum_{l=0}^{n}(-)^{i+k}\frac{c_{ik}q_{i-1}}{q_{k-1}}\frac{q_{k-1}}{q_l}\delta_{kl}(-)^{l+j}\frac{d_{lj}q_{j-1}}{q_{l-1}} \\ &= (-)^{i+j}\left\{\sum_{k=0}^{n}\frac{c_{ik}d_{kj}}{q_{k-1}q_k}\right\}q_{i-1}q_{j-1} \quad . \quad . \quad . \quad . \quad . \qquad (5) \end{aligned}$$

since $\delta_{kl} = 0$, unless $k = l$. The series here enclosed in brackets may be computed by means of the well-known expression

$$p_kq_{k-1} - p_{k-1}q_k = (pq)_k,$$

which evidently satisfies the relations

$$(pq)_k = a_k(pq)_{k-1} = \ldots = a_ka_{k-1} \ldots a_2a_1.$$

But $a_i = c_id_i$. Hence, by (3) and (5), we have for the leading element,

$$b_{00} = \frac{1}{q_0} + \frac{(pq)_1}{q_0q_1} + \frac{(pq)_2}{q_1q_2} + \ldots$$

$$=\frac{1}{q_0}+\left(\frac{p_1}{q_1}-\frac{1}{q_0}\right)+\dots+\left(\frac{p_n}{q_n}-\frac{p_{n-1}}{q_{n-1}}\right)=\frac{p_n}{q_n} \quad . \quad . \quad (6)$$

so that b_{00} is in fact the continued fraction, as Sylvester has shown. The actual expression (5) gives

$$b_{00}=\frac{1}{q_0}+\frac{a_1}{q_0q_1}+\frac{a_1a_2}{q_1q_2}+\dots+\frac{a_1a_2\dots a_n}{q_{n-1}q_n}. \quad . \quad . \quad (7)$$

Next let $(f-f_i)$ denote the residue when the first i terms of this last series are deleted, so that

$$f-f_i=f-\frac{p_i}{q_i}=\frac{a_1a_2\dots a_{i+1}}{q_iq_{i+1}}+\dots+\frac{a_1a_2\dots a_n}{q_{n-1}q_n}. \quad . \quad (8)$$

Then this is evidently the expression necessary for the computation of b_{ij} in general. The reciprocal matrix can now be stated:

$$M^{-1}=\begin{bmatrix} f, & -\frac{q_0}{d_1}(f-f_0), & \frac{q_1}{d_1d_2}(f-f_1), & -\frac{q_2}{d_1d_2d_3}(f-f_2), & \dots \\ -\frac{q_0}{c_1}(f-f_0), & \frac{q_0q_0}{a_1}(f-f_0), & -\frac{q_0q_1}{a_1d_2}(f-f_1), & \frac{q_0q_2}{a_1d_2d_3}(f-f_2), & \dots \\ \frac{q_1}{c_1c_2}(f-f_1), & -\frac{q_1q_0}{a_1c_2}(f-f_1), & \frac{q_1q_1}{a_1a_2}(f-f_1), & -\frac{q_1q_2}{a_1a_2d_3}(f-f_2), & \cdot \\ -\frac{q_2}{c_1c_2c_3}(f-f_2), & \frac{q_2q_0}{a_1c_2c_3}(f-f_2), & -\frac{q_2q_1}{a_1a_2c_3}(f-f_2), & \frac{q_2q_2}{a_1a_2a_3}(f-f_2), & \dots \\ \dots & & \dots & & \dots \end{bmatrix}, \quad (9)$$

where $a_i=c_id_i$. The entrance of each successive factor $(f-f_i)$, according to the *gnomon* pattern, is an interesting feature of the result. Indeed, the same feature may be seen in the suffix notation of the original matrix M, where the suffix may be said to define the gnomon in which the element stands.

The general value of the ijth element of M^{-1} is therefore given by

$$b_{ij}=(-)^{i+j}\frac{q_{i-1}q_{j-1}}{c_1c_2\dots c_id_1d_2\dots d_j}(f-f_r),$$

where $r=\max(i+1, j+1)$. that is, r is the larger of $(i+1)$ and $(j+1)$.

§ 4. DIFFERENTIATION OF A CONTINUED FRACTION.

Let

$$f(x)=\frac{1}{b_0+x}-\frac{a_1}{b_1+x}-\frac{a_2}{b_2+x}-\dots-\frac{a_n}{b_n+x} \quad . \quad . \quad (1)$$

be a rational function of x, constructed by adding x to each element b_i of the above continued fraction, so that $f(0)$ would denote the original fraction. Then evidently $f(x)$ is associated with a certain matrix,

$$M(x)=M+I_n \quad . \quad . \quad . \quad . \quad . \quad (2)$$

obtained by adding x to each principal diagonal element of the original M, where as before I is the unit matrix. Also, from the result of the last paragraph,

$$\frac{1}{M+I_x} = (M(x))^{-1} = \begin{bmatrix} f(x), & \cdots \\ \cdot & \cdot \end{bmatrix}. \qquad (3)$$

If this is differentiated with regard to x, then the result may be written down by the rules of ordinary algebra, seeing that the constant matrices M and I commute. Thus

$$-\frac{1}{(M+Ix)^2} = \begin{bmatrix} f'(x), & \cdots \\ \cdot & \cdot \quad \cdot \end{bmatrix}, \qquad (4)$$

that is, $f'(x)$ is the leading element in the square of the reciprocal of $M(x)$, accompanied by change of sign.

But the leading element of the square of the matrix M^{-1} in § 3 (9) is at once found by multiplying the ith element of the top row by the ith element of the first column and adding the results for each value of i. This gives the formula

$$f'(x) = -\left\{f^2 + \frac{q_0^2}{a_1}(f-f_0)^2 + \frac{q_1^2}{a_1 a_2}(f-f_1)^2 + \cdot \qquad + \frac{q_{n-1}^2}{a_1 a_2 \ldots a_n}(f-f_{n-1})^2\right\}, \qquad (5)$$

where f, f_i, q_i now denote respectively the continued fraction, the ith convergent and the denominator of the ith convergent when the variable x has been inserted throughout. Thus

$$f'(x) = -\{f(x)\}^2 - \sum_{i=0}^{n-1}\left\{\frac{(q_i(x))^2}{a_1 a_2 \; \cdot\cdot\, a_{i+1}}\left(f(x) - \frac{p_i(x)}{q_i(x)}\right)^2\right\}. \qquad (6)$$

This result once obtained can be proved anew by induction, by extending the continued fraction to the left. For let

$$\frac{P_{n+1}}{Q_{n+1}} = \phi(x) = \frac{1}{b+x-a_0 f(x)} = \frac{1}{b+x-}\,\frac{a_0}{b_0+x-} \cdot \qquad \frac{a_n}{-b_n+x}. \qquad (7)$$

The formula is true for $n=1$, and will therefore follow for $\phi(x)$ (the case for $n+1$), if it is true for $f(x)$ (the case for n). Now, assuming the result for $f(x)$, we have

$$\begin{aligned}\phi'(x) &= \{\phi(x)\}^2 (a_0 f'(x) - 1) \\ &= -\left(\phi^2 + a_0\phi^2 f^2 + \frac{a_0 q_0^2 \phi^2}{a_1}(f-f_0)^2 + \;\ldots\right),\end{aligned}$$

where

$$\phi = \phi(x), \qquad f = f(x), \text{ etc.}$$

Furthermore, if $\phi_i = P_i/Q_i$, then evidently

$$\frac{P_{i+1}}{Q_{i+1}}=\phi_{i+1}=\frac{1}{b+x-a_0f_i}=\frac{q_i}{(b+x)q_i-a_0p_i}.$$

Hence

$$P_{i+1}=q_i, \qquad Q_{i+1}=(b+x)q_i-a_0p_i. \qquad . \quad . \quad . \quad (8)$$

It follows that

$$\phi-\phi_{i+1}=\frac{a_0(f-f_i)P_{n+1}P_{i+1}}{Q_{n+1}Q_{i+1}}=\frac{a_0q_i(f-f_i)\,\phi}{Q_{i+1}}. \qquad . \quad . \quad (9)$$

Hence, by substitution,

$$\phi'(x)=-\left\{\phi^2+\frac{Q_0^2}{a_0}(\phi-\phi_0)^2+\ .\ .\ .\ +\frac{Q_n^2}{a_0a_1\ .\ .\ .\ a_n}(\phi-\phi_n)^2\right\}$$

which is the required result, so that the induction is complete.

If all the elements a_i and b_i are real, then naturally $f(x)$ and $f'(x)$ are real with x. If, further, all the a_i are positive, then $f'(x)<0$, as is evident from the actual series. Hence *the continued fraction $f(x)$ is a monotonic decreasing function of x, for all real values of the b_i, provided that all the a_i are positive.*

It can further be shown in this case ($a_i>0$) that the function $f(x)$ possesses exactly $(n+1)$ distinct simple poles, let us say at the values $x=\lambda_0, \lambda_1, \ldots, \lambda_n$, separating the curve $y=f(x)$ into $(n+2)$ distinct branches each running from $y=+\infty$ to $-\infty$. This is done by rendering the matrix M symmetric.

In fact, when $a_i>0$, let $c_i=d_i=\sqrt{a_i}$, so that the matrix is real and identical with M' its transposed, and

$$M=\begin{bmatrix} b_0 & c_1 & . & \cdots \\ c_1 & b_1 & c_2 & \cdots \\ . & c_2 & b_2 & \cdots \\ . & . & . & \cdots \end{bmatrix}=M'. \qquad . \quad . \quad . \quad . \quad (10)$$

As is well known,[4] there always exists a real orthogonal matrix H (such that $H'H=I$, and so $|H|\neq 0$), which reduces M to a pure diagonal form:

$$H'MH=\begin{bmatrix} \lambda_0 & . & . & \cdots \\ . & \lambda_1 & . & \cdots \\ . & . & \lambda_2 & \cdots \\ . & . & . & \cdots \end{bmatrix}, \qquad . \quad . \quad . \quad . \quad (11)$$

where the λ_i are the latent roots of M, being the zeros of the characteristic function

$$|\lambda I-M|\equiv(\lambda-\lambda_0)\ .\ .\ .\ (\lambda-\lambda_n),$$

all of which are real. Hence also

$$\begin{aligned} H'(M+xI)H &= H'MH+xI \\ &= \operatorname{diag}(\lambda_0+x,\ \lambda_1+x,\ \ldots,\ \lambda_n+x)=\operatorname{diag}(\lambda_i+x). \end{aligned}$$

But

$$H'^{-1}=H \quad \text{and} \quad H^{-1}=H'.$$

Hence, reciprocally,

$$H'(M+xI)^{-1}H=\operatorname{diag}\left(\frac{1}{\lambda_i+x}\right);$$

that is,

$$(M+xI)^{-1}=H\operatorname{diag}\left(\frac{1}{\lambda_i+x}\right)H'.$$

Now the continued fraction $f(x)$ has remained unchanged by taking $c_i=d_i$, since $c_i d_i$ remains unaltered and equal to a_i. Hence it is given by the leading element in this last product matrix: and this element in turn depends only upon the first row

$$[h_{00}, \ h_{01}, \ \ldots, \ h_{0n}] \qquad . \quad . \quad . \quad (12)$$

of the real orthogonal matrix H, and upon the first column of its transposed H'. Hence *the product yields the identity*

$$f(x)=\frac{h_{00}^2}{\lambda_0+x}+\frac{h_{01}^2}{\lambda_1+x}+ \ \ldots \ +\frac{h_{0n}^2}{\lambda_n+x} \qquad . \quad . \quad . \quad (13)$$

where the h_{0i} and the λ_i are real constants, provided that the a_i are positive and the b_i are real.

It is also impossible for this series of partial fractions to fall short of $(n+1)$ terms. For since

$$p_nq_{n-1}-p_{n-1}q_n=a_1a_2 \ldots a_n$$

it follows that the polynomials p_n and q_n cannot contain a common factor involving x, else the product of non-zero constants $a_1 \ldots a_n$ would do so too. Hence also the symmetrical determinant $|M+xI|$ and its leading principal minor have no common factor, since these are equal to q_n and p_n respectively: that is, the characteristic function $|\lambda I-M|$ has no repeated factor (4). But it has $(n+1)$ real linear factors: and these must accordingly be all distinct, as in the expression (13) above.

§ 5. INFINITE CONTINUED FRACTIONS.

In the case when $n \to \infty$ it is interesting to inquire whether the series for $f'(x)$ converges when the continued fraction itself converges. One form of the condition, as given by Pringsheim[5], is that the inequality

$$|b_n+x| > |a_n|+1 \qquad . \quad . \quad . \quad . \quad (1)$$

should hold for all n greater than a certain value n_0. Evidently $f'(x)$ will converge if

$$\sum u_n \equiv \sum_{n=1}^{\infty}\frac{q_{n-1}^2}{a_1a_2 \ldots a_n}(f-f_{n-1})^2 \qquad . \quad . \quad . \quad (2)$$

converges. To examine this let the nth remainder of the continued fraction be introduced, namely,

$$\theta_n = \frac{a_{n+1}}{b_{n+1}+x-}\,\frac{a_{n+2}}{b_{n+2}+x-}\cdots \quad . \quad . \quad . \quad . \qquad (3)$$

As Pringsheim has remarked, the convergence of $f(x)$ does not necessarily depend upon that of θ_n If θ_n converges, then $f(x)$ is said to be conditionally convergent; otherwise $f(x)$ is unconditionally convergent.

It will here be shown that *a sufficient condition for the absolute convergence of the series* Σu_n *is that*

$$|\,\theta_n\,|\,/\,|\,\sqrt{a_{n+1}}\,| \rightarrow l < 1, \qquad as \quad n \rightarrow \infty . \quad . \quad . \qquad (4)$$

The proof is straightforward and depends upon the value of u_{n+1}/u_n We have $f_{n-1} = p_{n-1}/q_{n-1}$,

$$u_n = \frac{q_{n-1}{}^2(f-f_{n-1})^2}{a_1 \;.\; a_n}, \quad \text{and} \quad f = \frac{p_{n-1}-\theta_{n-1}p_{n-2}}{q_{n-1}-\theta_{n-1}q_{n-2}}.$$

Hence

$$u_n = \frac{\theta_{n-1}{}^2 a_1 a_2 \ldots a_{n-1}}{a_n(q_{n-1}-\theta_{n-1}q_{n-2})^2},$$

so that

$$\frac{u_{n+1}}{u_n} = \frac{\theta_n{}^2 a_n{}^2 (q_{n-1}-\theta_{n-1}q_{n-2})^2}{\theta_{n-1}{}^2 a_{n+1}(q_n-\theta_n q_{n-1})^2}.$$

But consecutive remainders, θ_{n-1} and θ_n, are connected by the manifest relation

$$\theta_{n-1} = \frac{a_n}{b_n + x - \theta_n}.$$

and when this is substituted in the above and the value of q_n is given by means of its recurrence relation, the result is

$$\frac{u_{n+1}}{u_n} = \frac{\theta_n{}^2}{a_{n+1}}$$

This at once yields the condition for convergence as stated.

Such series exist, as, for example, when

$$y = f(x) = \frac{1}{x+}\,\frac{1}{x+}\cdots\frac{1}{+x+}\cdots, \qquad a_i = -1,\ b_i = 0.$$

The conditions for convergence are here

$$\text{(i)}\ |\,x\,| = |\,b_n + x\,| > |\,a_n\,| + 1 = 2.$$

$$\text{(ii)}\ \left|\frac{\theta_n{}^2}{a_{n+1}}\right| = |\,f(x)^2\,| < 1.$$

Now

$$y=\frac{1}{x+y}, \quad \text{and} \quad 2y=-x+\sqrt{x^2+4}.$$

Both conditions are satisfied if x is real and greater than 2.

§ 6. SECOND TYPE OF CONTINUED FRACTION.

Very much the same method may be applied to another type of continued fraction, which is sometimes met,

$$\frac{p_n(x)}{q_n(x)}=f_n(x)=\frac{1}{1+}\,\frac{a_1x}{1+}\,\frac{a_2x}{1+}\cdots\frac{a_nx}{1}, \qquad (q_n(x)\neq 0) \qquad . \qquad (1)$$

and which again denotes a rational function of x.

Curiously enough the most convenient method here is to employ an irrational matrix,

$$N=\begin{bmatrix} 1 & \sqrt{a_1x} & & \cdots \\ -\sqrt{a_1x} & 1 & \sqrt{a_2x} & \cdot\;\cdot \\ \cdot & -\sqrt{a_2x} & 1 & \cdot\;\cdot \\ \cdot\;\;\cdot\;\;\cdot & & \cdot\;\;\cdot\;\;\cdot & \cdot\;\;\cdot \end{bmatrix}=I+A\sqrt{x}, \quad . \quad . \quad (2)$$

but one which still possesses the characteristic property that the leading element in its reciprocal is equal to $f_n(x)$. Regarded as a function of x this possesses coefficient matrices I and A which commute: and hence the matrix N can be manipulated, for purposes of differentiation with regard to x, as if it were an ordinary scalar function. Thus

$$\frac{1}{N}=\begin{bmatrix} f_n(x), & \cdots \\ \cdot & \cdot\;\;\cdot \end{bmatrix}, \qquad \frac{-A}{2\sqrt{x}(I+A\sqrt{x})^2}=\begin{bmatrix} f_n'(x), & \cdots \\ \cdot & \cdot\;\;\cdot \end{bmatrix},$$

so that the value of $f_n'(x)$ is given by the leading element in the matrix

$$-\frac{1}{2\sqrt{x}}AN^{-2}.$$

Now, by hypothesis, the matrix A consists of two equal diagonals (of opposite sign) obtained from

$$D=\operatorname{diag}(\sqrt{a_1}, \sqrt{a_2}, \ldots, \sqrt{a_n}, \sqrt{a_{n+1}})$$

and such that

$$A=DU-U'D,$$

where U, the auxiliary unit matrix,[4] translates D one place to the right, and U', the transposed matrix, translates D one place downwards. The leading element in AN^{-2} is consequently equal to the difference of those in DUN^{-2} and $U'DN^{-2}$. But the latter matrix necessarily has a zero top row, owing to the fore-factor U': and hence it contributes nothing to the leading element. Also the leading element of DUN^{-2} is $\sqrt{a_1}$ times that of UN^{-2}, that is $\theta\sqrt{a_1}$, *where* θ *is the first element in the second row of* N^{-2}, owing to the fore-factor U.

The element θ can now be calculated from the matrix N^{-1} which is the same in form as (20), that for M^{-1}, but with $c_t = \sqrt{a_t x} = -d_t$. This gives

$$\theta = -\left\{\frac{q_0}{c_1}f(f-f_0)+\frac{q_0^2}{c_1^2 d_1}(f-f_0)^2+\ .\ .\ .\right\};$$

whence

$$\begin{aligned} f_n'(x) &= \tfrac{1}{2}\left\{\frac{f(f-f_0)}{x}+\frac{q_0^2}{a_1x^2}(f-f_0)^2-\frac{q_1^2}{a_1a_2x^3}(f-f_1)^2+\ .\ .\ \right\} \\ &= \frac{1}{2x}f(f-f_0)+\sum_{r=0}^{n-1}(-)^r\frac{q_r^2(f-f_r)^2}{2a_1a_2\ .\ .\ .\ a_{r+1}x^{r+2}}, \qquad (39) \end{aligned}$$

where

$$f=f_n(x), \qquad f_r=f_r(x).$$

A proof by induction is also possible. It will be seen that this series is of the same general character as that for the derivative of the original continued fraction, so that it admits of similar treatment when n tends to infinity.

§ 7. GENERALIZATIONS.

It is natural to consider also the matrix

$$A+Bx\equiv C, \qquad (1)$$

where A and B are each of the type M and are independent of x. As before, the leading element of the reciprocal matrix C^{-1} is a continued fraction, whose derivative is given by the leading element of the matrix

$$\frac{d}{dx}(A+Bx)^{-1}.$$

Except, however, in the case when A commutes with B, the work is very complicated, although a formal expression may be given as follows:—

If B is non-singular, then (since $(PQ)^{-1}=Q^{-1}P^{-1}$)

$$(A+Bx)^{-1}=B^{-1}(AB^{-1}+Ix)^{-1}=B^{-1}\left(\frac{I}{x}-\frac{AB^{-1}}{x^2}+\frac{AB^{-1}AB^{-1}}{x^3}-\ .\ .\ .\right),$$

where $|x|$ must exceed the modulus of each latent root of the product matrix AB^{-1}. In this case the series may be differentiated, term by term, so that

$$\begin{aligned} \frac{d}{dx}(A+Bx)^{-1} &= B^{-1}\left(-\frac{I}{x^2}+\frac{2AB^{-1}}{x^3}-\frac{3AB^{-1}AB^{-1}}{x^4}+\ .\ .\ .\right) \\ &= -B^{-1}(Ix+AB^{-1})^{-2} = -(A+Bx)^{-1}B(A+Bx)^{-1}. \qquad (2) \end{aligned}$$

This proof of the formula fails if B is singular, in which case if A is non-singular the result may be given by

$$\frac{d}{dx}(A+Bx)^{-1} = -A^{-1}B(A+Bx)^{-1}A(A+Bx)^{-1} \quad . \quad . \quad (3)$$

or equally well by

$$-(A+Bx)^{-1}A(A+Bx)^{-1}BA^{-1} \quad . \quad . \quad . \quad (4)$$

both of which follow from the identity

$$(A+Bx)^{-1} = (I+A^{-1}Bx)^{-1}A^{-1}.$$

Once more, if both A and B are singular, the problem retains significance provided that C itself is non-singular. If then C is non-singular at a value x_1 of x, it may be written

$$C = (A+Bx_1)+B(x-x_1) = A_1+B(x-x_1), \qquad |A_1| \neq 0.$$

Hence, as before,

$$\frac{dC}{dx} = -(A+Bx)^{-1}A_1(A+Bx)^{-1}BA_1^{-1} \quad . \quad . \quad . \quad (5)$$

§ 8. Historical Note.

Several generalizations of continued fractions have from time to time been suggested by Jacobi,[6] Perron, Maunsell, Paley, Ursell, and Milne-Thomson. It may be shown without difficulty that these generalizations are connected with continuant matrices which are analogous to the continuant M, but of more complicated structure. The whole theory may be unified, as above, by a rational reduction to a diagonal matrix, as suggested by Smith's theorem.

REFERENCES TO LITERATURE.

1. Sylvester, 1853. *Phil. Mag.*, (4), vol v, p. 446, and vol. vi, p. 297. *Collected Works*, vol. i, pp. 609, 641

2. Whittaker, E T., 1916 *Proc. Roy. Soc Edin*, vol xxxvi, pp. 243–255.

3. Smith, H. J. S., 1861. *Phil Trans.*, vol. cli, pp. 293–326. *Collected Works*, vol. i, pp. 367–406.

4. Turnbull and Aitken, 1932 (Glasgow) *Canonical Matrices*, p. 106; p. 62.

5 Perron, 1913 (Teubner). *Die Lehre von dem Kettenbrüchen*, p. 254.
Pringsheim, 1898. *Münchener Sitzungsberichte*, vol. xxviii.

6. Jacobi, 1868. *Journal für reine und angew Math.* . . ., vol lxix, pp. 29–64.
Perron, 1907. *Math Annalen*, vol lxiv, p. 1.
Maunsell, 1929. *Proc. Lond. Math. Soc.*, (2), vol. xxx, p. 127.
Paley and Ursell, 1929. *Proc. Cambridge Phil. Soc.*, vol. xxvi, pp. 127–144.
Milne-Thomson, 1933. *Proc. Edin. Math. Soc.*, (2), vol. iii.

(*Issued separately June 23, 1933.*)

XIII.—The Formation of Rock Joints and the Cleat of Coal.

By **Emeritus Professor P. F. Kendall,** D.Sc., F.R S., and **Professor Henry Briggs,** D.Sc., Ph.D. (With a Plate.)

(MS received March 18, 1933 Read May 15, 1933)

PART I.

GENERAL CHARACTERISTICS OF THE PROBLEM.

THE occurrence in coal seams of planes of division, generally approximately at right angles to the stratification, has been known for centuries. Owing to the important part these planes play in the lay-out of workings, miners have found it necessary carefully to pay heed to their direction and character. They long ago observed, for example, that, in the majority of seams, there are main divisions or "cleats" and secondary divisions or "cross-cleats," and that these systems intersect at an angle of generally 80 to 90 degrees It was also seen that cleats are always best developed in the bright parts of bituminous coal, and that they are less distinct, more widely spaced, or even non-apparent in durain and in cannel. When the dull and bright parts are finely laminated, however, the cleat planes pass through the mass without interruption Sparry inclusions (ankerites) are very common in the cleats of bright coal and equally rare in dull coal; occasionally the inclusions extend a little way into the roof, and then the existence of associated cleat in the roof is particularly easy to detect.

A more specialised study of cleat has made clear a number of features of which the most important is the general uniformity in its direction over wide areas, and the general agreement in the polarity of the cleat in seams that may be separated by great thicknesses of rock.

Another remarkable attribute of cleat is its consistence and ubiquity: no band of bright coal, however thin, and no lenticle, however small, has been too insignificant to escape We have seen well-formed cleat in films of coal about one-five-hundredth of an inch in thickness; in coal-lenses a hand's breadth across occurring in Scottish oilshale (Lower Carboniferous); in the thin, coaly exteriors of fossil trees lying prostrate at all angles, and in coal pebbles embedded in conglomerates. These pebbles, torn off during the erosion of their parent beds and redeposited in the midst of other detritus, have been recorded in strata of widely varying ages in many districts of Great Britain and other countries.* Some are a few inches in diameter, while others are mere flakes or pellets; but all exhibit cleating. Moreover, when the beds in which they lie are situated among coal seams, the direction of cleat in the pebbles is found to agree with that of the seams. Evidently the cleat was induced in the pebbles subsequent to their deposition in the conglomerates

---

* X Stainier, *Ann. Soc Scient. Bruxelles*, vol. l, 1930, B, p. 34.

Though uncleated coals exist, cleating is a normal structure in coals of all ages, including the most recent. In the Isle of Wight lignite with a definite cleat lies among soft, Lower Eocene clays and sands. A lignite at Craig-a-hulyer, Ireland, exhibits both cleat and cross-cleat, the former coursing about N. 21° W.* Another Tertiary lignite exposed at Shorne Wood, Kent, also has cleat and counter-cleat, the mean direction of the main divisions being N. 24° W. A lignite in the Rift depression at Toro, Uganda, is late Tertiary; again, double cleating has been developed, the principal direction being N. 87° W. magnetic (1928) Such evidence points to the formation of cleat at no long period after the bed was deposited and at a relatively early stage of coalification.

In the majority, the roofs of coal seams have been jointed in the same direction as the cleat of the coal, though in all coalfields roof-joints are to be found that lie at other angles with the meridian, and occasionally the divergence is marked.† Sometimes, however, the agreement is absolute, and the cleat-planes of the coal can be traced upwards without a break or bend into the roof, and it is then impossible to allow of any explanation other than that of simultaneous division. Simultaneous cleating of the coal and one or more of the superjacent strata predicates that the coal remained uncleated at least as long as was required for the companion beds to be laid down and consolidated. On the other hand, when the lines are disparate the probability is that the roof received its divisions later than those of the coal, since it seems likely that the forces that were effective in that regard had changed their direction of action. It should not, however, be overlooked that the same system of forces may cause shearing in different directions owing to a difference in the strength properties of the rocks concerned.

The fact that the Isle of Wight lignite has obtained a cleat before the associated deposits are consolidated provides a reason for the non-simultaneity of cleating that must be of wide validity.

These considerations of the simultaneity or otherwise of the cleating of a coal seam and the adjacent beds, and of the agreement or otherwise of the directions, do not affect the important conclusion that cleat is a form of jointing—a conclusion that has been accepted by qualified observers for many years. The common features of conjugate directions of break, the regularity of the divisions, their uniformity of direction, and the fact that, despite numerous exceptions, the orientation of the cleat in coal and of the joints in other rocks is in general agreement, argue strongly for joints and cleat being identical in character and in origin. The difference between the jointing of rocks and the cleat of coal is one of dimension: joints in sandstone or limestone are usually separated by distances of a few feet or a few yards (though we have seen them only a few inches apart) while the cleat planes of coal are near together, half a dozen occasionally lying within the space of an inch.

---

* All bearings are to the true meridian, unless otherwise stated.

† P. F. Kendall, "On Cleat in Coal Seams," *Geol. Mag.*, February 1914, p. 49.

Beginning as an investigation of cleat, we have found it advisable to extend the inquiry into the larger problem of jointing, realising that, in a subject that offers many difficulties and inconsistencies, the wider survey is the more likely to provide the truer perspective.

The foregoing observations indicate that joint and cleat formation is a progressive operation: a bed, or a series consisting of a few beds, is laid down on strata already jointed; joints are induced in it, and it is succeeded by other beds which in turn acquire jointing. It is therefore evident that the forces causing jointing were continuing forces The further fact that recent coals are also cleated makes it clear that those influences were not special to any particular period, but that they have persisted through geological time and are therefore to be looked for among those incident on the earth's crust at the present day.

Polarity of Cleat and Jointing —An early observation on the regularity of cleat is that of E. Mammatt; * writing in 1834 of the Leicestershire coalfield, he stated:

"The extraordinary uniformity in the direction of the slynes [cleats] and of the partings of the rocky strata [joints] seems to have been determined by the operation of some law not yet understood. . . . Wherever these slynes appear, their direction is 23° W. of N. by the compass, whatever way the stratum may incline " Allowing for the magnetic declination of the time and place, the direction given is equivalent to N. 47½° W from the true meridian.

A few years later Milne,† describing the Lothians coalfield, remarked that the cleat was independent of the direction of dip of the strata. "I have observed at a great number of places," he continues, "the directions of the *backs* or lengthways joints, and also of the *cutters* or end-joints. The former appear to me to run everywhere in this district in a direction between N. and W. by compass. The cutters, of course, therefore run between E. and N." Milne tabulated forty bearings of joints and cross-joints, and most of these values have been incorporated on the map, fig. 2.

---

* "Geological Facts to Elucidate the Ashby de la Zouch Coalfield," 1834.

† *Coal Fields of the Lothians*, Edin., 1839, p. 16.

In 1849 E. Hopkins * recorded the general parallelism of joints in the primary rocks from the Straits of Magellan to the Caribbean Coast. He claimed that, in the isthmus of Panama and in Mexico and North America, whatever the contortions of the rocks, "the planes pass on without much deviation, running in lines in a direction a few degrees east of magnetic north," and added that similar phenomena were to be found in Australia, Asia, and Europe. There were many local aberrancies, he remarked, but the general course was well maintained So far as we know, this was the first time the parallelism had been recognised as being of far more than local or even than regional significance.

Though remarkably little on the subject is to be found in the literature of the last fifty years, jointing attracted the attention of some of the older school of British geologists, notably Sedgwick, de la Bêche, J. Phillips, Jukes, King, and Prestwich Phillips referred to cleat in coal as maintaining an approximately uniform direction, in spite of fractures or changes of dip, over the whole of a great Carboniferous region, and observed that much the same north-westerly trend applied to the coalfields of Northumberland, Durham, Yorkshire, Lancashire, and Derbyshire.† As early as 1836 this astute observer published an account of cleat in Yorkshire, and by means of a dial-diagram proved two directions to predominate, these lying at right angles ‡ Two years later Jukes, inquiring of cleat in the Nottinghamshire coalfield, was informed by a collier that it "faced two o'clock sun, like as it does all over the world, as ever I heered on " § If broadside to the sun at 2 p.m. the course of the planes would be about N. 60° W.; and the statement is evidence that the uniformity of bearing was recognised by miners at that time.

We shall have occasion, in Part II of this paper, to refer to the conclusions advanced in 1879 by the French geologist A. Daubrée in regard to the mechanics of jointing.|| The most important contribution on the theory of the subject of the nineteenth century, Daubrée's treatment will always deserve the closest study. Taking the river systems of France as his example, he demonstrated the connection between rock-jointing and physical geography; he insisted on the constancy of polarity of jointing,

---

* "On the Structure of Crystalline Rocks," *Mining Almanack*, 1849.

† *Brit. Assocn. Report*, 1856, p. 369.

‡ *Illustrations of Yorkshire*, vol. ii, 1836.

§ J. B. Jukes, *Student's Manual of Geology*, 2nd ed., 1862, p. 207.

|| *Étude Synthétiques de Géologie Expérimentale*, Paris, 1879, p. 303

both over large areas and through great thicknesses of rocks of different kinds; and he had much of interest to say in regard to the influence of faults on jointing.

The dial-diagram introduced by Phillips has been used by us on previous occasions for depicting the bearings of cleat in certain of the English coalfields.*

Thanks to the cordial co-operation of many friends and correspondents in many parts of the world, we are now able to report on a larger number of observations on cleat and jointing than has been possible hitherto. Though most of the values relate to British coalfields, a by no means insignificant proportion of the 2000 bearings we have collected come from abroad. Some of these, and particularly those from the United States, have been extracted from geological and other memoirs and reports, but for the majority we are indebted to a great many mine agents, managers, and surveyors. Nearly all the data furnished by these observers refer to the main cleat of coal; but latterly the inquiry has been extended to include other rocks and to gain information on the subsidiary planes of jointing.

The dial-diagram is no longer suitable to illustrate with sufficient definition the distribution of these large numbers of values, and we have adopted another and clearer system in fig. 1. That chart is a frequency diagram setting forth the *percentage* distribution of the geographical bearings of the principal planes of cleat and jointing. It indicates separately, and for every two degrees of arc from N. 90° W to N. 90° E., the relative frequency of occurrence of bearings for certain British coalfields and for that of West Virginia. It was constructed from 1150 values, though many of them are actually the means of several—and sometimes of several dozen—readings in the same mine or quarry.

The chart brings out clearly the great preponderance of directions lying between north and west. In some coalfields—West Virginia and Yorkshire, for example—the polarity of the great majority of the planes is between relatively close limits; in others, such as Scotland, the distribution is wider; but even there the bulk of the readings occur within that same quadrant.

When a main cleat is recorded in the north-east quadrant it is frequently found to be a cross-cleat that has become locally accentuated: on moving away from that area it is seen to resume its secondary character.

Judging from the Geological Memoir,† the cleat in the Arkansas coalfield is more regular than that for any of the regions whose names appear on fig. 1, it being stated to point to "11 o'clock sun" (S. 15° E. or N. 15° W.), with only insignificant variation. The main cleat of the Indiana field also appears to exhibit a marked uniformity of direction; according to G. H. Ashley * it runs N. 28° to 30° W., and is unaffected by irregularities of the beds.

---

* P. F. Kendall, *loc. cit*; Kendall and Wroot, *Geology of Yorkshire*, 1924; H. Briggs, *Colliery Engineering*, January 1932.

† A. J. Collier, "The Arkansas Coal Field," *U S. Geol. Surv. Mem.*, 1907, p. 48.

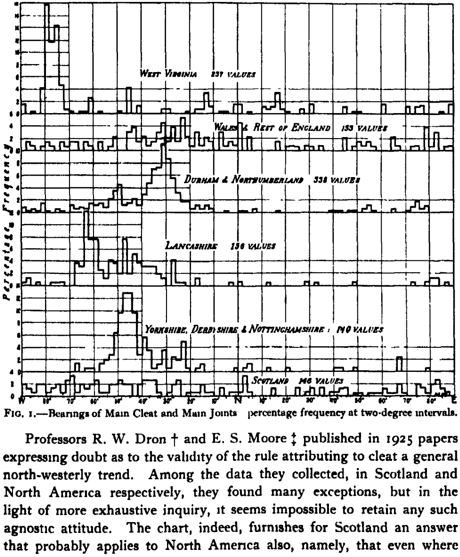


FIG. 1.—Bearings of Main Cleat and Main Joints percentage frequency at two-degree intervals.

Professors R. W. Dron † and E. S. Moore ‡ published in 1925 papers expressing doubt as to the validity of the rule attributing to cleat a general north-westerly trend. Among the data they collected, in Scotland and North America respectively, they found many exceptions, but in the light of more exhaustive inquiry, it seems impossible to retain any such agnostic attitude. The chart, indeed, furnishes for Scotland an answer that probably applies to North America also, namely, that even where

* *23rd Ann. Rep. Dept. of Geol. and Nat. Resources, Indiana*, p. 25.

† "Notes on Cleat in the Scottish Coalfields," *Trans. Inst. Min. Engs.*, vol. lxx, p. 115.

‡ *Coal Age*, vol. xxvii, p 576.

the exceptions are numerous, a sufficiently extensive survey upholds the rule. In general, the cleat at any given place runs in virtually the same direction in all the seams; yet, as Dron and Moore again observed, there are exceptions. A difference of 65° exists, for instance, between the cleat of the Wilsontown Main Coal and that of the overlying Woodmuir Main Coal at the Wilsontown Colliery, Lanarkshire. Many similar examples could be cited; yet the occasional wide discrepancy must not be allowed to mask the fact that, through great thicknesses of rock, the agreement in direction is often remarkably close, and that, for every exception to the rule, quite a dozen cases of conformity with it could be brought forward.

The following facts of observation may, then, be regarded as fully established:—

(1) In every part of the world "all rocks, stratified or unstratified, igneous, aqueous, and metamorphic are traversed by numerous planes of division." * (2) In general, the joints adhere to a quadrangular arrangement. Sometimes there are three systems, but generally two; and, of these two, one is the master-joint or main cleat and the other the cross-joint or cross-cleat. (3) The prevailing directions are approximately north-west and approximately north-east (4) In most Carboniferous districts areas can be found in which the master-joints run north-east, and occasionally, as in Nova Scotia, that direction is dominant for the main cleat over a large region; but, when all the available information is examined, it becomes decisively clear that there is one principal orientation—namely, north-west. Even with igneous rocks the trend of the chief joints is commonly west of north.

Mid and East Lothian Coalfield—The ready assistance we have received from the colliery agents and managers, and from Messrs G. D. Dott and R. M'Adam, has made it possible for us to study jointing and cleat in the Mid and East Lothian coalfield more intensively than circumstances have so far permitted elsewhere. The particulars are set forth on the map, fig 2 Mr Dott worked assiduously on the limestones in the south and east of the area, and the results that are shown for those rocks are mean values derived from several hundreds of his measurements

Several features combine to make the coalfield of unusual interest. The occurrence of a large number of workable seams both above and below the Millstone Grit, the great range of dip (0° to 83° in the beds examined); the rapid changes in dip and frequent changes in strike; the numerous faults; the proximity on the western side of the great, reversed Pentland fault, and the existence of many quarries, together make the field especially suitable for the elucidation of the problems under review.

---

* Jukes, *op. cit*, p. 207.

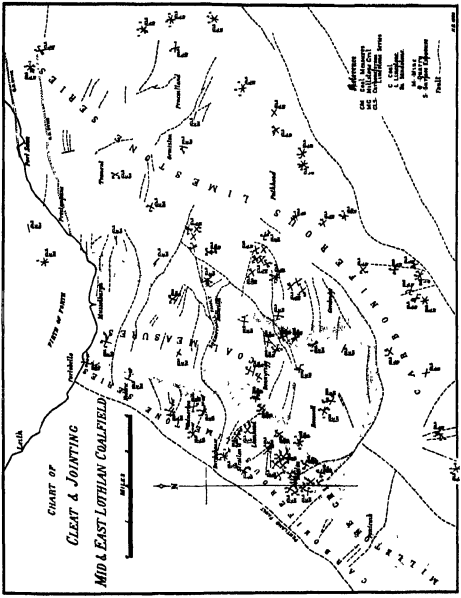


FIG. 2.

The map demonstrates in the first place that, on the whole, cleating and jointing has little or no connection with the dip. Secondly, it reveals the tendency, especially among the limestones, to develop triple jointing, a tendency that appears to be increased in the neighbourhood of faults.

Thirdly, it shows that the most persistent and, generally, the most prominent divisions * are those coursing approximately W.N.W., no matter what may be the character of the rock or its geological position. Fourthly, it throws light on the effect on the jointing of the Pentland and other faults, or, more strictly, of the forces that caused the faults.

Jointing and Faults.—The jointing of beds may be influenced by faults in any one of four ways:

(1) The first and probably the commonest effect is for the joints to be slewed round near the fault. Usually they become more parallel with it, though occasionally they are twisted in the opposite sense. The swing may be as much as 30°. The prevalence of east-and-west faults throughout the Scottish coalfields provides, indeed, an explanation for the many bearings in that region lying at a large angle from the meridian The slewing is most pronounced when the fault courses at about 45° to the normal direction of jointing.

Not infrequently the beds close to a fault are tilted steeply, and, as we shall see (p. 181), high inclination is itself a cause of joints becoming more parallel with the strike; but we do not know of any instance in which the slewing may entirely be attributed to this purely geometrical effect.

An excellent example of the swinging of joints near faults is furnished by the Great Scar Limestone near the Craven faults, Yorkshire.†

(2) In other cases, the forces that produced the fault have accentuated one or other of the normal planes of jointing that happened to be parallel with the fault. This effect may be studied with advantage in coals and sandstones over a wide strip parallel with the Pentland fault (fig. 2). The course of the fault agrees closely with that of the normal cross-jointing, and in the seams of Niddrie and Loanhead, as well as in the Grit along the North Esk, the cross-cleats and cross-joints have in consequence become the most prominent divisions. The absence of that effect at Roslin and Gilmerton appears to be due to the proximity of other faults roughly parallel with the ordinary north-westerly trend.

(3) When neither of the two normal directions conforms with that of the fault a third plane of division may be induced parallel with the fault. The rock then becomes triple jointed. Well-developed triple cleating is occasionally seen in coal, but, so far as our experience goes, the phenomenon is more usual, or more easily discerned, in limestone. An excellent example is the Burdiehouse limestone at Burdiehouse mine (fig. 2); there are three joints, and the master-joint is parallel with the Pentland fault.

---

* The order of importance of the planes is indicated on the map by the numerals 1, 2, and 3.

† L. R. Wager, *Q.J.G.S.*, vol. lxxxvii, 1931, p. 392.

Naturally enough, joints of this character are more frequent near the larger faults; they were recognised as a separate class by Goodchild,* who named them "fault joints."

(4) Lastly, faults may traverse strata without any observable effect on the joints. Here again the Burdiehouse mine provides an interesting example. The workings are bounded on their northern side by the Sheriffhall fault (fig. 2)—a normal fault having a maximum throw of about 500 feet The actual contact with the fault is to be seen at three or four places in the mine at the present time. The joints of the limestone extend to that contact without alteration either in spacing or in direction Large though the dislocation has been, there is no sign of movement in the joint planes nor have the cheeks of the joints been opened apart.

Cleat in the Southern Hemisphere —In contrast with the large number of readings available from Carboniferous regions north of the equator, our information concerning cleat in the southern hemisphere is scanty. In all we have only 23 bearings, and of these 14 lie in the north-west and 9 in the north-east quadrants. The values of greatest "weight," namely, those relating to the Natal coalfields † and to the district of Witbank, Transvaal,‡ definitely assign north-westerly bearings to the main cleat of these regions; yet it is clearly inadvisable to generalise on such slight evidence.

PART II.

THE DYNAMICS OF ROCK-JOINTING, WITH ESPECIAL REFERENCE TO THE CLEAT OF COAL.

Theories of Joint Formation.—The mechanical origin of joints is so generally admitted, and so clear from the evidence, that it is unnecessary to do more than make a brief mention of the other theories that have been advanced from time to time. One of the earliest was that joints are the result of shrinkage, and, in regard to cleat, that explanation is still current in some quarters. The meandering course of shrinkage cracks, forming, say, in mud, is altogether different from the incisive, straight, and definitely oriented line of cleat So far as coal is concerned, the only observations supporting that theory are the occurrence of ankerite in the cleats of bright coal, and the existence of open cleats as shown by X-ray examination.§ When those open cleats are traced to a thick bank of durain the gap disappears. The durain may be cleated, though usually at less frequent intervals; but the cheeks of the joints in dull coal are closer together and seldom hold sparry inclusions. If a layer of cannel is sandwiched between bands of bright coal the difference is still more marked: the cleats extend up to or down to the cannel and there abruptly stop Yet the cannel, though apparently uncleated, reveals on trial a propensity to break in the same direction as the well-formed cleats of the neighbouring bright layers. It is therefore clear that the spar-filled and gaping cleats of clarain and vitrain are merely a consequence of differential shrinkage, and that the shrinkage took place after the cleats were formed.

---

* *Geol. Mag.*, vol. x, 1883, p. 397.

† Data collected by Mr F. S Steart.

‡ Data collected by Mr A. C. Graham.

§ Briggs and Kemp, *Trans. Inst. Min. Engs.*, vol. lxxvii, 1929, p. 5; *Colliery Engineering*, January 1932, p. 15.

King,* while accepting the mechanical origin of slaty-cleavage, rejected it for jointing. He saw in the latter the effects of an agency akin to that producing cleavage in minerals; he also suspected a connection between the chief suite of joints and the magnetic meridian on the ground that the range of the meridian (about 23½° on each side of geographical north) was about the same as that of the joints whose directions he had measured. Other writers, both before and after his time, have also attempted to couple jointing and the earth's magnetism. Unfortunately for that theory, the more extensive data now available assign a bearing more nearly north-west than north for the mean direction of the principal series. The same observer, alluding to Fox's experiments on clays, in which divisions were produced by passing an electrical current through them, hazarded the guess that east-and-west earth currents may be found responsible for rock-jointing.

Prestwich, likewise influenced by the occasional similarity of the angles between joints and those between the cleavage planes of such minerals as calcite and felspar, was also of opinion that jointing was directed by a "condition of crystallization" in both igneous and sedimentary rocks.† Yet the joints (cleats) of coal, a non-crystalline substance, are better developed than, probably, in any other material. Again, in a table (p. 276) compiled by him from various sources, the directions of the main joints in granites lie in the north-west quarter in twelve out of the thirteen instances recorded—a degree of coincidence impossible to reconcile with the crystalline cleavage theory.

The same objection has equal force against W. O. Crosby's hypothesis that joints are due to seismic tremors. So sporadic and capricious an agency cannot have been responsible for divisions oriented, as these are, preponderatingly in one quarter.

---

* *Trans. Roy. Irish Acad.*, vol. xxv, 1875, p. 605.

† *Geology*, vol. i, 1886, p. 283.

James Geikie, though recognising mechanical stress as the chief agent, favoured the view that solar heat may have been a contributory cause of jointing.[*] The expansion of a surface bed exposed to the sun's rays during the day could, he observed, make the bed buckle upwards and even separate from the layer beneath No doubt in some strata and some climates the action he indicated is able to produce irregular horizontal divisions near the surface; but that it could be the means of inducing conjugate systems of joints extending vertically from top to bottom of beds hundreds of feet thick may safely be disallowed.

The most elementary and least satisfactory of the mechanical theories is that attributing joints to tensile stress. Dating back to Phillips (1856), the view is held to-day by some American writers. So strong, indeed, is the emphasis laid in some quarters on the effect of tension (whether pure tensile stress or tension as a component of shear) that even the definition of a joint may be made to uphold the theory For example, in his recent book Dr C. M. Nevin [†] defines a joint as a type of fracture along which there has been little if any movement, and in which "both the relief and major movement have been at right angles to the surface of fracture and not along or parallel with it." Such a definition implies that the cheeks of the fracture opened apart when the rock broke; it postulates an open crack as the normal condition of jointing, and it precludes shear as that term is understood by engineers.

We do not deny the existence of cracks due to tension, but they are the exception and not the rule; they are, indeed, of so special and unimportant a type that, in any general discussion of jointing, they may be left out of account We also allow that open joints are often to be seen at or near outcrops, where they have been under the influence of frost, of dilatation due to relief of load, and, in limestone, of solution; but an underground inspection is sufficient to establish the fact that, in the great majority of instances, the sides of joints are tight together. Even in folded ground, where gaping as a consequence of bending would not have been unexpected, the joints generally reveal themselves to be close structures. The smooth, regular and occasionally slickensided surfaces of joints are in themselves a sufficient refutation of the tension hypothesis. Again, in conglomerates—often well jointed—the divisions do not wriggle their way in between the pebbles as tension fractures would do, but pass indifferently through both hard pebbles and softer matrix in sharp, undeflected lines.[‡]

---

[*] *Structural and Field Geology*, 1908, p. 152.

[†] *Principles of Structural Geology*, New York, 1931, p. 132

[‡] See also J. B. Jukes, *op. cit.*, p 210.

The field evidence, regarded from both the geologist's and the engineer's standpoint, is in entire support of Daubrée's conclusion that true jointing is the effect of shear. The slight movement associated with jointing has been for the most part in the plane of the joint; not normal to it.

We have now to consider the nature of the stresses that caused the beds to shear in this special manner

The problem is essentially one calling for the broadest view It cannot be stated, let alone decided, from the polarity of the cleats and joints in a single district. Local anomalies are numerous; they present interesting and perhaps important issues; but to emphasise them unduly, as is occasionally done, is to render a just perspective impossible. The exceptions in this region or that cannot disturb the general conclusion, brought out in Part I of this paper, that, for the northern hemisphere at any rate, the dominant direction of the cleats in coal and the joints in other sedimentary rocks is north-westerly.

We are, then, concerned with a phenomenon of super-continental range. The effect under consideration is, we believe, unique in geological experience in its regularity over so large a geographical extent and over so enormous a period of time.

Character of the Stresses producing Jointing.—Exceptis excipiendis, true joints in sedimentary rocks possess the following characteristics:—

1. They never occur singly, but always as an extensive series of parallel divisions that maintain regularity of form and direction over wide areas

2. They habitually occur in two conjugate sets crossing each other at a large angle. (A third set may appear, though less often in coal than in other rocks.)

3. Generally they are at right angles to the bedding planes.

4. Their surfaces are close together, smooth and sometimes slickensided.

5. They differ from faults in not disturbing the continuity of the beds.

6. They differ from cleavage in being actual instead of merely latent planes of division (There is, nevertheless, a close relationship between jointing and fracture-cleavage. For example, in a compound seam of coal, jointing (cleating) may have been produced in the bright bands and a coarse fracture-cleavage in a layer of cannel.)

Many workers in the field of dynamical geology, from Daubrée * onwards, have studied the mechanism of faulting. It is well agreed that faults are shear fractures, and that every description of fault—normal, reversed, and transcurrent—is capable of being caused by non-rotational stress systems differently arranged. The common occurrence of inclined striæ on fault surfaces suggests the action of torsional stress, and no doubt torsion plays a part in faulting; no doubt, also, there are subsidiary features still to be elucidated; yet the central fact stands clear that all the chief features of *faulting* have been satisfactorily accounted for without invoking the action of any other than non-rotational stresses.

---

* See, *e g*, Daubrée, *Études Synthétique de Géologie Expérimentale*, Paris, 1879; E M. Anderson, "The Dynamics of Faulting," *Trans. Edin. Geol Soc*, vol. viii, 1905, p. 38; C. K. Leith, *Structural Geology*, New York, 1913; Briggs and Morrow, "The Rationale of Faulting and Subsidence," *Trans. Inst. Min. Engs.*, vol. lxxiii, 1927, p. 465; C. M. Nevin, *op. cit*

On the other hand, the numerous attempts to explain *jointing* by the same process, and by reference to the same stress systems, have failed. None of them is able to account for the two prime characteristics of joints· their great number, and their absence of throw. The fact has usually been overlooked that a non-rotational system, represented in the ordinary way by three "principal stresses" operating in directions mutually at right angles, cannot attain equilibrium by shearing without a permanent displacement of the shorn parts—in other words, without creating a fault. Moreover, these stresses can, and often do, obtain relief at a single shear fracture; thus a fault is often a solitary structure; but a solitary joint is unknown.

Though both are the consequence of shear, the differences between joint and fault are of so essential a nature that we must perforce abandon the non-rotational type of stress as the cause of the former and examine the only alternative—rotational stress or torsion.

Daubrée (*op. cit.*, p. 307) was the first to give expression to the torsional theory of joint formation, and his famous experiment with sheets of glass has often been repeated by ourselves and others.* So important is that experiment in its bearing on our problem that we deem it advisable to attempt an analysis of the stresses imparted to the glass, and to inquire into the changes in the manner of fracture when those stresses are modified in various ways.

The following facts are apparent when a pane is broken by *simple torsion* imposed by applying a twist to the shorter edges of the rectangle (Pl. fig. 3):—

1. The fractures are numerous, the whole surface being intersected by them.

2. The breaks are in two sets, crossing each other at angles of from 70° to 90°.

3. The breaks make an angle of 45° or thereabouts with the longer edges.

---

* *E.g.*, Versey, *Proc. Yorks. Geol. Soc.*, N.S., vol. xxi, p. 203.

4. There are main or master fractures (which may be of either series), extending from one edge of the pane to the other, and shorter subsidiary fractures that often extend only from one main break to the next.

5. The relative movement between two sides of a break is at right angles to the surface of the glass, or principally in that direction, and that movement is small.

6. The cracks commence at the boundaries of the plate and extend inwards.

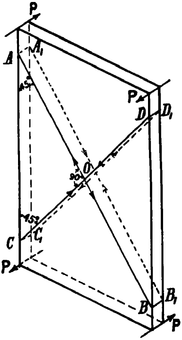


FIG 4.- Illustrating the forces acting on a thin sheet subjected to simple torsion

7. When the thickness of the plate is small compared with its area, the cracks are approximately normal to the surface, though considerable local variations from that rule are to be observed

8. The system of cracks bears a strong resemblance to rock-jointing, and especially to the cleat of coal.

An analysis of the state of strain of the glass sheet sufficiently complete for the present purpose may now be presented. Fig. 4 represents the pane resisting the tractions, *P*, impressed in the sense of the arrows at each corner. If the sheet be made of india-rubber, and a small circle be drawn on it having *O* at centre, the warping of the prism under the torsional stress will be found to cause the circle to be transformed into an ellipse; the major and minor axes of the ellipse will be seen to coincide respectively with *AB* and *CD*—two lines crossing at 90° and each making an angle of 45° with the edges.[*] So far as the surface turned towards the observer is concerned, then, a maximum tensile stress is operative in the direction of *AB* and a maximum compressive stress in the direction of *CD*.[†] If the other side be examined in like manner, C_1D_1 will be seen to be a line of maximum tension and A_1B_1 of maximum compression

---

[*] G. F. Becker, *Trans. Amer Inst Min. Engs*, vol. xxiv, 1895, p. 136.

[†] Nevin, an advocate of the tension theory of joint formation, argues (*op cit.*, p. 143) that the sheet ought to crack along *CD*, *i.e.* at right angles to the direction of maximum tension. Clearly, he omitted to look at the other side of the panel The glass, which persists in breaking in both directions, should itself have been sufficient to demonstrate the defectiveness of both the argument and the theory it seeks to uphold.

It follows that, in these conjugate directions, the material is subject to maximum shear strain. Bearing in mind that, when they occur, the fractures start at the edges and that the movement is approximately in the direction of such short lines as BB_1 or DD_1, it is evident that the stress along the edges of the sheet is the decisive agent. Now the shearing force at the edges is everywhere the same round the rectangle and is of the magnitude $P/2$. When the thickness is small compared with the other dimensions, it is therefore reasonable to conclude that the glass shatters when the shearing force at the edge becomes too great for the material to sustain, and that the shear-planes develop inwards along lines of maximum strain.

Treble Jointing.—The occasional appearance of three sets of cleats in coal has been remarked on in this paper and elsewhere, and is not so uncommon as we had once supposed.* The existence of three sets of high-angled joints in limestone has also been noted by many observers, and several examples are indicated on the map of the Mid and East Lothian coalfield (fig. 2). A well-known instance is that of the Miocene gypsum beds of Chaumont, Montmartre, described by Jukes † as having well-defined and equidistant vertical joints spaced 120° apart. A specimen of coal casually picked out of the scuttle as this was being written possesses two cleats at right angles and a third crossing the others at 45°, all have spar and pyrites between their faces; all are well developed, but the third is the clearest. It is unlikely that the third break in coal or rock was formed later than the others, since a bed already divided by planes 60° or 90° apart would, we suggest, be incompetent to withstand a new onslaught of stress: it would gain relief by yielding along the existing joints instead of fracturing afresh

But if the three systems were formed simultaneously, as seems probable, some additional factor not so far taken into account must have been present at the time. An extension of Daubrée's experiment, in which the glass rectangle (Pl. fig. 5) is loaded with a simple transverse shear stress as well as with torsion applied at the ends, indicates one method of producing a triple cleat. As the photograph shows, the three joints are developed in the neighbourhood of the additional shear.

Cleat Anomalies in Basins.—That a basin-like character in a coalfield is connected with variations in the polarity of cleating is recognised by some mining engineers. Referring to the Radstock district of Somerset, one of our correspondents writes: "The coalfield was more or less basin-shaped, and therefore the bearing of the cleat varied throughout the area."

---

* Briggs, *Colliery Engineering*, January 1932, p 11.

† *Manual of Geology*, 2nd ed., 1862, p. 214.

Another attributed the irregularity of the cleat at Lassodie Colliery, Dunfermline, Fife, to the "several basin-shaped portions" in which the seams occur.

Wide deviations from the normal course seem to be limited to smaller basins. In large ones, such as the Durham coalfield, the cleat preserves a remarkably regular bearing

Two tentative and alternative explanations of the aberrancies in small basins occur to us: either the coal was laid down in a depression of the land surface, when the strains producing cleating would probably be modified, and the cleat distorted, by the local irregularity in the strata; or the beds were being flexed at the time the cleat developed, and the bending moment thus imparted to the beds was able to change the course of the cleat

Some support for the latter suggestion is obtained from tests carried out on glass plates broken by a combination of twisting and bending. The effect of a simple bending moment is illustrated by Pl. fig. 6. The lower end of the pane was clamped and the other forced upwards until fracturing ensued. The widely spaced breaks parallel to the shorter sides bear no resemblance to cleat. In fig. 7 the ends of the pane were gripped and a combined twist and bending force impressed across the middle as the arrows on the key-diagram indicate. A marked change in the resulting manner of fracture is evident, the cleating being both simpler and less regular, and the breaks lying at greater angles to the long sides of the rectangle In a bed, the bending force would produce folding, and if that force were operative when the cleat was induced the experiment indicates the likelihood of the cleats being swung round so as to make a smaller angle with the axis of the fold.

Cleat and Dip.—The warping of the cleats in consequence of folding lends a measure of support to the statement, frequent in the literature, to the effect that the cleat of coal and the strike of the seams usually coincide. As a matter of fact, there are many instances in which a rough agreement is apparent; but the exceptions are too numerous and important to make it possible to trace any general rule to that effect. The cleat in Yorkshire and Derbyshire maintains its north-westerly course despite the fact that the measures have an easterly strike near Leeds and Sheffield, and a northerly strike near Huddersfield and Chesterfield. Again, in Northumberland and Durham the cleat persists in virtually the same course although the general strike of the strata is north-north-east. Dron (*loc. cit.*) could discover no connection between strike and cleat in Scotland, while a century ago Mammatt (*loc. cit.*) commented on the uniformity in bearing of the "slynes" (cleats) of the Leicestershire coalfield "whatever way the stratum may incline."

When the line of cleat is undeflected by folding we may conclude with reasonable probability that jointing preceded folding On the other hand, where strike and cleat coincide, we lean towards the opinion that the forces producing the folding were active when the cleat was created.

It cannot, however, be overlooked that when a bed is bent up at a high angle the polarity of any pre-existing jointing is altered. The influence is purely a geometrical one and may be left out of account in all but the steepest measures. Great inclination has the effect of deflecting the cleat towards the strike. The deflection is zero when the cleat runs normal to the folding, and again when it was originally parallel with it; it attains a maximum when the directions of cleat and strike meet at about 45°. For example, if a seam was cleated at N. 45° W. when it lay horizontal, and was then bent up or down at 45° along an east-and-west axis, the cleats are swung 10° and are given a bearing of N. 55° W.

Anomalies near Faults —There are numerous examples of divergence from the prevailing direction of the cleat or jointing in the neighbourhood of faults Attention has already been drawn (p 172) to instances in Mid and East Lothian.

Not infrequently, the beds are tilted in the vicinity of a fault, and, as we have seen, high inclination is itself a cause of joints being slewed round into a direction more nearly parallel with the strike. Some of the deflections may be accounted for in this way, but only a few of the observed anomalies near faults can be so explained.

Discussing the bearing of the problem on the jointing of the Great Scar Limestone of Craven, L. R. Wager * suggests that the angular swing of the joints near the Craven faults was due to either (1) a lateral or transcurrent movement of the sides of the faults after the joints had been formed, or (2) the pre-existence of the faults, or of lines of weakness in the earth's crust, which effected a modification of the direction of the joints in their neighbourhood.

Let us consider the second hypothesis first. Now our interpretation of the whole process of jointing differs from Wager's. Wager, who does not subscribe to the torsional theory of joint formation, regards joints as shear planes produced by forces acting parallel to the bed. We believe that joints are caused by torsional stress, and therefore that the shearing force acts approximately perpendicular to the plane of the bed If our reading of the evidence is correct, some indication that a pre-existing fault or zone of weakness could deflect the joints would have been apparent from the experiments with glass sheets. The longer edges may be regarded as representing pre-existing faults; they are free lines beyond which the sheet has no support; and, if the hypothesis "holds water," the cracks near them ought to curve round and run more nearly parallel with them. A glance at the photographs (Pl. figs. 3 to 7) is sufficient to show that nothing of that kind takes place.

---

* *Q.J.G.S.*, vol. lxxxvii, 1931, p. 392.

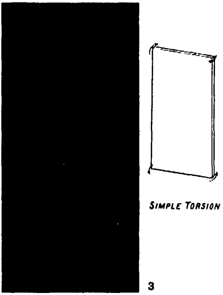
As he offers it, Wager's second hypothesis appears, then, to fail; but, we may now ask, is it possible to restate it with regard to a stress system active in the same zone at the period of jointing instead of to pre-existing faulting or weakness? The existence of the fault proves that, at some time or other, the strata near it were under shear stress. That stress, as the parent of the fault, must have developed before it. Can it, then, be true that, where joints are deflected near a fault, the stress subsequently producing the fault was in action at the moment of jointing? The experiments with panes of glass that have already assisted in elucidating other ambiguities may, one would think, help in suggesting an answer to this problem also.

We have made many trials in which, in addition to torsion, shear forces up to 60 lb were applied either normal to or parallel with the sheet of glass, and the majority of the results did not furnish any indication that complex stresses of these kinds give rise to the effect in question. Thus we fail to discover any evidence worthy of the name in support of the second hypothesis of Wager, whether that hypothesis be as propounded by him or stated in this modified form in which a co-existing stress system is substituted for the pre-existing fault or line of weakness.

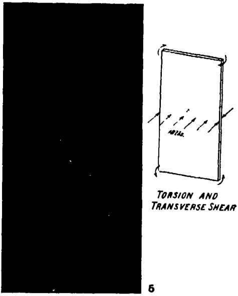
His first suggestion to the effect that the change in polarity of the joints near a fault may be due to lateral movement subsequent to faulting takes advantage, as it has every right to do, of the transcurrent or tearing movements that are recognised as a regular concomitant of normal faulting. It has, however, one grave objection in predicating the side-long movement as occurring after faulting, since in so doing it requires the bed to behave first as a brittle medium to the stress causing faulting and then as a plastic medium under a continuation of the same stress-brittle when the fault occurred, and plastic during the subsequent twisting round of the joints.*

---

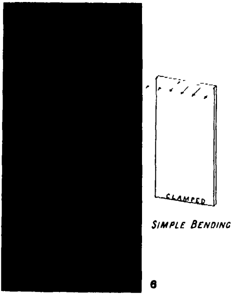
* It appears from Wager's map (*loc cit*) that on the north side of the North Craven fault, the joints are rotated parallel with the fault near Ingleborough and at right angles to it near Grassington The circumstances preclude large lateral movements in opposite directions on the same side of the fault, and at first sight the observation appears to give the death-blow to the theory of transcurrent shift. We have, however, to bear in mind, first, that the limestone possesses two sets of joints at 90°, and, second, that the subsidiary set may locally assume the rôle of the "master" series. It seems not unlikely then that, at Grassington, the joints depicted by Wager are "end" joints abnormally developed.



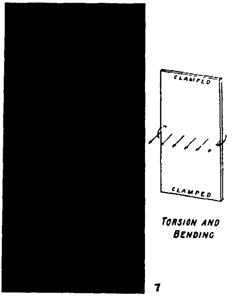
3



5



6



7

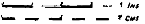


Fig 3 —A sheet of glass fractured by torsion

Fig 5 —Triple fractures produced in a sheet of glass by torsion combined with shear

Fig. 6 —A sheet of glass fractured by simple bending.

Fig. 7 A sheet of glass fractured by torsion combined with bending

An explanation of the phenomenon more in keeping with the known behaviour of rock-masses under stress is that the joints were formed first; that horizontal stress then developed; that the bed sought relief from the stress by horizontal flow in the zone of maximum strain—a flow having as one of its effects the re-orientation of the joints in that zone—and that eventually faults with (initially) large transcurrent components occurred in the same zone.

Plasticity and Jointing.—The phenomenon of plastic flow, to which reference has just been made, is to be observed in any rock subjected to a stress-difference for a sufficient length of time Quite a small inequality in the stresses is enough to cause a rock to flow if it be persistent.* One of the indications of plastic deformation in any stressed material is the formation of strain figures, bands of flow, or Luders' lines, and these, with their conjugate systems crossing at right angles, bear a superficial resemblance to cleat. In so far as they both occur in the direction of maximum shear, Lüders' lines and cleats or joints are related effects. The similarity, however, ceases there The sharp, incisive fractures of jointing are just as much indicative of a brittle condition in the bed as Lüders' lines are of the plastic state No rock can undergo plastic deformation and at the same time develop jointing On the contrary, a considerable degree of flowage is apt to eradicate any jointing or cleavage that the rock may possess. As Leith succinctly states: "If a rock with fracture-cleavage comes into the zone of rock-flowage the structure is obliterated." † The effect is well brought out by contrasting the highly developed cleat of most seams of bituminous coal which, in general, show few signs of flowage, with the relative absence ‡ of cleat in anthracite where such signs are abundant. As the means of distorting jointing or of destroying it, plastic flow has, then, a definite and sometimes an important influence; but it plays no part in the *formation* of the particular species of division with which we are here concerned.

Alternating Stress.—We have now to ascertain how it comes about that coal, a material especially given to flowage under appropriate conditions, has been able to behave as a brittle substance at the time the cleats were created. Now the known effects of a direct § stress-difference on a rock-mass may be summarised as follows:—

---

* See Nádai's *Plasticity* (McGraw-Hill Co., 1931), the latest and most authoritative treatise on the mechanics of matter in the plastic state.

† C. K. Leith, *Structural Geology*, New York, 1913, p. 63.

‡ Few anthracites possess a true cleat; but occasionally in South Wales, and more frequently in Pennsylvania, anthracite seams are to be found that still retain the cleat of the lower-rank coals from which they were derived.

§ The words "direct" or "continuous" are used as in electrical terminology in contradistinction to "alternating."

1. A stress-difference of sufficient magnitude suddenly applied causes faulting by shear.

2. A stress-difference of small magnitude persistently applied causes flowage.

3. A continuous * and persistent stress-difference, especially if of increasing intensity, may cause flowage first and faulting afterwards.

Whether the stresses are rotational or non-rotational is, for the moment, immaterial. The point to observe is that none of these effects bears any resemblance to jointing. The fact emerges that no combination of direct stresses is capable of providing an explanation of that mode of fracture. Thus we are compelled to break new ground by considering the possibility of alternating stress as the active instrument.

The conception of the beds being under the influence of alternating stress eradicates one grave difficulty of the theory of direct stress—namely, that connected with plastic flow. Any substance—notably any rock—may exhibit plasticity to direct stress; but if it fail under a rapidly alternating stress it breaks in the manner of a brittle material. Experience with metals indicates, indeed, that a persistent reciprocating loading induces brittleness The hypothesis offers a satisfactory explanation of the sharply cut character of the cleats of that relatively plastic material, coal.

If now one add to that hypothesis the conclusion already reached, to the effect that torsional stress is the most likely type to induce the special arrangement of joints in rocks—in other words, if we examine the effect of an *alternating torsional stress*—still another difficulty, namely, that of explaining the unfaulted character of the typical joint, also disappears. A direct or continuous torsion of the kind applied to the glass plate of fig. 3 would, at the moment of fracture, leave the bed in a warped condition, and the joints—especially the main joints—would become so many minute faults; a continuance of the twisting motion would increase the warping and amplify the faulting at the joints. Now joints exhibiting a small throw, though by no means unknown, are exceptional. To restore the original flatness of the broken glass plate and remove the displacement at the cracks, the pane has to be bent back again. Direct torsion, then, fails to satisfy the observed conditions; but alternating torsional stress does so, in that the initial movements are cancelled out by the reverse phase and the bed is returned to its original undislocated form. Moreover, an alternating movement is the only kind that can slickenside a face of break without producing throw.

---

* The words "direct" or "continuous" are used as in electrical terminology in contradistinction to "alternating."

We have been induced to accept the theory of alternating torsional stress as the cause of jointing, because it, and it alone, is able to account for all the six characteristics of jointing set forth on p. 176.

Failure through Fatigue.—When a material fractures under a rapidly alternating and persistent stress, it fails owing to fatigue. Stressed in this manner, even malleable media, such as the metals, break by shearing, though the magnitude of the stress be greatly below that required to cause failure by direct (non-alternating) load. Brittle substances, like rocks or concrete, are still less competent to withstand a persisting alternating load. Given, then, that the period of action is long enough, a rock will break under a stress that would be insignificant if direct, but which induces fatigue and eventual failure if alternating.

In explaining jointing, previous workers have dealt only with direct loading, and have been obliged to call to their assistance forces large enough to fracture the beds under that manner of loading. The doctrine of alternating stress relieves us from the necessity of invoking large forces: the effect we are seeking to elucidate can be the consequence of small stresses, providing they are persevering, of high frequency, and of uniform orientation. As Nevin* states: "The effect on geologic structures of long-time stresses of small magnitude cannot be over-exaggerated, and deformation because of so-called fatigue is a most frequent form of failure."

Tidal Action as the Fundamental Cause.—The conclusions so far reached—partly from observation and partly by a process of elimination of those explanations of the phenomena of jointing that are unable to satisfy the conditions of the problem—may be summarised thus:

1. Joints in a bed are formed not long after the bed is deposited

2. The forces that cause jointing have acted without intermission throughout geological history.

3. The regularity in direction of jointing makes it certain that the forces act with an equally regular orientation.

4. The species of stress most capable of producing joints is torsional stress.

5. Direct (non-alternating) stress of any kind is incapable of giving rise to a typical jointing system. Alternating stress alone satisfies the facts.

6. An alternating stress of small intensity (providing it is of short frequency and lengthy duration) is effective inasmuch as it results in failure of the rock-mass by fatigue.

The fifth of these conditions makes it necessary to rule out such considerations as continental drift or the secular slowing down of the earth's rotation consequent on tidal friction.

---

* *Op. cit.*, p. 7.

In 1922 one of us * suggested that the origin of the cleat of coal was to be sought in tidal action. The analysis of the dynamics of rock-jointing attempted in this paper supports that view. So far as we are aware, no other hypothesis is capable of satisfactorily explaining even a small proportion of the data.

The fact that the combined gravitational pull of the moon and sun produces an elastic response of sensible magnitude in the earth's crust is well known, and the movements have been measured and analysed by Schweydar,† Michelson and Gale,‡ and others. From W. D. Lambert's recent review § of the state of knowledge as to earth-tides, it appears that the earth responds with but a brief lag to the tidal impulses; that the amplitude of movement of the earth's surface caused by the lunar semi-diurnal tide (the principal influence) is about 8 inches at the equator, and that the oscillation is considerably increased near seaboards by the shifting of weight of the water-mass under the same influences. The secondary effect due to the water, he states, is such that at Pasadena, California, the actual value of one component of the earth-tide is two and a half times the theoretical value, and at a place in Korea about four times. As a covering of water of varying depth is a necessary condition for the deposition of a stratified series, the amplification of the earth-tide to which a bed or rock would in this way be subject seems to be a matter of importance

The sweep of the earth-tide in an east to west direction round the earth sets up a torsional wave of stress in the crust. Though small in magnitude, the stress is repeated endlessly with—in its major component—a semi-diurnal period. The force has thus the alternating character required to induce fatigue in a layer of rock. Operating approximately parallel with the equator, its sense is that inducing maximum shear stress in the north-westerly and north-easterly directions—the two chief directions of jointing.

Tidal movement is a maximum at the equator and diminishes towards the poles. The torsional stress developed in a stratum of given area by the wave-like impulse of the earth-tide depends, however, not on the extent of vertical movement, but on the rate of change of the movement. As

---

* P. F Kendall, "Physiography of the Coal Swamps," Presidential Address, Geology Section, *Brit. Assoc. Report*, 1922, p 71.

† *Zentralbureau der International Erdmessung*, Berlin, No. 38, 1921.

‡ *Jour. Geol.*, vol. xxvii, 1919, p. 585.

§ *Bull. Nat. Research Council*, Washington, No. 78, 1931, p. 68.

a first approximation, the movement may be regarded as proportional to the cosine of the latitude; the rate of change with latitude is accordingly proportional to the sine of the latitude. it is a minimum at the equator and a maximum at the poles. This corollary provides an explanation for the fact that, in general, cleat is more definite and more regular in its course in British and North American coal seams than it is, say, in those of India.

In the light of these arguments, we suggest the process of jointing to be as follows: A bed or series of beds is laid down on strata that are already jointed. At first soft enough or incoherent enough easily to adjust itself to the tidal wave without breaking, it is sooner or later covered up and gradually becomes consolidated by pressure, loss of moisture, inflow of cementing material, and, with coal, by chemical changes. Losing its initial plasticity and gaining brittleness, it then finds itself called upon to flex as a continuous elastic sheet under the alternating tidal torque. Eventually it fails through fatigue along the planes of maximum shear, and the joints thus come into existence.

(*Issued separately June* 28, 1933.)

XIV.—Studies in Genetic Psychology: The Intellectual Resemblance of Collateral Relatives.

By **J. L. Gray**, M.A., and **Pearl Moshinsky**, B.Sc. (From the Department of Social Biology, University of London.) *Communicated* by Professor LANCELOT HOGBEN, M.A., D.Sc.

(MS received March 18, 1933 Read May 15, 1933)

1. INTRODUCTION.

THE introduction of standardised methods for describing that characteristic of social behaviour denoted by the word intelligent is of comparatively recent date. Therefore it is not feasible to undertake a study of the intellectual resemblance of relatives along the ancestral line. In two classes of collateral relatives, however, such resemblance can be made the subject of inquiry. These are (*a*) members of the same fraternity—twins or ordinary sibs—and (*b*) first cousins. This investigation, which was prompted by a previous inquiry into the intellectual resemblance of twins carried out in the Department of Social Biology, is confined to a study of ordinary sibs and first cousins. Though resemblance between both sibs and first cousins has already been investigated, there are in each case special reasons which justify further inquiry.

(a) *Sibs.*—The resemblance between ordinary brothers and sisters is of pre-eminent importance because, with the exceptions of Thorndike (1905) and Herrman and Hogben (1933), no previous investigators who have studied the response of twins to intelligence tests have recorded a control group of ordinary sibs tested under similar conditions at the same time. From the standpoint of genetic psychology a comparison between fraternal twins and ordinary sibs is not less significant than a comparison between fraternal and identical twins. For, with a possible exception to be noted below, there is no difference between the genotypic correlations for fraternal twins and for sibs conceived at different times. If the variance of ordinary brothers and sisters is significantly greater than that of fraternal twins, in other words, if the correlation for sibs is lower than that for fraternal twins, we have some presumptive evidence for the influence of environmental differences associated with birth-rank in determining differences in the Intelligence Quotient. But such evidence would not be wholly without ambiguity from the biological standpoint.

Crossing-over is known to vary with age. If linkage values change as the age of the parent advances, we might suspect an increase in the frequency of rare combinations of genes, and in consequence greater variability among offspring separated by an interval of some years, as compared with offspring conceived at the same time. However, it is unlikely that such a source of increased variability would be very important, for reasons which will be discussed later. In the same way the greater variance of fraternal as compared with identical twins is not entirely unambiguous. As pointed out by Stocks (1930), Holzinger (1929), and by Herrman and Hogben, the greater variance of fraternal twins with respect to physical characteristics may bring them into situations in which the effective environment is less homogeneous than that of two identical twins.

In agreement with a previous investigation by Lauterbach (1925), Herrman and Hogben found that the correlation between fraternal twins of mixed sex with respect to intelligence is much the same as the average for a large number of physical measurements given by Pearson and Lee (1903), viz. about 0·53. This value does not differ significantly from the theoretical genotypic correlation for sibs deduced on the assumptions (*a*) that the environment is homogeneous, (*b*) that mating occurs at random, and (*c*) that there are no dominance deviations. In contrast with this value, Herrman and Hogben find that in the case of sibs of different age the value r is about 0·35.

In view of the conflicting statements of other investigators who have studied correlations between sibs, it is important to confirm or controvert this finding. Thus, while Pintner (1924) gives the same value as Herrman and Hogben, Hildreth (1925) assigns a value as high as 0·68. Between these two extremes Miss Elderton (1923) gives for Kate Gordon's data 0·53, Hart (1924) gives 0·45, and Thorndike (1928) gives 0·60. These figures are based on different tests. The age-groups of different workers are not in all cases the same, a fact of considerable importance where there exists a significant correlation between age and I.Q. Furthermore, the method of selection is not identical in all cases. Thus we may offer an explanation of the equivalence of Elderton's value to that which has been assigned to the intellectual resemblance of fraternal twins. Elderton's figure is based upon a group of orphanage children. In such a case differences of birth-rank might well have a psychological effect quite negligible compared with what they would have when children are brought up within a small family unit. Further, this method of selection eliminates differences between the social environment of different pairs.

(b) *First Cousins.*—The theoretical interest which pertains to the study of the resemblance between first cousins arises partly from the fact that a comparison of cousins, classified with respect to the sex of members of a given pair and of their related parents, provides an excellent method of deciding whether differences of intelligence may arise in virtue of gene differences involving sex-linked gene substitutions. The theoretical treatment of this issue is given in a recent paper by Hogben (1932).

There is another reason why the resemblance between first cousins is of special interest. The values given by Elderton (1907) for the correlations between first cousins with respect to physical and psychological characteristics are all of an exceptionally high order. The genotypic correlation for cousins, calculated on the assumptions described above, is one-quarter the fraternal correlation, *i.e.* 0·125. Since the fraternal correlation of many physical characteristics is about 0·52, it is somewhat surprising that Elderton obtains such high values for cousins as those given in Table I

TABLE I —ELDERTON'S VALUES FOR THE CORRELATION OF FIRST COUSINS AND SIBS.

	Intelligence.	Health.	Temper	Hand-width.	Eye-colour	Hair-colour
Cousins	34	·31	·20	·34	43	·29
Sibs .	·46	·53	·50		·50*	. .

* Pearson and Lee.

Miss Elderton's correlations for mental characteristics were based upon arbitrary standards of assessment which antedated the more modern methods of testing. Only one investigator has recorded a correlation for the I.Q. of first cousins. Dexter (1924) gives the value 0·22, based on 131 pairs. In this case also the methods employed do not satisfy the requirements of modern technique in the testing of intelligence. Three different tests were given to children classified in three different age-groups. In the absence of information respecting the correlation between the various tests and between age and I.Q. in the group studied, it is not possible to compare Dexter's results with those of the present investigation.

2. CRITERIA OF RELIABILITY.

The children of this investigation were pairs of sibs (brothers and sisters of different birth-rank) and pairs of first cousins taken from elementary schools conducted by the London County Council. The ages of the children tested were between nine and twelve years six months. The test was the Otis Advanced Group Test (Form B), used by Herrman and Hogben and also by Holzinger (1929) in their investigations into the I.Q. of twins. The method of selecting children and the range of ages were also the same as in the twin inquiry of Herrman and Hogben. Therefore, all figures given in this investigation are strictly comparable with those of Herrman and Hogben. The actual routine of testing was standardised by the kindness of Dr Herrman, who accompanied the senior author in the testing of many of the schools. In all cases both members of a pair of either (*a*) sibs or (*b*) first cousins were tested simultaneously in the same group.

In the interpretation of the significance of the correlations between sibs or first cousins calculated from these data, three issues demand separate investigation. These are: (*a*) the correlation of age and I.Q , (*b*) the existence of a residual correlation between unrelated pairs selected from the same schools, and (*c*) the reliability of the test itself in the sense in which the word reliability is used by educational psychologists.

(a) *Age and I.Q.*—Ideally the correlation between age and I Q in a group should be zero. In fact, the constancy of the I Q. for age is not so perfect as to yield zero correlation for wide ranges of age. Various statistical devices have been proposed to correct for the influence of an appreciable correlation between age and I Q. To the present writers, however, it seemed more desirable to arrange the conditions of the investigation in such a way as to reduce the correlation between age and I.Q. to negligible dimensions, than to introduce statistical corrections which may involve assumptions of somewhat dubious validity. In a group-test such as the Otis or Terman, for which norms only cover a period of from eight to eighteen years, it is clear that the age-group between eight and nine will tend to give an unduly high mean I.Q , and children of over, let us say, fourteen an unduly low value. In other words, all children of exactly eight years of age who are below the norm for that year are excluded from the test, because there are no published norms for below eight years of age. Again, a bright child of fourteen may be able to attain a score equivalent to the norm for eighteen years. Since the upper limit for the test score norms is at eighteen years of age, the same child, if tested at fifteen years of age, will evidently be assigned a lower I.Q. than if the test had been carried out a year earlier, even though he or she might obtain a higher raw score at fifteen. Apart from these considerations educational selection introduces another complicating factor. An appreciable number of bright children tend to leave elementary schools between the ages of eleven and twelve, when they are drafted into secondary or central schools. Herrman and Hogben actually found that the constancy of the I.Q. for the Otis Test is only satisfactory when the group tested is confined to the limits set in this investigation. We have studied the question more fully, using data collected for their investigation ("A"), which they have kindly placed at our disposal, and combining it with the data ("B") provided by our own investigation. In Table II two samples of each group of data, "B" and "A" plus "B" are given, constructed so that no two related individuals are in the same table and the age composition of each sample is identical

TABLE II.—CORRELATION FOR AGE AND I.Q.

		r	No. of Pairs.
"B" data	Sample I	·005±·04	610
	Sample II	·02 ±·04	611
"A"+"B" data	Sample III	·03 ±·04	775
	Sample IV	·02 ±·04	776

	r
Mean of I and II .	·0125
Mean of III and IV .	·025
Mean of all . . .	·019

The I.Q.'s for Table II and elsewhere in this communication were calculated on the basis of published norms for over 25,000 American school-children. The negligible correlation between age and I.Q. determined on this basis shows that within the range of nine to twelve and a half years the American norms are satisfactory for London elementary school-children.

(b) *Correlation between Unrelated Pairs.*—On the average only twenty pairs of cousins and sibs of the requisite age could be obtained from a single school. Thus the data presented in what follows are based upon a sample which is made up of a large number of groups in which all individual pairs have the same school environment and tend to be alike in social status. Hence if there were a measurable correlation between social status or school environment and I.Q., it would seriously bias the values obtained for correlations between relatives selected in this manner. This possibility is easily controlled in the following way. The resemblance between unrelated pairs chosen in exactly the same fashion was determined by constructing correlation tables for pairs of unrelated individuals. These were made by combining one member of one pair of sibs or cousins with one member of another pair of sibs or cousins taken from the same school. A random age distribution was maintained in each table. Two values were then determined as follows:—

−0·05±·07 (200 unrelated pairs),
0·01±·07 (200 unrelated pairs).

This shows that the correlations for sibs or first cousins are not significantly affected by the fact that different groups of pairs are taken from different schools, and possibly from somewhat different social levels.

(c) *Reliability.*—The question of reliability is one which merits very careful scrutiny. By reliability educational psychologists may mean one of three things: (i) The correlation coefficient obtained by testing each individual in a group with alternative forms of the same test on two different occasions. (ii) The correlation obtained when each individual of a group is tested on different occasions with one and the same form of the test. (iii) The correlation obtained by comparing the test scores, or an I.Q. calculated from the test scores, when odd and even items in any one form of the test are separately assessed ("split-half reliability"). It is clear that these three criteria of reliability do not measure one and the same thing. In the first case differences may arise because of differences in learning capacity. Such differences do not enter into the third. If such learning capacity is not perfectly correlated with whatever we measure by the I Q., reliability as judged from successive application of alternative forms of the same test will not agree with reliability measured by the third method. Again, the part played by memory is more important when the second method is used, than when either of the other two are employed.

It is therefore desirable to ask in what sense the issue presents itself in such an investigation as this. It is clear, in studying the correlation of relatives for I.Q., that three sources of error may affect our estimate of their resemblance between different members of the same pair. The first is defective administration of the test. This corresponds in a physical experiment to the accuracy of the measuring instrument employed. The second relates to variations in the response of the subject arising from the external conditions under which the test is made In physical science this would be analogous to matching colour standards under different conditions of illumination. The third concerns the way in which the responses of different members of a pair are affected by their reactions to events antecedent to the performance of the test. It is obvious that any difference between the response of one and the same individual to successive forms of the test, arising from differences of learning capacity and memory, have no relevance to an investigation such as the present, in which both members of a given pair were always tested simultaneously by the same person in the same room under the same conditions of illumination, ventilation, humidity, and temperature. Furthermore, all members of a group of pairs so tested were subject to the same technique of administration. For this reason reliability as measured by the testing of a group on two successive occasions does not seem to us to be an appropriate form of assessment for an investigation such as this.

It is possible that a coefficient of correlation approaching unity might be obtained in any of the following three cases: (*a*) The practice or learning effect apparent in the score obtained on a second occasion might be diminished where the children have had previous experience of similar types of test. (*b*) Where a large proportion of the children on whom the reliability test is carried out are at the age of fourteen or more, and where I Q.'s are being compared, those obtaining on both occasions a score in excess of the maximum for which norms are published will necessarily yield the same I.Q. each time. (*c*) Much will depend upon the size of the interval between successive administrations of the test. For instance, it is likely that the longer the interval the less will be the disparity in scores due to the effects of learning or memory. It is claimed by others (*cf.* Pintner, p. 90) that the practice effect is eliminated by lengthening the intervals between successive applications of a test up to a limit of about twelve months. Presumably it is on this basis that Kelley's (1927) figure of 0·97 for the reliability of the Otis Test in the first sense defined above is determined.

If, for the reason stated, reliability measured by correlation for successive testing by alternative forms of the same test introduces considerations which have no bearing on this inquiry, analogous considerations apply with equal force to the use of the method of split-half reliability. The latter may be a useful instrument for investigating the internal homogeneity of a test, when our object is to inquire how far we are entitled to regard it as a measure of a single element of social behaviour. This issue does not emerge in the present inquiry. The problem dealt with here does not assume a unitary view of the nature of intelligence. Two issues arise in making comparisons between the resemblance of pairs of individuals selected in one way, *e.g.* fraternal twins, and pairs of individuals selected in another way, *e g.* sibs, when we test both individuals of a given pair under the same conditions, and arrange the inquiry so that roughly similar numbers of pairs belonging to each group are tested simultaneously. These two issues can be included under one general question: Is the relationship signified by the classification of pairs in a group unique? It may not be unique if (*a*) the age differences between pairs belonging to one group, or the age distribution of the group as a whole, are not the same as for other groups. The only question which then arises is whether the test yields an index which is not significantly correlated with age throughout the range covered by the data. (*b*) It may not be unique for another reason. The distribution of environmental variables between two groups distinguished in terms of genetic relationship may not be uniform. In this inquiry we have eliminated the possible significance of differences arising from the school environment and social level of the pairs studied by showing that there is zero correlation for genetically unrelated pairs selected in the same way. We have therefore eliminated all environmental variables other than those of the intra-familial class, as, for example, birth-rank.

Having established these two points, it seems to us that there is only one valid test of the reliability of conclusions which are primarily concerned with the relative magnitude of resemblances of the kind with which we are dealing. This is whether independent investigation, carried out in the same way with the same test, yields corresponding numerical values. As will be seen, we have applied this check to part of our data, when we compare the correlation coefficient for sibs ascertained from our investigation with that obtained in the previous study by Herrman and Hogben

3. AGE DISTRIBUTION OF THE DATA.

The age distribution of all the children in the group of sibs investigated in this inquiry, classified with respect to sex, is given in Table III

TABLE III —AGE DISTRIBUTION OF SIBS

Age, in Years and Months

	9·0–9·11.	10·0–10·11.	11·0–11·11.	12·0–12·11.	Total
Male pairs	72	52	65	27	216
Female pairs	62	41	55	30	188
Mixed pairs	70	51	51	18	190
Total individuals	204	144	171	75	594

In connection with subsequent analysis of the influence of birth-rank, it is of interest to place on record the size of the age interval between older and younger members of a pair in each age-group Table IV shows that in approximately 70 per cent. of the pairs of sibs examined the age interval was less than two years, and in over 95 per cent. the interval was less than three years.

TABLE IV.—AGE INTERVAL OF SIBS.

Age of Younger	*Difference between Younger and Older (in Years).*				
	0–0·11	1–1·11	2–2·11	3–3·11	Total.
9·0– 9·11 . .	2	128	67	6	203
10·0–10·11 . .	4	69	8	.	81
11·0–11·11 . .	2	11	..	.	13
Total pairs . .	8	208	75	6	297

The age distribution of all the children in the group made up of pairs of first cousins is given in Table V, which is constructed in the same way as Table III.

Table V.—Age Distribution of Cousins.

Age, in Years and Months.

	9·0–9·11.	10·0–10·11.	11·0–11·11.	12·0–12·11.	Total.
Male pairs	53	93	84	26	256
Female pairs	49	70	54	11	184
Mixed pairs	48	71	49	18	186
Total individuals	150	234	187	55	626

Table VI sets out the age intervals between the younger and older members of the cousin pairs. In nearly 60 per cent of the pairs of cousins examined the age interval between the two members of a pair was less than a year. In over 90 per cent. the interval was less than two years. Thus, apart from the negligible correlation between age and I.Q. within the range 9–12½ years exhibited in Table II, the cousin data of this investigation were selected in such a way as to minimise any error in the determination of the correlation coefficient for intelligence which might arise from extensive variations between the ages of members of the same pair.

Table VI—Age Intervals of Cousins.

Age of Younger.	*Difference between Younger and Older (in Years).*				
	0–0·11.	1–1·11.	2–2·11	3–3·11.	Total.
9·0– 9·11	43	62	24	1	130
10·0–10·11	87	31	5	.	123
11·0–11·11	50	6	.	..	56
12·0–12·11	4	..	..	..	4
Total pairs	184	99	29	1	313

Since one of the objects of this inquiry has been to seek for evidence pointing to sex-linked gene differences affecting intelligence, it is desirable also to give the age distribution of the cousin group classified with respect to the sex of their related parents. This is given in Table VII, from which five pairs were excluded because the relationship of the parents was not checked.

Table VII—Age Distribution of Cousins with Respect to the Sex of Their Related Parents.

Age, in Years and Months.

	9·0–9·11.	10·0–10·11	11·0–11·11.	12·0–12·11.	Total.
Maternal pairs	68	109	75	30	282
Paternal pairs	24	43	40	7	114
Mixed parent pairs	55	77	70	18	220
Total individuals	147	229	185	55	616

4. RESEMBLANCE OF SIBS.

In this investigation 297 pairs of sibs were tested. In view of the effect of sex-linked genes upon the observed correlation between sibs, according as we take female pairs, male pairs, or pairs of mixed sex, we have treated the data first as a whole and then classified with respect to sex. In Table VIII the correlation coefficients calculated from the observed data are given in the left-hand column, and in the right-hand column will be found, for purposes of comparison, the genotypic correlations for autosomal and sex-linked genes in a system of random mating.

TABLE VIII.—CORRELATION FOR SIBS ("B").

	Observed Values (I Q)		Genotypic Correlation	
	r.	No.	(*a*) Autosomal	(*b*) Sex-linked.*
All pairs	·35±·05	297	·50	·40
Female pairs	·47±·08	94	·50	·75
Male pairs	·33±·09	108	·50	·50
Mixed pairs	·26±·10	95	·50	·35

* Hogben (1932) and Charles (1933).

Two features of the new data embodied in Table VIII deserve consideration. The first is the striking agreement between our own value for the entire group of 297 pairs of sibs, and that given by Herrman and Hogben for their smaller group of 103 pairs selected and tested in the same way and belonging to an almost identical age-group. The second is the fact that the correlation coefficients of the pairs classified with respect to sex diminish in the same order as the genotypic correlations for sex-linked genes.

Lest too much importance be attached to this parallelism, we have calculated the coefficients for a larger group composed both of our own data and those of Herrman and Hogben (Table IX).

TABLE IX.—CORRELATIONS FOR SIBS ("A" + "B").

	r.	No.
All pairs	·37±·04	391
Female pairs	·41±·08	122
Male pairs	·33±·07	138
Mixed pairs	·37±·07	131

It will be seen that although the female pairs still yield a higher value than male pairs or pairs of mixed sex, the male pairs yield a lower value than the pairs of mixed sex. It is possible that the higher correlation for female pairs is due to greater uniformity in the early family environment of female sibs, as compared with male sibs or sibs of mixed sex. If it were due to sex-linked genes, we should expect male pairs to be more highly correlated than pairs of mixed sex, not only because the genotypic correlation for male pairs is higher than for pairs of mixed sex when inheritance is sex-linked, but also because the absence of dominance deviations in the somatic correlation of brothers tends to magnify the difference still further. The main result, therefore, of this part of our inquiry is to confirm the conclusion that the correlation for a group of sib pairs is significantly lower than that for a group of fraternal twins, when these are tested for intellectual resemblance under similar conditions by the same method of assessment.

5. THE INFLUENCE OF BIRTH-RANK ON THE I.Q. OF SIBS.

The conclusion last stated raises an issue already mentioned in the Introduction. What influence has birth-rank on the I.Q. of members of the same family? Herrman and Hogben found from their data that the mean I.Q of first-born children of small fraternities was distinctly higher than that of later members; but the numbers examined were too few to justify a confident statement concerning the significance of this result. Our own mean values for all individuals belonging to the sib group and to the group of first cousins, classified with respect to birth-rank, are compared with those of Herrman and Hogben in Table X. A striking agreement between the two is manifest.

TABLE X.—MEAN I.Q. OF CHILDREN OF DIFFERENT BIRTH-RANK (SIBS).

	First	Second.	Third	Fourth and Fifth	Sixth and Later
Gray and Moshinsky .	112·8 ± 1·3 (275)	108·8 ± 1·1 (320)	106·6 ± 1·4 (181)	105·6 ± 1·4 (197)	104·6 ± 1·4 (173)
Herrman and Hogben	117·5 ± 1·5 (126)	111·8 ± 1·4 (116)	108·5 ± 8·0* (11)		

* Third and later-born children

To see the significance of this in its correct perspective it is necessary to consider two points The first arises from the fact that first-born children constitute an older age-group. This leads us to ask: Could their higher mean arise from the small positive correlation between age and I Q recorded in Table II? If this were so, we should expect the difference between first- and later-born children to be not greater than that between younger and older children, when the data are divided into two equal age-groups as in Table XI. This table is based on the entire cousin group, where we should necessarily expect to find a larger proportion of first-born children than in the sib group, and where, in consequence, we should obtain a more equal distribution of first- and later-born children for purposes of comparison.

TABLE XI.—MEAN I.Q. AND PERCENTAGE OF FIRST-BORN CHILDREN IN YOUNGER AND OLDER AGE-GROUPS (COUSINS).

Age-group.	No.	Mean I.Q	No. of First-born	Percentage of First-born.
9·0 –10·9	350	107·7±1·1	113	31%
10·10–12·6	265	106·7+1·3	86	31%

This table shows that in the case of cousins, among whom first-born children constitute an equal proportion of both the younger and older age-groups, there is no difference between the mean I.Q. of younger and older children.

The second issue which demands consideration is a possible negative correlation of fertility with I.Q. This would be a serious source of error if an appreciable proportion of individuals belonging to the group of first cousins were members of single fraternities Such individuals might augment the group of first-born children by the addition of representatives of families of low fertility. As a matter of fact, very few such children are included in our census of cousins and sibs; the proportion is less than 5 per cent. of our entire data. In any case the issue can be tested by comparing first- with later-born children in families of different sizes. From Table XII it will be seen that in fraternities of different sizes the mean value for the I.Q. of first-born is generally higher than that for later-born children. In the absence of very decisive evidence to the contrary, we are entitled to disregard the possibility that parental fertility and intelligence are negatively correlated within the restricted social group which forms the subject of this inquiry.

TABLE XII —MEAN VALUES OF I Q. FOR FIRST- AND LATER-BORN CHILDREN IN FAMILIES OF DIFFERENT SIZE.

Size of Family.	First.	Second.	Third	Fourth and Fifth.	Sixth and Over	Mean
2	115·4±2·5 (90)	108·4±2 2 (65)		.	..	112·5±1·7 (155)
3	113·2±2·4 (84)	109·4+2·1 (89)	107·4±3 0 (37)	..	..	110·6±1 4 (210)
4–5	110·4±2·2 (76)	111·9±1·8 (113)	107 6±2·1 (82)	104 6±2·3 (70)	..	109·0±1·0 (341)
6 and over	109·2±4·2 (25)	101·5+2·6 (53)	104·7±2·3 (62)	106·2±1·7 (127)	104 6±1·4 (173)	105·5±0 9 (440)
Mean	112·8±1·3 (275)	108·8±1·1 (320)	106·6±1·4 (181)	105·6±1·4 (197)	104·6±1·4 (173)	

6. THE RESEMBLANCE OF FIRST COUSINS.

We may classify first-cousin pairs into such as are both male, both female, or of different sex; and again into such as are the offspring of two brothers, two sisters, or brother and sister. When the related parents are mixed sibs, the genetic theory of sex-linked inheritance necessitates our drawing a further distinction between the case in which the male parent of the male is the brother of the female parent of the female and the case in which the female parent of the male is the sister of the male parent of the female. Combining both methods of classification, we have thus ten classes, viz. female pairs, male pairs, and mixed pairs of both maternal and paternal cousins, together with male or female and two classes of mixed pairs of cousins whose related parents are mixed sibs.

For the detection of sex-linked genes, the theory of the ideal method is given by Hogben (1932). Hogben's values for the theoretical correlations, when transmission is sex-linked and mating occurs at random, are given in Table XIII

TABLE XIII.—GENOTYPIC CORRELATIONS FOR FIRST COUSINS (AFTER HOGBEN).

	Heterozygote Intermediate.	Dominance Complete.
A. *Maternal Cousins.*		
Female pairs	·1875	·125
Male pairs	·375	·375
Mixed pairs	·250	·133
B. *Paternal Cousins.*		
Female pairs	·250	·167
Male pairs	·000	000
Mixed pairs	·000	·000
C. *Mixed Cousins.*		
Female pairs	·125	·083
Male pairs	·000	·000
Mixed pairs (*a*)*	·000	·000
Mixed pairs (*b*)*	·167	·067

* (*a*) Male parent of male cousin is brother of female parent of female cousin.
(*b*) Female parent of male cousin is sister of male parent of female cousin.

Unfortunately, the division of a group into ten classes of dimensions adequate for confident treatment is possible only when a very large body of data is available. C. Gordon has given the corresponding theoretical values appropriately weighted for the proportions of the several classes examined in this investigation, as classified in Table XIV. The theoretical values are exhibited on the right-hand side of the table; the values calculated from the data of this investigation are given on the left-hand side.

TABLE XIV.—OBSERVED CORRELATIONS FOR I Q. COMPARED WITH GENOTYPIC CORRELATIONS.

	Observed Correlation for I Q.		Genotypic Correlations.	
	r.	No.	Autosomal	Sex-linked *
A. *All Cousins*	·16±·06	313	·125	·112
B. *Pairs Classified with respect to Sex Only*				
Female pairs	·17±·10	92	·125	·172
Male pairs	·16±·09	128	·125	·047
Mixed pairs	·10±·03	93	·125	·104
C *Pairs Classified with respect to Sex of Related Parents.*				
Parents sisters	·15±·07	141	·125	·281
Parents brothers	·03±·13	57	·125	·042
Parents mixed	·24±·09	110	·125	·063

* Calculated for these data by Mr C. Gordon

If we take into consideration only the data presented in Table XIV, we are at once impressed with the fact that with one conspicuous exception the order of magnitude of the correlation coefficient for the several classes, like that in Table VIII, exhibits a striking parallelism with what would be expected if sex-linked gene substitutions contribute conspicuously to observed differences in intelligence. However, as before, we must emphasise the need for caution in estimating the significance of this correspondence. The most convincing evidence of sex-linked gene differences would be an exceptionally high correlation for the male pairs of maternal cousins which appear in Table XIII. In this investigation we obtained sixty-one pairs of this class. The correlation for this small group was practically zero. For this reason we are not entitled to attach great significance to the order of size in the coefficients exhibited in Table XIV.

7. DISCUSSION.

Three main issues of interest to the biologist and psychologist emerge from the new data presented in this communication. In order to facilitate their discussion we have tabulated the recorded values for the correlation between collateral relatives with respect to intelligence, as measured by the Otis Group Test employed in this investigation. These are given in Table XV.

TABLE XV.—OTIS GROUP TEST CORRELATIONS.

Relationship	*r*	No.	Investigator.
Identical twins . . .	·88 ± ·02	50	Holzinger (1929).
	·84 ± ·04	65	Herrman and Hogben (1933).
Fraternal twins of like sex .	·62 ± ·06	52	Holzinger.
	·47 ± ·08	96	Herrman and Hogben
Fraternal twins of mixed sex	·51 ± ·06	138	Herrman and Hogben.
Sibs	·32 ± ·09	103*	Herrman and Hogben.
	·35 ± ·05	297	Gray and Moshinsky.
First cousins . .	·16 ± ·06	313	Gray and Moshinsky
Unrelated pairs from same school.	—·02	(Mean of two values for two samples of 200)	Gray and Moshinsky

* Sibs between 8½ and 12 years.

(i) Although there has been complete unanimity amongst previous investigators concerning the conclusion that there is greater intellectual resemblance between identical twins than between fraternal twins of like or mixed sex, the relative resemblance of fraternal twins and ordinary sibs has not hitherto received satisfactory treatment. As already stated, Herrman and Hogben recorded a significantly lower correlation for sibs than for fraternal twins; but their value for sibs was not based upon a very large sample. Moreover, it was decidedly lower than the values obtained by other investigators employing other tests. Merriman (1924), who did not himself test a control group of sibs, assembled from an unpublished study by Miss Rensch figures which range from $r = \cdot 34$ to $r = \cdot 61$, based on samples of less than 200. On this basis he asserts that "sibling resemblance is universally stated to be very close to $r = \cdot 5$," and adds: "These figures furnish convincing evidence for the relation between fraternal twins and siblings." The fraternal twins, he states in the same paragraph, "give practically the same amount" for the Stanford-Binet Test. On the other hand, the Beta and N I.T. Tests give much higher values for fraternal twins. Merriman thus arrives at a mean for sibs which differs very little from the correlation of fraternal twins found by Lauterbach and by Herrman and Hogben. We are entitled to question the validity of this procedure for a reason which may be illustrated thus. A biologist investigating the effect of pituitary removal upon the blood calcium of a species of frog, whose blood calcium is subject to a considerable seasonable variation, would not compare a set of values derived from hypophysectomised animals taken in one month of the year with the mean of a large number of independent observations made with methods of different accuracy at different times of the year.

If we are to examine differences between the degree of resemblance shown by relatives of different types, it is of the utmost importance to compare values obtained in the same test on samples selected in the same way under the same conditions and, as far as possible, by the same investigators. In the present study the technique of administration may be taken to be identical with that employed in the investigation of Herrman and Hogben, who have examined the largest sample of twins recorded up to the present. With a larger sample of sibs, selected from the same type of school and so chosen as to eliminate any appreciable effect of age upon I.Q. differences, our results confirm those of Herrman and Hogben. In our view, they justify the conclusion that the difference between the resemblance of fraternal twins and ordinary sibs is as firmly established as the difference between the resemblance of identical and fraternal twins.

It remains to inquire what significance can be attributed to this difference. From the standpoint of genetic psychology the most obvious answer is that two fraternal twins share the same uterine environment and the same stage in the family fortunes, and that they are more likely to share the same type of physical and social routine than sibs born at different times. Sibs born at different times may come at different periods in the family fortunes, benefit to a different extent from the experience of their parents, and may be housed in a uterine environment conditioned by different states of maternal health. Indeed, it would not necessarily be surprising if studies of sib resemblance based on samples from different communities or different social levels led to different results. For we must remember that the comparative constancy of the I.Q. between the ages of, let us say, five and fifteen affords no presumption concerning the differential influence of environmental agencies which operate *in utero*, or during the earliest and, it may well be, most formative years of childhood. In a period of prosperity, or in a social group experiencing an increasing standard of living, the environment of later-born children may be more favourable than that of first-born children. On the other hand, in times of depression, or in a group with no prospects of economic advancement, the environment may be less favourable for later-born children. Taken in conjunction with the fact that we have no other data for sib resemblance based on the use of the Otis Test, the somewhat higher values for the correlation of sibs obtained by American workers, with the exception of Pintner (1924), do not appear to us to be remarkable.

While there are obvious reasons why the environment of fraternal twins should be more homogeneous than that of two sibs separated by a period of years, we must not neglect an alternative possibility which might increase the variance within the latter group. Haldane and

Crew (1925) have shown that crossing-over varies with age for the barred and silver sex-linked mutations in the domestic fowl. If an analogous phenomenon occurs in mammals, we might expect greater variance in offspring separated by a long period of years, on account of the increase in the frequency of combinations of genes located on different members of the same pair of chromosomes in the maternal parent. There are two reasons why we do not think it at all likely that this explanation accounts for the difference between the resemblance of fraternal twins and sibs with respect to intelligence The first is that in this investigation the pairs of sibs examined were in no case separated by a period equivalent to a large fraction of the child-bearing period. The second is that the number of human chromosomes is comparatively large and the disparity between their sizes not great, as it is, for example, in the domestic fowl. It would be indeed remarkable, taking both these facts into consideration, if the observed disparity between fraternal twins and ordinary sibs with respect to intellectual resemblance were exclusively, or even mainly, due to a genetic mechanism.

(ii) The second point of interest which emerges from the present investigation concerns the relative magnitude of the correlation for first cousins and ordinary sibs. In so far as environmental differences may distinguish the two members of a cousin pair even more strikingly than members of a pair of sibs, it is reasonable to suppose that the significance of the resemblance between the correlations in each case would not be easy to estimate. This raises an issue which is still controversial. Two views exist concerning the use of correlation technique in genetic inquiries. One is that put forward by Dr Fisher (1918), who holds that the genetic theory of correlation permits us to ascribe to observed variance in a population numerical fractions attributable to gene differences and differences due to environment. The other view is advanced in his recent William Withering Memorial Lectures by Professor Lancelot Hogben, who contends that the problem of nature and nurture so stated is inherently ambiguous. He maintains that correlation technique, when applied to data collected in a way which conforms to the conditions of a controlled experiment, can be used to *detect* gene differences or differences due to environment, but not to indicate which of the two is more important. Whichever view we take, it must be admitted, as Haldane points out in his recent book, *The Causes of Evolution*, that the very high correlations for physical characteristics recorded by Elderton in her studies upon the resemblance of first cousins are exceedingly difficult to explain in terms of a purely genetic theory. Indeed, Dr Fisher himself was foremost to admit this difficulty.

In this investigation the correlation of first cousins for intellectual resemblance (·16) was much lower than the value recorded by Elderton (·34), and does not exceed the genotypic correlation theoretically deduced for autosomal genes in a system of random mating (·125) by a quantity appreciably different from the difference between observed fraternal coefficients for physical characteristics (·53), and the genotypic correlation coefficients calculated upon the same assumption (·50) On the other hand, the correlation for sibs here recorded (·35) is much lower than the fraternal coefficient for physical characteristics (·53) recorded by Pearson and Lee In fact, the ratio of the correlations for first cousins and sibs with respect to intelligence, as determined in this inquiry (·16–·35), is roughly the same as the recorded ratio of the same coefficients for physical characteristics (·35–·53) This clearly calls for an explanation, and we do not find it easy to see how genetic theory can provide it. On the other hand, it would be easy to explain why the ratio of first-cousin correlation coefficients to coefficients for sibs might be lower than that predicted by genetic theory, if, as Hogben points out in a forthcoming publication, significant inter-familial differences of environment tend to raise fraternal correlations more than correlations between first cousins.

Tentatively, we might advance the following explanation for what is actually found, though we fully recognise that it offers a more plausible interpretation of the data derived from the study of social behaviour than of the data obtained from the study of physical characteristics This is that when we are examining a comparatively homogeneous social group, the differences which distinguish the environment of individuals of different birth-rank within the same family are of greater significance than differences of environment which distinguish individuals of the same birth-rank in different families Such a conclusion is not wholly repugnant to views which have been advanced by leading psychiatrists like Freud and Adler, though not always on the basis of evidence which would commend itself to a scientific investigator.

(iii) In dealing with a species such as Man, in which the number of chromosomes is large and their sizes do not vary greatly, there is no *prima facie* reason why differences with respect to any characteristic should be appreciably affected by sex-linked gene substitutions. On the other hand, such a result is not essentially impossible, since we might encounter a considerable series of multiple allelomorphs. There is a common belief that conspicuous intellectual ability is more frequently inherited through the mother than through the father. Whether this belief has any foundation in fact, there are no quantitative data to justify. In any case, if there were evidence associating ability of offspring with the maternal rather than the paternal parent, it would be obviously open to a different construction, since in the formation of a child's social environment the influence of the mother may well be more important than that of the father. In this investigation we have attempted to apply what means we have at our disposal to identify differences due to sex-linked genes. The results are suggestive, but we do not regard them as at all conclusive, and offer them merely as a starting-point for further investigation.

8. SUMMARY.

I. The correlation for pairs of sibs differing by not more than three years of age and between the ages of nine and twelve and a half years, tested by the Otis Advanced Test, is ·35±·05.

II. For pairs of first cousins the coefficient is ·16±·06.

III. This investigation establishes the existence of a significant difference between the correlation of fraternal twins and ordinary sibs with respect to intelligence as measured by a group-test.

IV The possible existence of sex-linked gene differences with respect to intelligence is discussed.

ACKNOWLEDGMENTS.

Acknowledgment is made to the Education Officer of the London County Council and to the Principals of the schools visited for their cordial co-operation; to Dr Louis Herrman for advice and help; to Professor Lancelot Hogben for valuable encouragement and assistance; and to the Rockefeller Research Fund of the London School of Economics for a grant in aid of the research.

REFERENCES TO LITERATURE.

CHARLES, 1933 "Collateral and Ancestral Correlations for Sex-linked Transmission Irrespective of Sex," *Journ. of Genetics*, vol xxvii.

DEXTER, 1924 "On Family Resemblance beyond the First Degree of Relation," *School and Society*, p. 19.

ELDERTON, 1907 "On the Measure of Resemblance of First Cousins," *Eugenics Laboratory Memoirs*, vol. iv.

ELDERTON, 1923. "A Summary of the Present Position with regard to the Inheritance of Intelligence," *Biometrika*, vol. xxvi, No. 3.

FISHER, 1918. "The Correlation between Relatives on the Supposition of Mendelian Inheritance," *Trans. Roy. Soc. Edin.*, vol. lii.

KATE GORDON, 1919. "Psychological Tests of Orphan Children," *Journ. of Delinquency*, p. 18.

HALDANE and CREW, 1925. "Change of Linkage in Poultry with Age," *Nature*, vol. cxv, p. 641.

HART, 1924. "Correlations between Intelligence Quotients of Siblings," *School and Society*, p. 20.

HERRMAN and HOGBEN, 1933. "The Intellectual Resemblance of Twins," *Proc. Roy. Soc. Edin.*, vol. liii, pt. ii

HILDRETH, 1925. "The Resemblance of Siblings in Intelligence and Achievement," *Teachers College Contributions to Education*, No. 186

HOGBEN, 1932. "Filial and Fraternal Correlations in Sex-linked Inheritance," *Proc Roy. Soc. Edin.*, vol. lii, pt iii (No. 19).

HOGBEN, 1932. "The Correlation of Relatives on the Supposition of Sex-linked Transmission," *Journ. of Genetics*, vol. xxvi, No. 3

HOLZINGER, 1929. "The Relative Effect of Nature and Nurture Influences on Twin Differences," *Journ. Educational Psychology*, vol xx, No. 6.

KELLEY, 1927. "The Interpretation of Educational Measurements"

LAUTERBACH, 1925. "Studies in Twin Resemblance," *Genetics*, vol x, No. 6

MERRIMAN, 1924. "The Intellectual Resemblance of Twins," *Psychological Review Monographs*, p. 152.

PEARSON and LEE, 1903. "The Law of Ancestral Heredity," *Biometrika*, vol ii, pt. iv.

PINTNER, 1924. "The Mental Indices of Siblings," *Psychological Reviews*, No 25

STOCKS, 1930 "A Biometric Investigation of Twins and their Brothers and Sisters," *Annals of Eugenics*, vol. iv, pts. i and ii.

THORNDIKE, 1928. "The Resemblance of Siblings in Intelligence," *27th Year-Book of the National Society for the Study of Education*, pt. i, chap iii.

(*Issued separately July* 3, 1933)

XV.—Matrices and Continued Fractions, II. By Professor H. W. Turnbull, F.R.S.

(Revised MS. received May 13, 1933 Read May 15, 1933)

INTRODUCTION.

THIS communication is a sequel to a former work (*Proc. Roy. Soc. Edin.*, vol. liii, 1933, pp. 151–163) An explicit, and apparently new, form is given for the rational reduction of a matrix to diagonal form, applicable to symmetric matrices and also to continuants. An account follows which co-ordinates several existent theories of generalized continued fractions, and concludes with a few determinantal theorems as corollaries.

§ 1. APPLICATION OF SCHWEINS' THEOREM.

Let

$$A=[a_1, b_2, \ldots, m_n]=[j_i]$$

denote a matrix of n rows ($i=1, 2, \ldots, n$) and n columns ($j=a, b, \ldots, m$) from which any non-zero minor determinant of r rows is chosen According to a classical identity of Schweins[1] (1825), the ratio of a second to this first r-rowed minor may be expressed as a series of continually simpler terms, provided that the minors differ only in respect of one row (or one column). For instance, if $(abc)_{123}$ is the minor of the first three rows and columns,

$$\frac{(abd)_{123}}{(abc)_{123}}=\frac{(ab)_{12}(acd)_{123}}{(ac)_{12}(abc)_{123}}+\frac{a_1(cd)_{12}}{c_1(ac)_{12}}+\frac{d_1}{c_1}. \qquad (1)$$

The series has r terms, fixed as follows· from the given r-rowed minor (which necessarily has a non-zero minor of $(r-1)$ rows) a non-zero minor $(ac)_{12}$ is selected. From this again a non-zero c_1 is selected: and so on until a single element is reached in r steps. Inspection will now show how the denominators, and in turn the numerators, are constructed in this, and in the r-row, case.

The proof by means of Jacobi's theorem on the adjugate is simple. In fact, we have

$$(abd)_{123}(ac)_{12}=(ab)_{12}(acd)_{123}+(abc)_{123}(ad)_{12}. \qquad (2)$$

On transferring the leading term on the right to the left of (1), and using (2), we obtain on the left

$$(ad)_{12}/(ac)_{12};$$

and by repeating the process the identity is proved.

A modified form of the same theorem is illustrated by the following identity:—

$$\frac{(abcd)_{1234}}{(abc)_{123}}+\frac{(abc)_{124}(abd)_{123}}{(abc)_{123}(ab)_{12}}+\frac{(ab)_{14}(ad)_{12}}{(ab)_{12}a_1}+\frac{a_4d_1}{a_1}=d_4, \quad . \quad . \quad (3)$$

which expresses any element d_4 of the given array as a series of terms, whose denominators are based on a set of non-zero minors, such as

$$a_1,\ (ab)_{12},\ (abc)_{123},\ \ldots \neq 0. \quad . \quad . \quad . \quad (4)$$

This (and all such identities) is proved telescopically by replacing the two leading terms by a single term as given by an identity of type (2) Equally well the simplification may proceed telescopically from right to left, beginning with the term d_4.

If conditions (4) hold, then any given matrix A can be expressed in the form

$$A=PDQ, \quad |P|=|Q|=1,. \quad . \quad . \quad (5)$$

where P and Q are non-singular matrices with unit determinants, and D is a diagonal matrix. Namely,

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}=\begin{bmatrix} 1 & . & . \\ \frac{a_2}{a_1} & 1 & . \\ \frac{a_3}{a_1} & \frac{(ab)_{13}}{(ab)_{12}} & 1 \end{bmatrix}\begin{bmatrix} a_1 & . & . \\ . & \frac{(ab)_{12}}{a_1} & . \\ . & . & \frac{(abc)_{123}}{(ab)_{12}} \end{bmatrix}\begin{bmatrix} 1 & \frac{b_1}{a_1} & \frac{c_1}{a_1} \\ & 1 & \frac{(ac)_{12}}{(ab)_{12}} \\ . & . & 1 \end{bmatrix}. \quad (6)$$

In proof of this, or of its n-rowed analogue, it is only necessary to perform the indicated multiplications. The resulting n^2 equations, obtained by equating corresponding elements on the left and right, are then seen to be so many examples of Schweins' theorem.

The reciprocals of the matrices P and Q are as follows:—

$$P^{-1}=\begin{bmatrix} 1 & . & . \\ -\frac{a_2}{a_1} & 1 & . \\ \frac{(ab)_{23}}{(ab)_{12}} & -\frac{(ab)_{13}}{(ab)_{12}} & 1 \end{bmatrix}, \qquad Q^{-1}=\begin{bmatrix} 1 & -\frac{b_1}{a_1} & \frac{(bc)_{12}}{(ab)_{12}} \\ . & 1 & -\frac{(ac)_{12}}{(ab)_{12}} \\ . & . & 1 \end{bmatrix}. \quad . \quad (7)$$

The verification that the product PP^{-1}, as also QQ^{-1}, is in fact the unit matrix I, once again depends directly upon the same theorem. The actual values of p_{ij}, q_{ij}, the typical elements belonging to the i^{th} row and the j^{th} column h in P and in Q are given by quotients of determinants as

$$p_{ij}=\frac{(ab\ldots h)_{12\ldots(j-1)i}}{(ab\ldots h)_{12\ldots j}},\quad (i>j);\qquad q_{ij}=\frac{(ab\ldots dh)_{12\ldots i}}{(ab\ldots de)_{12\ldots i}},\quad (i<j),\quad (8)$$

where $a, b, \ldots, h, \ldots, m$ are the n columns in their actual order in A.

§ 2. *Application to Singular Case.*

When the matrix A is of rank r ($< n$), at least one minor of r rows does not vanish, whereas each minor of more than r rows vanishes. In this case the formula $A=PDQ$ continues to apply, only that a zero now replaces every term which otherwise would have an $(r+k)$-rowed minor in its numerator *or* denominator. For example, if $(ab)_{12}\neq 0$, $(abc)_{123}=0$, $r=2$,

$$\begin{bmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \\ a_4 & b_4 & c_4 & d_4 \end{bmatrix} = \begin{bmatrix} 1 & & . & . \\ \frac{a_2}{a_1} & 1 & . & . \\ \frac{a_3}{a_1} & \frac{(ab)_{13}}{(ab)_{12}} & 1 & . \\ \frac{a_4}{a_1} & \frac{(ab)_{14}}{(ab)_{12}} & . & 1 \end{bmatrix} \begin{bmatrix} a_1 & . & . & . \\ . & \frac{(ab)_{12}}{a_1} & . & . \\ . & . & . & . \\ . & . & . & . \end{bmatrix} \begin{bmatrix} 1 & \frac{b_1}{a_1} & \frac{c_1}{a_1} & \frac{d_1}{a_1} \\ . & 1 & \frac{(ac)_{12}}{(ab)_{12}} & \frac{(ad)_{12}}{(ab)_{12}} \\ & . & 1 & . \\ . & . & . & 1 \end{bmatrix}, \quad (1)$$

and the proof is essentially the same.

It is still true that

$$A=PDQ,\qquad |P|=|Q|=1,$$

but in this case the diagonal matrix D contains just r non-zero elements. It has been assumed for neatness that the leading element a_1 and the leading principal minors, up to the order r, are non-zero. In the contrary case, when some other r-rowed non-zero minor has to be selected the formula is modified by suitably deranging the rows and columns of both P and Q.

This result is in keeping with, but differs from, the fundamental theorem of H. J. S. Smith,[2] in which a matrix is reduced to a diagonal of r non-zero elements Δ_p/Δ_{p-1}, where the Δ_p are the H.C.F.'s of the r complete sets of minors of each particular order $p(< r)$.

It will be seen that, in the case when A is symmetrical ($A=A'$), then the above formulæ virtually give the reduction of a symmetrical matrix to rational canonical form by congruent transformation, since the matrices P and Q are evidently transpositions of each other. That is, if $A=A'$, then

$$A=PDQ,\qquad P'=Q,\qquad Q'=Q.$$

Again, if A is of rank r, and A_1, its leading submatrix of r rows and columns, is assumed to be non-singular, then let A be written in partitioned form

$$A = \begin{bmatrix} A_1 B_1 \\ A_2 B_2 \end{bmatrix}, \qquad |A_1| \neq 0. \quad . \quad . \quad . \quad . \qquad (2)$$

Here B_2 has $(n-r)$ rows and columns, B_1 has r rows and $(n-r)$ columns, and A_2 has $(n-r)$ rows and r columns. This leads to the following result.

THEOREM.—*If* A *is of rank* r, *and* A_1 *is a non-singular submatrix of rank* r, *then*

$$B_2 = A_2 A_1^{-1} B_1 \quad . \quad . \quad . \quad . \quad . \quad . \qquad (3)$$

identically.

Proof.—Let $D = P_1 A_1 Q_1$ be the diagonal form of the non-singular A_1, where P_1 and Q_1 are the reciprocals of the P and Q factors in formula § 1 (6) (for r rows and columns). Consider the continued product

$$\begin{bmatrix} P_1 & \cdot \\ P_2 & I \end{bmatrix} \begin{bmatrix} A_1 & B_1 \\ A_2 & B_2 \end{bmatrix} \begin{bmatrix} Q_1 & Q_2 \\ \cdot & I \end{bmatrix}$$

consisting of three matrices similarly partitioned, I being a unit matrix of $(n-r)$ rows and columns. If

$$P_2 A_1 + A_2 = 0, \qquad A_1 Q_2 + B_1 = 0, \qquad |A_1| \neq 0,$$

then P_2, Q_2 are uniquely given by $-A_2 A_1^{-1}$, $-A_1^{-1} B_1$: and the continued product is

$$\begin{bmatrix} P_1 A_1 Q_1, & \cdot \\ \cdot \quad , & B_2 - A_2 A_1^{-1} B_1 \end{bmatrix}.$$

But this must be of the same rank as A, since the other two factors of the continued product are non-singular. Also the rank of $P_1 A_1 Q_1$ is r, which is that of A. Hence the other submatrix must be zero; which proves the result.

A similar result holds if A_1 is any r-rowed non-singular square submatrix of A. Next if

$$x = \{x_1, x_2, \ldots, x_r\}, \qquad y = \{y_1, y_2, \ldots, y_{n-r}\}$$

denote sets of parameters, supposed to be arranged in *column* (as indicated by the convention { }), then the matrix product

$$A\{x, y\} \equiv \{z\}$$

also denotes a column, of n elements $z_1, z_2, \ldots, z_n$. Let these, equated to zero, be regarded as n linear equations for the x and the y.

Then

$$A\{x, y\} \equiv \begin{bmatrix} A_1 & B_1 \\ A_2 & B_2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0.$$

That is,

$$A_1 x + B_1 y = 0, \qquad A_2 x + B_2 y = 0.$$

But $| A_1 | \neq 0$. Hence x may be found as $-A_1^{-1}B_1 y$; so that

$$(B_2 - A_2 A_1^{-1} B_1) y = 0.$$

By the above theorem this coefficient of y is zero, so that y may be arbitrary. This gives an explicit form of the well-known result that the solution of n linear homogeneous equations for n unknowns x, y involves exactly $(n-r)$ arbitrary parameters y.

The resultant matrix $(B_2 - A_2 A_1^{-1} B_1)$ is incidentally an example of a non-commutative determinant introduced by Richardson.[3]

§ 3. *Continued Fractions.*

The preceding methods, based upon the fundamental theorem of Schweins, are really the underlying principles in the theory of continued fractions. In fact they unify the theory not only of simple but of generalized continued fractions, all of which may be looked upon as ***matrices*** either in extended A, or in diagonal D, form The algorithm of Schweins is the ***general***, of which all the algorithms of continued fractions are ***particular*** representations.

All such particular cases are included among matrices called ***continuants***, of the ***open*** or ***closed*** types.

Open Continuant.

$$\begin{array}{ccccccc} a_1 & -1 & & & & & \\ a_2 & b_2 & -1 & & & & \\ a_3 & b_3 & c_3 & -1 & & & \\ a_4 & & c_4 & b_4 & -1 & & \\ & & c_5 & b_5 & a_5 & -1 & \\ & & & b_6 & a_6 & c_6 & -1 \end{array}$$

Closed Continuant.

$$\begin{array}{ccccccc} a_1 & -1 & & & & & \\ a_2 & b_2 & -1 & & & & \\ a_3 & b_3 & c_3 & -1 & & & \\ & b_4 & c_4 & a_4 & -1 & & \\ & & c_5 & a_5 & b_5 & -1 & \\ & & & a_6 & b_6 & c_6 & -1 \end{array}$$

To the right of the over-diagonal $(-1 = b_1 = c_2 = d_3 = \ldots)$ every element is zero: to the left there are arbitrary elements arranged in s sets a, b, c, ... (The illustrations have three sets.) Suffixes refer to rows; and each row below that on which the set starts has one letter of each set. A set starts on the principal diagonal: and if the set is broken into several columns, then at each column it starts on the principal diagonal, which must be completely filled with elements.

The characteristic of the ***closed*** continuant is that each column should contain s elements of the sets. Hence the elements are confined to s

consecutive and parallel diagonals: and, according to the rules laid down, a cyclic repetition, of period s, is apparent in the rows. In a closed continuant the elements are consecutive upon each row: in an open continuant this is not necessarily the case, as shown, for example, in the gap between a_4 and c_4.

The closed continuant of two sets ($s=2$),

$$A=\begin{bmatrix} a_1 & -1 & & \\ a_2 & b_2 & -1 & \\ & b_3 & a_3 & -1 \\ & & & \cdots \end{bmatrix}, \quad . \quad . \quad . \quad . \qquad (2)$$

is associated with an ordinary continued fraction

$$\frac{p_n}{q_n}=a_1+\frac{a_2}{b_2+}\,\frac{b_3}{a_3+}\,\frac{a_4}{b_4+}\cdots, \quad . \quad . \quad . \quad . \qquad (3)$$

where the successive numerators p_i can be written as a vector

$$p=\{1,\ p_1,\ p_2,\ p_3,\ \ldots\} \quad . \quad . \quad . \quad . \qquad (4)$$

which is identically equal to

$$\{1,\ a_1,\ (ab)_{12},\ (abc)_{123},\ \ldots\} \quad . \quad . \quad . \quad . \qquad (5)$$

in the original notation of the matrix A This is a result due to Sylvester,[4] which may easily be verified. The recurrence relations satisfied by the p_i are comprehended in the simple matrix formula

$$Ap=0,$$

which again is easily verified The set q satisfies the same formula, except for the first row of A. From this point of view a generalized continued fraction is a set of $s-1$ ratios

$$p_n \quad q_n : \cdot \quad \cdot : r_n$$

(and therefore *one* ratio in the ordinary case, $s=2$), wherein each of the sets $p=\{p_i\}$, $q=\{q_i\}$, . . . satisfy the same recurrence relations

$$A_s p=0, \quad A_s q=0, \quad \ldots, \quad A_s r=0,$$

where A_s means the matrix A modified by writing zeros throughout the first $(s-1)$ rows. While this modification is necessary in order to introduce suitable initial conditions for determining the $(s-1)$ sets $p, q, \ldots, r$, the first set may be taken as satisfying the unmodified equation $Ap=0$.

It is interesting to note that, in the ordinary case, $s=2$, the diagonal form D of A has for its elements actual continued fractions since, for instance,

$$\frac{(abcde)_{12345}}{(abcd)_{1234}}=\frac{p_5}{p_4}=a_5+\frac{b_5}{b_4+}\,\frac{a_4}{a_3+}\,\frac{b_3}{b_2+}\,\frac{a_2}{a_1}.$$

Modifications and generalizations may now be examined according to the value of s. If $s=1$, there is one set a, and the case is relatively trivial. If $s=2$ there is the possible case

$$A \equiv \begin{bmatrix} a_1 & -1 & & & \\ a_2 & b_2 & -1 & & \\ a_3 & . & b_3 & -1 & \\ . & . & . & . & . \end{bmatrix},$$

where the set a is confined to the first column. If $Ap=0$, then p_n is the numerator of an *ascending* continued fraction,

$$a_1 + \cfrac{a_2 + \cfrac{a_3 + \dots}{b_3}}{b_2} \qquad \equiv a_1 + \frac{a_2}{b_2} + \frac{a_3}{b_2 b_3} + \dots$$

The closed case, as has been remarked, gives rise to the ordinary (or descending continued fraction), while all intermediate open cases furnish hybrids or mixed ascending and descending fractions.

§ 4 SECOND MATRIX REPRESENTATION.

The above discussion of open and closed continuants is a development of the initial work of Sylvester. A second useful representation of continued fractions by matrices was introduced by H. J. S. Smith (*Collected Works*, i, 184; B.A. Report (1860)), who pointed out the connection between the continued fraction

$$\frac{1}{-\mu_1 -} \; \frac{1}{\mu_2 -} \cdots \frac{1}{-\mu_n} \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

and the product of n two-rowed matrices

$$\prod_{r=1}^{n} \begin{bmatrix} . & -1 \\ 1 & \mu_r \end{bmatrix}.$$

Evidently the fraction (3) of § 3 is such that

$$\begin{bmatrix} 1 & a_1 \\ . & 1 \end{bmatrix} \begin{bmatrix} a_2 & . \\ b_2 & 1 \end{bmatrix} \begin{bmatrix} 1 & a_3 \\ . & b_3 \end{bmatrix} \dots \text{ to } (n+1) \text{ factors} = \begin{bmatrix} p_n & p_{n-1} \\ q_n & q_{n-1} \end{bmatrix} \text{ or } \begin{bmatrix} p_{n-1} & p_n \\ q_{n-1} & q_n \end{bmatrix} \quad (2)$$

according as n is odd or even, as is at once verified by induction. If $(pq)_n$ denotes $p_n q_{n-1} - p_{n-1} q_n$, then on taking determinants of each side we have

$$(-)^{n-1}(pq)_n = a_2 b_3 a_4 b_5 \dots \text{ to } n \text{ factors.} \quad . \quad . \quad . \qquad (3)$$

This illustrates the case of two sets a, b ($s=2$), wherein the sets appear in separate rows throughout the matrices, a in the first, b in the second row. The general case can best be exemplified by taking $s=3$, and using three types of matrix-factor:

$$A_i=\begin{bmatrix} a_i & . & . \\ b_i & 1 & . \\ c_i & . & 1 \end{bmatrix},\quad B_i=\begin{bmatrix} 1 & a_i & . \\ . & b_i & . \\ . & c_i & 1 \end{bmatrix},\quad C_i=\begin{bmatrix} 1 & . & a_i \\ . & 1 & b_i \\ . & . & c_i \end{bmatrix}, \quad . \quad (4)$$

and, in general, using s such types of s-rowed matrices. If P_i denote either A_i, B_i or C_i, then the continued product

$$P_1P_2\ldots P_n=\begin{bmatrix} p_\lambda & p_\mu & p_\nu \\ q_\lambda & q_\mu & q_\nu \\ r_\lambda & r_\mu & r_\nu \end{bmatrix} \quad . \quad . \quad . \quad (5)$$

defines a set of three convergents, the λ^{th}, the μ^{th}, and the ν^{th} to a generalized continued fraction. The exact values of the suffixes λ, μ, ν depend upon the types of the P_i, and indeed reflect the features of the open and closed continuants already considered. In fact, if the P_i are determined by the actual order of the letters in the principal diagonal of a continuant, then the elements p, q, r satisfy the recurrence relations $A_s p=0$, etc., as is proved without difficulty by induction. Thus, for instance,

$$A_1B_2C_3B_4A_5C_6\ldots,\qquad A_1B_2C_3A_4B_5C_6\ldots \quad . \quad . \quad (6)$$

define two generalized continued fractions, corresponding to the exemplified open and closed continuant. The former has been called by Perron[10] a *deranged*, the latter a *cyclic*, continued fraction.

If both sides of (5) are postmultiplied by P_{n+1}, then the result is evidently the same as adding unity to λ or to μ or to ν only, according as this new factor P is of type A, B or C. It follows that in, and only in, the cyclic case the suffixes λ, μ, ν are consecutive integers, as, for instance, in (2). The deranged case has been studied by Paley and Ursell,[5] and more recently by Milne-Thomson,[6] who has made essential use of the matrix methods of H. J. S. Smith.

A generalization of (3), which expresses the determinant $(pq\ .\ r)_{\lambda\mu\ldots\nu}$ as a product given by the small letter equivalent of (6), is at once obtained by taking determinants of (5).

Again, if each such determinant is unity, so that $a_i=1$ in A_i, $b_i=1$ in B_i, and $c_i=1$ in C_i, then the theory simplifies to that of continued fractions with unity for numerator at each new stage, as in (1), and to corresponding generalizations ($s>2$). This serves to explain the ***raison d'être*** of the above classifications. Such fractions may be generated arithmetically or algebraically by the well-known H.C.F. process: that of finding the H.C.F. of s numbers, gradationally by means of s sets *a*, *b*, *c*, . . . of partial quotients. To find the H.C.F. of 27, 8 and 5 we may proceed as follows:—

$$\begin{bmatrix} 27 \\ 8 \\ 5 \end{bmatrix} = \begin{bmatrix} 1 & & 5 \\ & 1 & 1 \\ & & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 1 & & 5 \\ & 1 & 1 \\ & & 1 \end{bmatrix} \begin{bmatrix} 1 & & \\ 1 & 1 & \\ 2 & & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & & 5 \\ & 1 & 1 \\ & & 1 \end{bmatrix} \begin{bmatrix} 1 & & \\ 1 & 1 & \\ 2 & & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & \\ & 1 & \\ & 1 & 1 \end{bmatrix} \begin{bmatrix} \cdot \\ 1 \\ \cdot \end{bmatrix}.$$

Here 27 and 8 are divided by 5, quotients appearing in the first square matrix (in the third column, because 5 is in the third original position), while remainders and divisor appear in the vector column. Next 3 and 5 are divided by 2, quotients appearing in the first column. Either unit may be chosen for third divisor. and the work ends with a vector of $(s-1)$ zeros and a non-zero, the H C.F.

In this way sets *a*, *b*, *c* are generated by means of three-rowed matrices $C_1A_2B_3$: and the order of factors—cyclic or otherwise—is determined by the choice of divisor at each stage. Thus the above H.C.F. process, applied simultaneously to *s* numbers determines successively columns of quotients *a*, *b*, *c*, . . ., which give rise to a continuant finite or infinite, open or closed, as the case may be. In the terminology of Paley and Ursell the leading diagonal elements a_1, b_2, c_3, b_4, . . . are "principal elements," while each element (essentially non-zero) which terminates a column a_4, b_3, c_4, . . . is called a "cofactor of the dot."

§ 5. EXTENSION OF WHITTAKER'S THEOREM.

Starting from Sylvester's expression of a continued fraction as the quotient of two determinants, Whittaker[7] gave a theorem for the derivative $f'(x)$ of a function expressible as a certain continued fraction. This result may be generalized in the following way:—

If f(x) *is defined as a leading element of a matrix by the relation*

$$\begin{bmatrix} f(x), & \ldots \\ . & . \quad . \end{bmatrix} = \frac{1}{A+Ix} = \frac{1}{A_x},$$

$A=[a_{ij}]$ *being a general matrix, each of whose elements is independent of* x, *then* f'(x) *is given by*

$$f'(x) = \left| \begin{array}{cc} \cdot & (UA_x) \\ (A_xU') & -I \\ \hline \cdot & A_x \\ A_x & -I \end{array} \right|, \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

where (UA_x) *denotes the matrix of* n *rows and* $(n+1)$ *columns formed by omitting the top row of the* $(n+1)$*-rowed matrix* A_x, *and where* (A_xU')

denotes (n + 1) *rows and* n *columns formed by omitting the first column of* A_x.

The numerator is a determinant of order $(2n+1)$, and the denominator one of order $(2n+2)$, which factorizes into $(-)^{n+1}\,|A_x|^2$. In proof of this it is only necessary to expand the numerator by a Laplace development with a partition horizontally just below the n^{th} row. For example, if $n=2$,

$$f'(x)=\begin{vmatrix} . & . & a_{21} & a_{22}+x & a_{23} \\ . & . & a_{31} & a_{32} & a_{33}+x \\ a_{12} & a_{13} & -1 & & \\ a_{22}+x & a_{23} & . & -1 & . \\ a_{32} & a_{33}+x & . & . & -1 \end{vmatrix} \div \begin{vmatrix} . & . & & a_{11}+x & a_{12} & a_{13} \\ & . & . & a_{21} & a_{22}+x & a_{23} \\ & & . & a_{31} & a_{32} & a_{33}+x \\ a_{11}+x & a_{12} & a_{13} & -1 & . & . \\ a_{21} & a_{22}+x & a_{23} & . & -1 & . \\ a_{31} & a_{32} & a_{33}+x & & . & -1 \end{vmatrix}$$

For if A_{ij} denotes the cofactor of the ij^{th} element in the determinant $|A_x|$ then this expansion yields at once

$$f'(x)=-(A_{11}A_{11}+A_{21}A_{12}+\quad.\quad+A_{n1}A_{1n})/|A|^2,$$

which agrees with the original result[8] as obtained by squaring the reciprocal matrix, $1/A_x$. On replacing A by M this evidently becomes Whittaker's Theorem on the derivative as it was originally presented.

§ 6. COMPOUND CONTINUANTS.

Some interesting determinantal theorems may be obtained from continuants by the process equivalent to that of condensing continued fractions. The procedure may be illustrated by utilizing the notation

$$(0)=1,\quad (i)=a_i,\quad (ij)=\begin{bmatrix} a_i & -1 \\ 1 & a_j \end{bmatrix},\quad (ijk)=\begin{bmatrix} a_i & -1 & . \\ 1 & a_j & -1 \\ . & 1 & a_k \end{bmatrix},\ .\ .\ .\ ,\quad . \qquad (1)$$

for closed continuants containing three diagonals in all. Let M_i be called the i^{th} compound of M_1 and be defined in the following way.—

$$M=M_1=\begin{bmatrix} a_1 & -1 & . & . & . \\ 1 & a_2 & -1 & . & \\ . & 1 & a_3 & -1 & \\ . & . & . & . & . \end{bmatrix}=\begin{bmatrix} (1) & -(0) & . & . & \cdots \\ (0) & (2) & -(0) & . & \\ . & (0) & (3) & -(0) & \\ . & . & . & . & . \end{bmatrix},$$

$$M_2=\begin{bmatrix} -(12) & (0) & . & . & \cdots \\ (4) & -(234) & (2) & . & \\ . & (6) & -(456) & (4) & \\ . & . & . & . & . \end{bmatrix},$$

$$M_3=\begin{bmatrix} (123) & -(0) & . & . & \cdot\cdot \\ (56) & (23456) & -(23) & . & \\ . & (89) & (56789) & -(56) & \\ . & . & . & . & . \end{bmatrix},\quad .\quad .\qquad (2)$$

and so on. These matrices are related to one another by the following property. *If*

$$p \equiv p_{(1)} = \{p_0, p_1, p_2, \ldots\}$$

is a column vector which is annihilated by M_1; *that is, if*

$$M_1 p_{(1)} = 0,$$

then for each positive integral value of i *the* i^{th} *compound continuant* M_i *annihilates the vector*

$$p_{(i)} \equiv \{p_0, p_i, p_{2i}, \ldots\};$$

that is,

$$M_i \, p_{(i)} = 0.$$

In proof of this it is only necessary to write down $(2i-1)$ consecutive equations from the array $M_1 p_{(1)} = 0$, and to eliminate $(2i-2)$ of the p's, none of whose suffixes is a multiple of i. Results of such elimination, when tabulated, can be made to assume the form $M_i p_{(i)} = 0$. A simple example is typical. The result of eliminating p_1 and p_3 from the second, third, and fourth rows of the array $M_1 p_{(1)} = 0$ is

$$\begin{bmatrix} p_0 - p_2 & a_2 & \\ a_3 p_2 & 1 & -1 \\ p_2 - p_4 & \cdot & a_4 \end{bmatrix} = 0.$$

Whence, after expansion by the first column

$$(4)p_0 - (234)p_2 + (2)p_4 = 0,$$

which is identical with the second row of the matrix equation $M_2 p_{(2)} = 0$.

Or again, the equation

$$(56)p_0 + (23456)p_3 - (23)p_6 = 0,$$

is the result of eliminating p_1, p_2, p_4, p_5 from the first five equations of the array $M_1 p_{(1)} = 0$: this leads to the third compound set. Similarly each row of each compound is constructed.

The Heilermann[9] identity,

$$x + \frac{b_1}{1+} \, \frac{b_2}{x+} \, \frac{b_3}{1+} \quad \ldots = x + b_1 - \frac{b_1 b_2}{x + b_2 + b_3 -} \, \frac{b_3 b_4}{x + b_4 + b_5 -} \ldots,$$

may be obtained by forming the second compound M_2 of a suitably chosen continuant M_1. These compound continuants must not, of course, be confused with the ordinary compounds of a matrix which involve all, and not merely a specially selected set of minors of each order.

Relations between the minor *determinants* of these matrices now follow.

Thus, if $(M_i)_r$ denotes the leading principal minor determinant of r rows and columns in the matrix M_i, it is evident from the above theorem that

$$p_r = (M_1)_r p_0.$$

Similarly

$$(2)(4)\ .\ .\ .\ (s)p_{2r} = (M_2)_r p_0, \qquad s = 2r - 2$$

The determinant of the first r *rows and columns of* M_k *is thus equal to that of the first* kr *rows and columns of* M_1, *multiplied by the continued product of the first* k *over-diagonal elements of* M_k, *accompanied by the sign* $(-)^\mathrm{k}$,

For example,

$$\begin{vmatrix} (123) & -(0) & \\ (56) & (23456) & -(23) \\ & (89) & (56789) \end{vmatrix} = (23)(56)(123456789)$$

REFERENCES TO LITERATURE.

1. SCHWEINS, 1825. *Cf.* Muir, *History of Determinants*, vol i, p. 174
2. SMITH, H. J. S., 1860. *British Association Report* (1860). *Collected Works*, vol. i, p. 408.
3. RICHARDSON, A. R., 1928. *Proc London Math. Soc*, (2), vol xxviii, pp. 395–420.
4. SYLVESTER, 1853. *Phil. Mag.*, (4), vol. v, p. 446 and vol. vi, p. 297 *Collected Works*, vol i, pp 609, 641.
5. PALEY and URSELL, 1929 *Proc Cambridge Phil. Soc*, vol xxvi, pp. 127–144.
6. MILNE-THOMSON, 1933. *Proc Edin. Math. Soc*, (2), vol. iii, pp 189–200.
7. WHITTAKER, E T., 1916 *Proc. Roy Soc. Edin*, vol. xxxvi, pp. 243–255.
8. TURNBULL, 1933. *Proc Roy. Soc. Edin.*, vol. liii, pp. 151–163.
9. HEILERMANN, 1845. *Journ für Math.*, vol xxxiii, p. 174.
10. PERRON, 1907. *Math Annalen*, vol lxiv, p 1, who gives many references to Jacobi and others.

(*Issued separately July* 13, 1933)

XVI.—Studies on the Reproductive System in the Guinea-Pig: Variations in the Œstrous Cycle in the Virgin Animal, after Parturition, and during Pregnancy.

By **Thomas Nicol,** M.B., Ch.B., F.R.C.S.E., Senior Lecturer in Anatomy, University of Glasgow.

(MS received March 16, 1933. Revised MS May 13, 1933. Read June 5, 1933)

IN the course of an investigation on the effects of intravitam staining with trypan blue on the uterus and ovaries of the guinea-pig at different periods of the œstrous cycle* in virgin animals and after parturition, it was found necessary to have definite information with regard to possible variations in the lengths of the cycles. Stockard and Papanicolaou (1917 and 1919) reviewed the earlier work on the œstrous cycle in this rodent, and introduced the vaginal smear technique by which successive cycles could be studied in the living animal. The results of their researches seemed to indicate that the non-pregnant female guinea-pig comes into "heat" regularly about every sixteen days, and that there are only slight time-variations in the periodic rhythm of a given female, and that among different individuals there is a limited variation in the lengths of the cycles. Practically similar results were obtained by Selle (1922), though Voss (1927) indicated that his cases did not confirm the very regular periodicity.

My trypan-blue material suggested that perhaps the œstrous rhythm of a given female might occasionally vary considerably from what could be taken as the normal for that animal, or that among different females the cycles might vary in length over a much greater range than observed by Stockard and Papanicolaou. It seemed therefore desirable to study the variations in this strain of animals, especially as the number of œstrous records which had been observed appeared sufficient for a reliable examination of the cycles.

The present paper is a study and analysis of 368 œstrous cycles recorded from 40 guinea-pigs, of which 36 were virgin animals, and 4 were females examined from the time of parturition and which had been separated from the male before littering. Observations on 4 pregnant animals, 2 of which showed apparent œstrous signs, are also recorded.

---

* The terms used in this paper to distinguish the different stages of the œstrous cycle are those originally proposed by Heape, *Quart. Journ. Micr. Sci.*, vol. xliv, 1900.

For the 36 virgin females, 328 cycles are recorded, and the consecutive cycles observed for both virgin and postpartum animals vary in number from 4 to 14, the average being about 10. Some of the virgins were examined from before the onset of sexual maturity. It will be seen that these data afford the opportunity of studying the range of variation between successive cycles in the same animal as well as between different animals, and, as far as I am aware, such a series of consecutive cycles has not been hitherto recorded and analysed for the guinea-pig.

PROCEDURE

The vagina, as is well known, is normally closed between the œstrous periods by what appears to be a thin glossy membrane uniting the anterior and posterior lips of the crescent-shaped vaginal orifice. That the closure of the vagina is not superficial but due to a solid epithelial cord was shown by Kelly and Papanicolaou (1927), who studied the mechanism and anatomy of this physiological process. At the time of sexual activity or œstrus a temporary opening is established between the vagina and the outside, the orifice remaining open for three or four days, then closing very tightly.

Stockard and Papanicolaou (1917) found the period of "heat" or œstrus to last about twenty-four hours,* during which the vagina contained an abundant quantity of fluid The microscopic examination of smears of the vaginal content showed that the constitutional elements changed in a definite manner from the beginning to the cessation of the œstrous flow, so that the period of œstrus could be divided into four stages These shortly stated are:

Stage I. Mucous secretion. Epithelial cells (cornified). Duration—6 to 12 hours.

Stage II. Vaginal fluid thick and cheesy. Great numbers of epithelial cells (nucleated). Duration—2 to 4 hours.

Stage III. Fluid thinner. Epithelial cells and polymorphonuclear leucocytes. Leucocyte containing cells strikingly typical. Epithelial cells gradually becoming less numerous Duration—4 to 6 hours.

Stage IV. Continuation of Stage III. Red cells due to slight hæmorrhage into uterus and vagina. Duration—1 to 2 hours.

Diœstrus: Leucocytes and atypical epithelial cells.

---

* It must be noted, however, that the female is "in heat," strictly speaking, only two to three hours usually near end of Stage I and beginning of Stage II (Stockard and Papanicolaou; Squier).

Ovulation occurs at the end of Stage II, or beginning of Stage III.

Selle (1922) showed that, as occurs also in the rat, Stage I of Stockard and Papanicolaou is preceded by one in which desquamation of the epithelial cells of the vagina occurs. This primary stage of Selle usually occurs while the vagina is still closed.

I have a number of times tested the serial order of these changes in the vaginal content during œstrus, and my observations agree in all essentials with these workers. It appears therefore that, since the duration and serial order of the changes is known, examination of the animals once a day will yield sufficiently reliable data on the cycle as a whole.

The animals in my series were accordingly examined once daily for their œstrous condition, always about the same hour, and the observations were taken continuously by myself during the months of May to January inclusive, except for a fortnight during July, when the observations were entrusted to an assistant. When the vagina was open a daily smear of the vaginal content was taken almost until the vaginal orifice became occluded. In taking the smears the technique of Stockard and Papanicolaou was followed. Hæmatoxylin and eosin were used as stains. For each animal examined a chart was made, on which were noted the day of examination, the presence or absence of the "vaginal closure membrane"; if the vagina was open and a smear therefore taken, the amount of the smear and the nature of its cells were also recorded. Only some of the animals were examined daily during the full nine months, though all were observed daily for at least several months.

At œstrus I most commonly found the vagina open for two days during which smears could be taken, practically closed on the third day, and completely closed on the fourth day, or perhaps not till the fifth day. This is in accord with the observations of other workers. The same animal, however, may in this respect show slight variations. Although most often closed on the fourth day, the vagina may remain open at different œstrous periods for three to seven or even nine days. In females kept separate from the male from before parturition the vagina was found to be completely closed about the fourth day, but at the second œstrous period postpartum the vagina may remain open for nine or ten days, though the orifice is closed at successive œstrous periods in about four or five days. This occurred in one of four animals after parturition. At the beginning of sexual maturity the vagina may remain open at about the first œstrus for as long as nine or ten days, though at successive œstrous periods the orifice in the same female is closed by the "membrane" on the fourth day. It is apparent from these facts that no conclusions regarding the presence of a state of heat can be arrived at from the opening of the vagina alone, unless the vaginal content is examined microscopically.

During each cycle typically corresponding changes were shown by Stockard and Papanicolaou (1917) to be occurring in the vagina, uterus, and ovary, a given stage in one of these organs closely accompanying parallel stages in the other two.

THE ŒSTROUS RHYTHM IN THE VIRGIN ANIMAL.

It has long been known that to maintain the œstrous rhythm the animals must be healthy, well fed, and under uniform environmental conditions (Allen, 1922). The guinea-pigs in our colony were therefore kept in separate spacious boxes, carefully looked after, and feeding was at regular intervals. The food consisted of green vegetables, chop and hay. Moreover, the breeding-room was well ventilated and the temperature was thermostatically controlled never to fall below 55° Fahrenheit. During the week the temperature was usually about 70° Fahrenheit, only perhaps falling to 55° during the week-end.

Table I records the cycles of thirty-six virgin guinea-pigs examined daily for their œstrous conditions. The animals varied from about two months to eight months old at the time of the first examination (Table I, column 2). The months during which the daily examination was carried out are shown for each animal by crosses in column 6. The lengths of the diœstrous cycles are shown in column 3, and these are calculated from the smears as being the length of time from the beginning of one "heat" period to the beginning of the next.

The animals have been divided into four groups: A, B, C, and D, according to age and special characteristics in the cycles which appeared to be of significance. Attention will now be drawn to some of the findings in these groups, but the main points will be treated later in the discussion.

Group A.—This group shows the lengths in days of consecutive cycles from twelve animals, which when first examined were sexually mature, and of ages ranging between four and eight months. The characteristic feature of the group is the regularity in the length of successive cycles in the same animal. It will be noted that among a long series of practically regular cycles an animal (*e.g.* No. 4) may have one cycle about twice as long as the others. In animal No. 10 a series of consecutive 17 to 18-day cycles is followed by one cycle of 16 days, then a longer one of 20 days, and the cycle becomes again of about 17 days, and in animal No. 12 a short cycle of 14 days is followed by eight 17 to 18-day cycles, then one of 13 days occurs. With regard to the mean length of the cycles for the different animals, eleven animals have mean cycles ranging in length roughly between 16 and 18 days, while the remaining animal has a regular 24-day cycle. The latter is undoubtedly an unusual case, but it may be stated that the vagina in this animal was open only for 3 to 4 days every 24 days, when smears of the vaginal content examined microscopically appeared to show quite definite indications of the state of œstrus.

The mean length of cycle for this group, which includes 124 cycles, is 17·18 days, which agrees roughly with the results of other workers.

Group B.—This group illustrates the irregular œstrous cycle rhythm. These females varied in age from six to eight months when first examined for their œstrous conditions. They have been arranged so that the irregularity observed in the series of cycles for each animal gets progressively worse from animal No. 13 to animal No. 19. In animal No. 15 no œstrous period was observed on one occasion for 42 days, when the vagina which had been closed during all this period opened, and a definite smear, microscopically examined, indicated a state of œstrus. Graphs were made for each animal in this group and compared, and it appeared that some animals, such as No. 18, have a totally irregular œstrous rhythm, while there is the suggestion in others (*e g.* No. 15) that the irregularities may be separated by a series of what appear to be normal cycles for that animal.

In Table I, column 4, the total number of cycles for this group is given as 81, the cycle of 42 days belonging to animal No. 15 having been omitted as considered an extreme exception. The mean length of the cycle for this group is 18·81 days, which is somewhat high, and no doubt due to those irregular animals having so many long cycles.

Group C.—Eleven virgin females examined from about the onset of sexual maturity at two to three months old are included in this group, which shows that at puberty the first few cycles may be longer than what is shown afterwards to be the normal for a given animal. There appear to be all degrees and variations of this. Animals Nos. 29 and 30, for each of which five cycles are recorded, show an irregular œstrous rhythm. A similar irregularity is shown in the first five periods of animal No. 28, and it seems interesting to note that this irregularity is apparently then replaced by four successive cycles of 16 days,* which may be considered the normal regular rhythm for this female. The total number of cycles examined in this group is 75, and the mean length of the cycle for the group is 17·97 days. This rather high result appears to be due to the fact that the mean length of cycle for some of the animals (Table I, column 5) is rather high, no doubt as a result of the unusually long cycles at the onset of sexual life.

---

* Only three of these 16-day cycles are shown in Table I, but a fourth cycle of 16 days has since been observed for animal No. 28.

TABLE I.—RECORDS OF 36 VIRGIN GUINEA-PIGS EXAMINED DAILY FOR THEIR ŒSTROUS CONDITIONS.

	Index Number of Animal	Age when first Examined	Diœstrous Cycle in Days.	Total Number of Cycles.	Mean Length of Cycle in Days	Month Examined.								
						May.	June	July	Aug	Sept	Oct.	Nov.	Dec.	Jan.
		Months												
Group "A"	1	×5	15, 16, 16, 16, 16, 17, 16, 16, 17, 16, 16	11	16·09		+	+	+	+	+	+	+	
	2	×4	15, 17, 16, 16, 16, 16, 17, 16, 17, 17, 16, 16, 16	13	16·23		+	+	+	+	+	+	+	+
	3	4	17, 16, 17, 16, 16, 16, 17, 17, 17	9	16·56				+	+	+	+	+	+
	4	5	17, 32, 17, 17, 17, 17, 18, 18, 17, 18, 17, 17, 16	13	18 31		+	+	+	+	+	+	+	+
	5	×5	16, 16, 16, 17, 15, 16, 16, 16, 17, 16, 17, 16	12	16·17			+	+	+	+	+	+	+
	6	×7–8	18, 18, 18, 18, 17, 17, 18, 16, 18, 18, 16, 16, 17	13	17 31	+	+	+	+	+	+	+	+	+
	7	×6	18, 17, 16, 16, 16, 17, 17, 17, 16, 16, 15	11	16·45			+	+	+	+	+	+	+
	8	4	16, 16, 16, 16, 16	5	16 00			+	+	+				
	9	7	24, 24, 24, 24	4	24·00			+	+	+	+			
	10	4–5	17, 18 17, 18 17, 18, 16, 20, 17, 17, 19, 18	12	17·67		+	+	+	+	+	+	+	+
	11	×6–7	18, 18, 18, 16, 20, 19, 18, 19, 17, 18, 18	11	18·09			+	+	+	+	+	+	+
	12	4	14, 18, 17, 18, 18, 18, 18, 17, 17, 13	10	16 80				+	+	+	+	+	+
				124	17 18									
	13	7	19, 19, 11, 18, 18, 17, 20, 18, 18, 17, 18, 18, 19, 18	14	17 71	+	+	+	+	+	+	+	+	+
	14	×7–8	18, 18, 17, 19, 21, 23, 19, 22, 21, 20	10	19 80	+	+	+	+	+	+	+	+	
	15	×7–8	23, 19, 19, 19, 19, 21, 21, 20 24, 10, 42*	10	19·50	+	+	+	+	+	+	+	+	+
Group "B"	16	6	21, 21, 20, 25, 18, 24, 15	7	20·57			+	+	+	+	+	+	
	17	×7	16, 17, 15, 17, 17, 21, 25, 17, 21, 20, 18, 20, 22	13	18·92	+	+	+	+	+	+	+	+	+
	18	×7–8	19, 22, 11, 18, 21, 20, 17, 18, 20, 22, 17, 17, 20	13	18 62	+	+	+	+	+	+	+	+	+
	19	×7–8	11, 17, 22, 13, 18, 17, 19, 17, 17, 23, 15, 24, 19, 19	14	17·93	+	+	+	+	+	+	+	+	+
				81	18·81									

The ages marked with a cross (×) are those furnished by the dealer from whom the animals were obtained The other animals were bred in our own animal house

* The 42-day cycle of animal No. 15 is omitted from the total number of cycles.

TABLE I—*continued.*

	Index Number of Animal.	Age when first Examined	Diœstrous Cycle in Days.	Total Number of Cycles.	Mean Length of Cycle in Days.	Month Examined.								
						May	June	July.	Aug	Sept	Oct	Nov	Dec.	Jan.
		Months												
Group "C"	20	2–3	32, 23, 17, 17, 17	5	21·20						+	+	+	+
	21	2–3	20, 19, 16, 16, 17, 16, 16, 16, 16	9	17·33				+	+	+	+	+	+
	22	2–3	19, 24, 16, 17, 16, 16, 16	7	17·71					+	+	+	+	+
	23	2–3	19, 16, 17, 17, 17, 16, 16, 17, 17	9	16·89				+	+	+	+	+	+
	24	2–3	19, 17, 17, 18, 17, 18, 17, 16, 17	9	17·33				+	+	+	+	+	+
	25	2–3	19, 17, 17, 18, 17	5	17·60						+	+	+	+
	26	2–3	19, 18, 17, 18, 17, 17, 16, 16	8	17·25					+	+	+	+	+
	27	2–3	32, 11, 19, 19, 21	5	20·40						+	+	+	+
	28	2–3	20, 11, 17, 11, 18, 16, 16, 16	8	15·63					+	+	+	+	+
	29	2–3	25, 20, 19, 15, 19	5	19·60						+	+	+	+
	30	2–3	26, 22, 17, 17, 21	5	20·60						+	+	+	+
				75	17·97									
Group "D"	31	2–3	17, 17, 17, 17, 17, 17, 16, 17, 17	9	16·89				+	+	+	+	+	+
	32	2–3	17, 17, 17, 17, 17, 17	6	17·00						+	+	+	+
	33	2–3	17, 17, 17, 16, 17, 16, 16, 17	8	16·63					+	+	+	+	+
	34	2–3	18, 18, 19, 18, 18, 18, 19	7	18·29					+	+	+	+	+
	35	2–3	15, 15, 15, 15, 16, 16, 17, 16, 17, 15	10	15·70				+	+	+	+	+	+
	36	2–3	16, 18, 20, 18, 17, 23, 17	7	18·43					+	+	+	+	+
				47	17·04									
				327	17·73									

Group D.—This group of six females examined from the age of two to three months shows that some animals from the onset of sexual maturity may maintain a regular œstrous rhythm The last animal in the group (No 36) for which 7 cycles are recorded appears to have an irregular rhythm. In the 47 cycles recorded in this group the mean length is 17·04 days.

THE COMPLETE SERIES OF THIRTY-SIX VIRGIN ANIMALS.

It may first be mentioned that in agreement with Voss (1927) I have observed that heat periods may vary in intensity, and that a long diœstrum may be followed by a less intense œstrus.

The mean length of the 327 cycles from the 36 females was found to be 17·73 days (Table I), with a probable error of ± 1·82 days One could therefore expect to have cycles ranging from roughly 12 days to 23 days, and those outside this range would be considered exceptional.

In order to ascertain the relative frequency of cycles of different lengths, the 327 cycles recorded were arranged in groups according to the length of the cycles. Table II shows this distribution, the top row indicating the length of cycle and the bottom row the number of cycles of a given length which occurred in the series observed.

TABLE II—FREQUENCY OF CYCLES OF DIFFERENT LENGTH IN VIRGIN ANIMALS

Length of diœstrous cycle in days .	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	32
Number of cycles observed .	1	6		2	1	13	72	99	53	28	15	11	6	5	8	3	1	3

From the above Table II there was constructed diagram 1, which shows the frequency curve of this distribution and indicates more clearly the relative frequency of the cycles of different length.

It is apparent from diagram 1 that in this series of 327 cycles from 36 virgin females the most frequent cycle is one of 17 days, and the next most frequent are at 16 and 18 days, the 16-day cycle being more common than the 18-day cycle. The frequency curve is not perfectly symmetrical since there is a greater tendency for cycles to be longer than the mode of 17 days, than to be shorter. The cycles can apparently range from 10 to 26 days and even longer, though these short and long cycles are undoubtedly random variations.

Considerable variation can also be seen between the mean lengths of the cycles of all the animals (Table I, column 5) when these are compared with each other. This was examined mathematically to find out whether the animals differed significantly from one another. In this connection it should be noted that the mean lengths of the cycles for the animals might not be strictly comparable in that all the animals were not observed during the full nine months. Examination of the differences between individual cycles for any one animal, however, showed no significant difference between the cycles; hence the mean lengths of the cycles (Table I, column 5) could be taken as the representative mean cycles of the animals, and to these mean cycles tests for significance were applied according to the variance method of Fisher (1928).

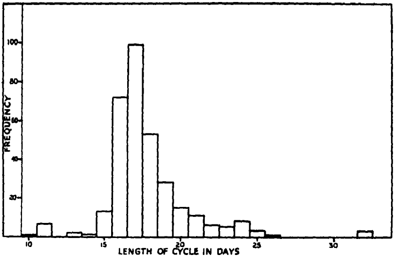


DIAGRAM 1.—Graph showing distribution of cycle lengths. (36 females—327 cycles)

It was found that the variation in the mean lengths of the cycles of the different animals is not due to chance, but is a "significant" characteristic for each animal.

Seasonal Variations.—The data under this head have a general biological interest, and as only the records of the virgin animals in my series were examined for seasonal variation it seems appropriate to discuss this question here. Stockard and Papanicolaou (1917) indicated that the length of the œstrous cycle is probably a little shorter during the warm time of the year, and a little longer during the cold weather, but admitted that the numbers of their cases were small and these slight seasonal variations might be more suggestive than demonstrative in importance. It therefore seemed desirable to examine the present larger series of cycles for variations due to season.

Some animals, for example No. 1, would appear to show no seasonal variation under these environmental conditions, but the irregularities of others make it desirable that the group as a whole should be examined. In order to examine for seasonal variations the cycles were first arranged in the various months. The rough monthly distribution of the cycles is shown in Table I, but as the mean length of cycle for all the animals taken as one group is 17·73 days, it can be seen that there will be many cases in which one cycle is spread over parts of two successive months. In fact, there were 118 such cases. Where one cycle fell partly in one month and partly in another, the cycle was allotted to the month in which the major portion lay. The result of this was that for any one month some animals were credited with two cycles and others with one, thus giving a bias with regard to length of cycle to the animals with the two cycles in that month. There are so many cases of overlapping between months, however, that the effect of this bias is small. Since there are not complete data for each animal for all the months, and as there is a significant difference in the mean lengths of the cycles for the different animals, it is not permissible in the usual way to take the total number of days occupied by all the cycles in one month, and by dividing by the total number of cycles for that month obtain a mean which could be compared for seasonal variation with other means similarly obtained for the other months. The following procedure was therefore adopted. The mean cycle for each animal was subtracted from the individual cycles of that animal, and the deviations thus obtained were considered free from the effect of the significant difference due to the animals. These deviations were then added for the respective months, and each total was divided by the number of cycles recorded for that month. The results are shown in the following table:—

TABLE III.

Month.	May	June	July	Aug	Sept.	Oct	Nov	Dec.	Jan
Mean deviation in days	−1·08	+0·33	−0·54	−0·06	+0·37	+0·71	+0·15	−0·52	−0·29
Number of cycles observed during each month .	6	9	28	30	47	52	55	57	43

An examination of these deviations for the various months as given in the above Table III shows that in some cases—for example, between October and January—there is a difference of one day. This, however, is practically accounted for by the fact that during October six animals at the beginning of their sexual life were included, and these had unusually long cycles. Table III was, however, examined for significance in relation to seasonal variation in the same manner as the differences between the animals were examined. The results showed no significant difference.

It may, therefore, be said that when a fairly uniformly warm temperature is maintained in the breeding-room during the winter months, there is no seasonal variation in the length of the sexual cycle in the guinea-pig. Moreover, no influence of season was noted with regard to the phases of the cycle.

DISCUSSION AND COMPARISON OF DATA WITH THOSE OF OTHER OBSERVERS.

From the present series of 327 cycles it has been again confirmed that the guinea-pig most commonly shows a remarkably regular periodicity in the length of consecutive cycles in a given female; a series of successive cycles may not vary at all, and if so the most common variation between any two consecutive cycles is usually one day or perhaps two days. This is in agreement with the records of 67 cycles from 26 females recorded by Stockard and Papanicolaou (1917) and of 111 cycles from 20 females recorded by Selle (1922)—in these tables the greatest variation between two consecutive cycles is two days.

However, the present animals also show that there exist curious instances where a longer or shorter cycle may, for no apparent reason, be interpolated among a series of practically regular cycles. Long and Evans (1920 and 1922) have also reported curious instances of such longer cycles for the rat

The series also shows that in some cases a period may be missed or so feebly expressed that the vagina remains closed. In the mouse Allen (1923) records that instead of the rapid recurrence of a second cycle a long diœstrous interval may intervene, and at times in such a cycle there may be slight indications of œstrus in the smears at the mid-interval, suggesting possibly that the failure of the expected heat is only partial.

Moreover, at the onset of sexual maturity the first one or two cycles may be prolonged, then the normal rhythm for the animal is abruptly established and maintained. Other animals have apparently a regular rhythm from the onset of sexual maturity. Prolongation of the first cycles of puberty are also reported for the guinea-pig by Ischii (1920), and for the mouse by Allen (1922) and Parkes (1926), and for the rat by Long and Evans (1920) and Ischii (1922).

The present series also shows that certain animals have almost a totally irregular rhythm, and this may be seen in some cases from the onset of sexual maturity. About one-fourth of the present animals show irregularities in their œstrous rhythm, but this is probably rather a high proportion, which would have been considerably less if a larger number of animals had been under observation It may be said, however, that the animals which showed irregular cycles, whether extreme or otherwise, never at any time, so far as could be observed, showed any obvious signs of illness or anything which might have been a causal factor related to the variations in the lengths of the cycles Irregularities in the œstrous rhythm were also observed by Voss (1927) when making a cell-type count of the vaginal contents during the diœstrous cycle in eight normal female guinea-pigs. One of his animals never had a heat period at all during two months, two other animals had no heat period in five and a half weeks, another animal showed the following series of irregular cycles—15, 20, 13, 18, 18 days, while the other animals did not vary so much. Long and Evans (1922) for the rat and Gostimirović (1931) for the white mouse show that in some of the animals there may occur a partially or totally irregular œstrous rhythm, and Hartman (1932) for the monkey records cases in which hardly two cycles are of the same length.

Having discussed the rhythm shown by the consecutive cycles in the individual animals, the variation in length of cycle without reference to any one animal will now be considered

It has been shown that the most frequent length of cycle recorded in my animals is one of about 16 to 18 days, and this is the length of cycle usually maintained by an animal with a regular rhythm. This agrees fairly accurately with the findings of Stockard and Papanicolaou (1917 and 1919)—16 days, ranging from 15 to 17 and, in exceptional cases, 18 days; Selle (1922)—16 days, ranging from 14 to 17 days; Hartmann and Olbers (1931)—15·4 days; and Ischii (1920)—15 to 16½ days, ranging from 14½ to 18 days. Other investigators give the length of the cycle as being 13½ to 19 days, Loeb (1918); 13 to 20 days, Voss (1927); and 10 to 18 days with an average of about 16 days, Evans and Swezy (1931). The previous literature on the length of the œstrous cycle in guinea-pig has been summarised by Long and Evans (1922). That my results tend to be slightly higher than, for example, those of Stockard and Papanicolaou may be due to the strain of animals used. In this connection it is to be noted that Allen (1923) for the mouse suggests that a genetic factor may be operating partially to determine cycle length Moreover, it has been shown for the present animals that some females in a series may show differences from one another which have been proved statistically significant, and such an unusual example as an animal showing a series of consecutive 24-day cycles can occur. It seems interesting to note that in the rat Long and Evans (1922) record two cases of spontaneous œstrous cycles of 39 days duration.

It must be stated that the smears of some of the very short cycles were not absolutely typical, but were here recorded as "heat" smears in view of the number of epithelial cells, their occurrence in groups, and the presence of some leucocyte containing cells, in addition to large numbers of leucocytes. Cycles of about 11 days seem rather too short for a true heat and ovulation to occur It may be noted, however, that for the mouse it has been shown by Parkes (1926 and 1927) and for the guinea-pig by Courrier (1928) that the presence of ripe follicles is not indispensable to the occurrence of heat; and also for the mouse Allen (1923) states that ovulation may occur at one period, but at the next the follicles may fail to rupture. Furthermore, it has been shown for the guinea-pig by Loeb (1911 and 1923) and Papanicolaou (1920) that removal of the corpora lutea after an ovulation accelerates the appearance of the next œstrus. Practically similar results were obtained in the rabbit by Nielsen (1921), and in the cow by Hammond (1927). It might be that influences such as these were related to the occurrence of the short cycles recorded for the present animals.

With regard to the occurrence of the very long cycle of 42 days and the long periods of anœstrus observed by Voss (1927), it may be noted that for the mouse Gostimirović (1931) has recorded cases of very long diœstrous intervals, and Parkes (1928) mentions that some animals may have apparent absence of any real cycle, this being perhaps correlated with interesting abnormalities of the ovary.

The diœstrum may be prolonged by underfeeding, Papanicolaou and Stockard (1920), and a one-sided diet can prevent heat symptoms for a time, as found by Ischii (Loeb, 1923). There does not appear any definite evidence that such factors account for the longer cycles in the present series of animals.

Marked variation in the length of cycle from the mode for a given animal has now been recorded for a number of animals, including the mouse by Allen (1923), Parkes (1928), Zamkof and Rokhlina (1929), Gostimirović (1931); for the rat by Long and Evans (1922), and Ischii (1922); for the monkey by Corner (1923), Allen (1927), Hartman (1932); for the opossum by Hartman (1923); and for the cow (see Hammond, 1927).

THE CYCLE AFTER PARTURITION.

The guinea-pig is similar to the rat and mouse in having an immediate postpartum œstrous period, but it differs from these two animals according to Loeb and Kuramitsu (1921) in that inhibition of œstrus is not found during the rest of lactation, the second postpartum œstrus in the guinea-pig occurring in about 16 days, *i.e.* at the normal interval.

Four animals from the time of littering are included in this group (Table IV). The males had been separated from the females a considerable time before parturition, and the young were allowed to remain and be suckled. The examinations were carried out as for the virgin animals.

TABLE IV —RECORDS OF FOUR GUINEA-PIGS EXAMINED DAILY FROM AFTER PARTURITION FOR THEIR ŒSTROUS CONDITIONS

Index No of Animal.	Age at Parturition	No of Litter.	Diœstrous Cycle in Days.	Total Cycles	Month Examined.							
					July	Aug	Sept.	Oct	Nov	Dec	Jan.	Feb
37	6–8 months	2	17, 22, 18, 18, 18, 36, 19, 19, 18	9		+	+	+	+	+	+	+
38	7 months	1	26, 19, 16, 18, 34, 17, 17, 17, 16, 18	10	+	+	+	+	+	+	+	
39	6 months	3	20, 20, 17, 18, 17, 15, 18, 16, 17, 17, 17	11	+	+	+	+	+	+	+	+
40	6–8 months	3	11, 14, 22, 17, 17, 17, 17, 17, 19, 18	10	+	+	+	+	+	+	+	

It can be seen from Table IV that with regard to the first two postpartum cycles these animals show characteristics which closely resemble the young virgin animals at the onset of sexual maturity. It will be noted that the first cycle may be of about normal length. On the other hand, the first two or three cycles may be longer or shorter than what is shown to be the normal for these animals, after which the normal rhythm is abruptly established. The later periods of these animals show the variations already referred to in the study of the normal virgin animals

Observations in only four postparturient animals do not provide data upon which general conclusions can be based. The examples submitted illustrate, however, the fact that in females shown afterwards to have a practically regular œstrous rhythm, the first one or two postpartum cycles may be irregular, and therefore cannot be relied upon for experimental investigations on the cycle. The data, so far as they go, also indicate, as against the statements of Loeb and Kuramitsu, the possibility that in some cases lactation does prolong the first postpartum cycles

THE VAGINAL SMEAR DURING PREGNANCY.

It is stated by Kelly and Papanicolaou (1927) that when the guinea-pig is pregnant the vulva remains closed and does not open until parturition is imminent.

Four pregnant animals were examined by me daily for œstrous signs, as in the case of the virgin animals. The males were removed from the females about a week after the daily examination commenced, and the females occupied separate boxes. Two of these females were observed daily only during the last 42 days of pregnancy, and in both cases the vagina remained closed up to and including the day before parturition.

The records of the other two pregnant animals are as follows:—

Animal No. 43.—Littered 3, 2nd November. Age at parturition 6 months. Taking 62 days as the average duration of gestation (Squier, 1932), it may be said that impregnation occurred about the beginning of September. This animal was examined daily from 3rd September up till day of littering, and therefore during practically the whole period of gestation. The vagina was open for about 5 days from 3rd September and again from 29th September, and apparently typical vaginal œstrous smears were obtained. Assuming that the first of these periods represented the time when impregnation occurred, vaginal œstrous signs returned after about 26 days. Twenty-five days later—that is, about 10 days before parturition, the vagina again opened, but this time only for 2 days, and the smear obtained showed what appeared to be vaginal œstrous signs of slight intensity.

Animal No. 44 —Littered 1, 9/10th July. Age at parturition 7 months. Impregnation therefore probably occurred about 8th May. This female was examined daily from 14th May up till date of littering, and therefore during practically the whole of gestation. The vagina was open for 4 days from 25th May and again from 21st June, during which apparently typical vaginal œstrous smears were obtained at intervals of 17 and 27 days. In addition, this female discharged a small amount of blood from the vagina on 1st June, the vagina being closed the previous and the following day.

These two cases show that the vagina does not always remain closed throughout pregnancy, and also that vaginal œstrous signs may occur once or twice during the period of gestation. During the entire duration of œstrus, which lasts about 24 hours, it must be remembered that the female is actually "in heat," strictly speaking, for 2 or 3 hours only, and it is of course not known whether or not these two pregnant females experienced true "heat" and would have accepted the male Ischii (1920) observed that there may occasionally be weak indications of heat even in the pregnant guinea-pig. For other rodents Nelson (1929) has reported the recurrence of œstrous cycles in a pregnant white rat, while two instances of copulation during pregnancy have been reported by Long and Evans (1922), though no evidences of the normal œstrous cycle were noted. For the mouse Parkes (1926) stated that in so far as cornification is a criterion of œstrus, it may be said that œstrus does not occur during pregnancy. Recurring menstrual bleedings during pregnancy have been recorded in the macaque monkey and chimpanzee (see Hartman, 1932, for references). It seems probable that in the guinea-pig* during early pregnancy the factors involved in the maintenance of the sexual cycle still operate though much dampened down, and this may be related to the occasional occurrence at that time of vaginal œstrous signs.

With regard to the blood in the vagina noted in animal No 44, which at that time was about 24 days pregnant, the number of pregnant animals examined is too small for any profitable discussion. Suffice it to say that this may be possibly the same phenomenon known as the "placental sign" in rat and mouse, and even in higher forms, *e g.* the monkey (rat—Long and Evans, 1922; mouse—Parkes, 1926; monkey—Hartman, 1932).

SUMMARY

1. 368 cycles from 40 animals are recorded and analysed.
2. The normal variation in a series of consecutive œstrous cycles in the guinea-pig is one or perhaps two days.
3. Curious instances occur of a longer or shorter cycle being interpolated among a series of practically uniformly regular cycles Occasionally a cycle may be apparently missed, or so feebly expressed that the vagina remains closed.
4. The first cycles at puberty may be prolonged, then the regular rhythm for a given animal establishes itself abruptly. Other animals have a regular rhythm from the onset of sexual maturity
5. Some animals have practically a totally irregular rhythm, and this may occur in some cases from the onset of sexual maturity.
6. The animals of a series may show differences from one another in respect to mean length of cycles which are statistically significant, and such an unusual example as an animal with a regular 24-day cycle is recorded.
7. In animals which suckle their young the first postpartum cycles may be irregular, and then the normal rhythm is established abruptly.
8. The normal cycle is usually one of 16 to 18 days, which agrees fairly accurately with the findings of Stockard and Papanicolaou (1917) and other workers.
9. The variations from the mode may, however, be considerable, and cycles from 10 to 26 days and longer are recorded.
10. No seasonal variation is found in the length of cycle when a fairly uniformly warm temperature is maintained in the breeding-room.

---

* See Hartman (1932) for comparison with the monkey

11. In some animals the vagina may open during pregnancy, and apparently definite vaginal œstrous signs be observed.
12. Variations are also noted in the time the vagina remains open at œstrus, though the usual time is verified as being about 4 days.

I desire to express my deep sense of obligation to my chief, Professor Bryce, F.R.S., for generous encouragement and helpful criticism, and to Mr Richard A. Robb, M.Sc., Mathematics Department, University of Glasgow, for the statistical analysis of the data.

REFERENCES TO LITERATURE.

ALLEN, E., 1922. "The Œstrous Cycle in the Mouse," *Amer. Journ. Anat*, vol. xxx, pp. 297–371.

——, 1923. "Racial and Familial Cyclic Inheritance and other Evidence from the Mouse concerning the Cause of Œstrous Phenomena," *Amer Journ. Anat.*, vol. xxxii, pp 293–304.

——, 1927. "The Menstrual Cycle of the Monkey, *Macacus rhesus*: Observations on Normal Animals, the Effects of the Removal of the Ovaries and the Effects of Injections of Ovarian and Placental Extracts into the Spayed Animals," *Carnegie Inst. Wash. Pub*, No. 380, pp. 1–44.

CORNER, G. W., 1923. "Ovulation and Menstruation in *Macacus rhesus*," *Carnegie Inst. Wash. Pub.*, No. 332, pp. 73–101.

COURRIER, R., 1928. "La Présence du Follicule mûr n'est pas indispensable à l'apparition de l'œstrus," *C R. Soc. de Biol.*, vol xcix, pp. 26–27.

EVANS, H. M., and O. SWEZY, 1931. "Ovogenesis and the Normal Follicular Cycle in Adult Mammalia," *Mem. Univ. Calif*, vol. ix, No. 3, pp. 119–224.

FISHER, R. A., 1928. *Statistical Methods for Research Workers*, London

GOSTIMIROVIĆ, D., 1931. "Geschlechtsphysiologische Studien. Von M. Borst, A. Döderlein und D. Gostimirović. V. Der Brunstzyklus als Kontrolle der Ovarialfunktion. Einteilung des Verhaltens des Zyklus bei der weissen Maus," *Münch. Med. Wschr.*, Nr. 14, pp. 572–574.

HAMMOND, J., 1927. *The Physiology of Reproduction in the Cow*, Cambridge.

HARTMAN, CARL, 1923. "The Œstrous Cycle in the Opossum," *Amer. Journ. Anat.*, vol. xxxii, pp. 353–421.

HARTMAN, C. G., 1932. "Studies in the Reproduction of the Monkey, *Macacus* (*Pithecus*) *rhesus*, with special reference to Menstruation and Pregnancy," *Carnegie Inst. Wash. Pub.*, No. 433, pp. 1–161.

HARTMANN, H., and H. OLBERS, 1931. "Die zyklischen Veränderungen des Cervixepithels beim Meerschweinchen," zugleich eine Bemerkung zur Arbeit von Hofbauer "Kausale Faktoren der genitalen präcancerösen Veränderungen im *Zbl Gynäk.*, 1930, Nr. 38," *Zentralbl. f. Gynäkol.*, Nr. 15, pp. 1314–1319.

ISCHII, O., 1920. "Observations on the Sexual Cycle of the Guinea-pig," *Biol. Bull.*, vol. xxxviii, pp. 237–250.

——, 1922. "Observations on the Sexual Cycle of the White Rat," *Anat. Record*, vol. xxiii, pp. 311–314.

KELLY, G. L., and G. N. PAPANICOLAOU, 1927. "The Mechanism of the Periodical Opening and Closing of the Vaginal Orifice in the Guinea-pig," *Amer. Journ. Anat.*, vol. xl, pp. 387–411.

LOEB, L., 1911. "The Cyclic Changes in the Ovary of the Guinea-pig," *Journ Morph.*, vol. xxii, pp. 37–70.

——, 1918. "Corpus Luteum and the Periodicity in the Sexual Cycle," *Science*, N.S , vol. xlviii, pp. 273–277.

——, 1923. "The Mechanism of the Sexual Cycle, with special reference to the Corpus Luteum," *Amer. Journ. Anat.*, vol. xxxii, pp 305–343

LOEB, L., and C. KURAMITSU, 1921. "The Influence of Lactation on the Sexual Cycle in the Rat and Guinea-pig," *Amer. Journ. Phys.*, vol lv, pp. 443–449.

LONG, J. A., and H. M. EVANS, 1920. "On the Attainment of Sexual Maturity and the Character of the First Œstrous Cycle in the Rat," *Proc. Amer. Assoc. of Anatomists, Anat Record*, vol xviii, pp 244–245

——, 1920. "The Œstrous Cycle in the Rat," *Proc. Amer. Assoc. of Anatomists, Anat. Record*, vol. xviii, pp. 241–244

——, 1922. "The Œstrous Cycle in the Rat and its Associated Phenomena," *Mem. Univ. Calif*, vol. vi, pp. 1–148.

NELSON, W. O., 1929. "Œstrus during Pregnancy," *Science*, N S., vol lxx, pp. 453–454.

NIELSEN, F., 1921. "Action exercée par le corps jaune sur la maturation des follicules et sur la chaleur de la lapine," *C.R. Soc. de Biol*, vol. lxxxv, pp 614–615.

PAPANICOLAOU, G. N , 1920. "Influence of Removal of Corpora Lutea and Ripe Follicles on the Œstrous Periodicity in Guinea-pigs," *Proc Amer. Assoc. of Anatomists, Anat. Record*, vol. xviii, p. 251

PAPANICOLAOU, G. N., and C R STOCKARD, 1920 "Effect of Underfeeding on Ovulation and the Œstrous Rhythm in Guinea-pigs," *Proc Soc. Exper Biol. and Med.*, vol xvii, pp. 143–144

PARKES, A. S., 1926. "Observations on the Œstrous Cycle of the Albino Mouse," *Proc Roy Soc*, B, vol. c, pp. 151–170

——, 1926. "On the Occurrence of the Œstrous Cycle after X-ray Sterilisation Part I Irradiation of Mice at Three Weeks Old," *Proc. Roy. Soc*, B, vol c, pp. 172–199

——, 1927. "On the Occurrence of the Œstrous Cycle after X-ray Sterilisation. Part II. Irradiation at or before Birth," *Proc. Roy. Soc.*, B, vol ci, pp 71–95.

——, 1927. "On the Occurrence of the Œstrous Cycle after X-ray Sterilisation. Part III. The Periodicity of Œstrus after Sterilisation of the Adult," *Proc. Roy. Soc.*, B, vol. ci, pp. 421–449.

——, 1928. "The Length of the Œstrous Cycle in the Unmated Normal Mouse Records of One Thousand Cycles," *Brit. Journ. Exper. Biol.*, vol. v, pp. 371–377.

SELLE, R. M., 1922. "Changes in the Vaginal Epithélium of the Guinea-pig during the Œstrous Cycle," *Amer. Journ. Anat.*, vol. xxx, pp. 429–449.

SQUIER, R. R., 1932. "The Living Egg and Early Stages of its Development in the Guinea-pig," *Carnegie Inst. Wash. Pub.*, No. 433, pp. 223–250.

STOCKARD, C. R., and G. N. PAPANICOLAOU, 1917. "The Existence of a Typical Œstrous Cycle in the Guinea-pig—with a Study of its Histological and Physiological Changes," *Amer. Journ. Anat.*, vol. xxii, pp. 225–283.

——, 1919. "The Vaginal Closure Membrane, Copulation, and the Vaginal Plug in the Guinea-pig, with further considerations of the Œstrous Rhythm," *Biol. Bull.*, vol. xxxvii, pp 222-245.

VOSS, H. E., 1927. "Über weibliche Sexualhormone (Thelytropine). XIV. Beiträge zur Physiologie der vaginalen Brunstvorgänge des Meerschweinchens," *Pflügers Arch*, vol. ccxvi, pp. 156–180.

ZAMKOF, A. A., and M. L. ROKHLINA, 1929. "La Durée moyenne du cycle sexuel des femelles chez les Souris blanches," *C R Soc. de Biol.*, vol. cii, pp. 1003–1007

APPENDED NOTE

(Addendum · Received June 15, 1933)

Since this paper went to press eight further cycles have been recorded for the animals which were examined from the onset of sexual maturity, and the irregularities shown by animals Nos 27, 28, 29, 30, and 36 have now been replaced by regular 16- to 18-day cycles This shows that early irregularities usually disappear as age advances.

(*Issued separately August* 4, 1933)

XVII.—The Effect of Consanguineous Parentage upon Metrical Characters of the Offspring.

By **Professor Lancelot Hogben,** M.A., D.Sc. (From the Department of Social Biology in the University of London.)

(MS. received April 6, 1933. Read July 3, 1933.)

1. INTRODUCTION.

THE theoretical treatment of consanguineous unions in human populations has been the subject of several publications, notably by Dahlberg (1927), by Haldane and Moshinsky (1933), and by the present writer (1933). In these attention has been directed to the concentration of discrete characters among the offspring of related parents in a community mating at random. This communication is a contribution to the more general problem of variability which can be expressed in metrical terms Its theoretical interest resides in the fact that deductions from the genetical theory of inbreeding are open to little, if any, ambiguity, when used as an instrument for detecting gene differences whose manifestation is considerably modified by differences of environment prevailing in human societies. Two criteria will be examined. The first is the effect of consanguineous parentage upon the correlation of their offspring both when autosomal and sex-linked transmission occur. The second is the comparison of the variance of a group of individuals with consanguineous parents with that of a group of individuals with unrelated parents taken from the same social environment. Since first cousin marriages constitute the type of relatively close inbreeding most commonly encountered in practice, the inquiry which follows is confined to the treatment of unions between first cousins.

2. NOTATION EMPLOYED.

The relations given in the ensuing section are based on the matrix notation set forth in a previous communication by the writer (*op cit.*). The frequencies with which the various genotypes in a system involving an autosomal single gene substitution mate with another is represented by the symmetrical product:

$$P = \begin{vmatrix} l & m & n \\ o & p & q \\ r & s & t \end{vmatrix}.$$

The contingency table for the frequency of different pairs of parent-child is represented by the correlate:

$$C_p = \begin{vmatrix} l+\frac{1}{4}m & \frac{1}{4}(m+n)+\frac{1}{8}p & 0 \\ \frac{1}{4}(m+n)+\frac{1}{8}p & \frac{1}{4}(m+p+q) & \frac{1}{4}(n+q)+\frac{1}{8}p \\ 0 & \frac{1}{4}(n+q)+\frac{1}{8}p & \frac{1}{4}q+t \end{vmatrix}.$$

For sibs produced by such a system of mating, the frequency of all pairs is represented by the correlate:

$$C_f = \begin{vmatrix} l+\frac{1}{4}m+\frac{1}{16}p & \frac{1}{4}m+\frac{1}{8}p & \frac{1}{16}p \\ \frac{1}{4}m+\frac{1}{8}p & \frac{1}{4}m+2n+\frac{1}{4}p+\frac{1}{4}q & \frac{1}{8}p+\frac{1}{4}q \\ \frac{1}{16}p & \frac{1}{8}p+\frac{1}{4}q & \frac{1}{16}p+\frac{1}{4}q+t \end{vmatrix}.$$

For marriages between first cousins in a system of random mating when transmission is autosomal the frequencies of the respective unions is represented by

$$P = y \,.\, [E(S_y \times Z_R)]^2.$$

This is the product

$$\begin{vmatrix} \frac{1}{4}p^3(1+3p) & \frac{1}{4}p^2q(1+6p) & \frac{3}{4}p^2q^2 \\ \frac{1}{4}p^2q(1+6p) & \frac{1}{4}pq(1+12pq) & \frac{1}{4}pq^2(1+6q) \\ \frac{3}{4}p^2q^2 & \frac{1}{4}pq^2(1+6q) & \frac{1}{4}q^3(1+3q) \end{vmatrix}$$

When transmission is *sex linked* the mating system is represented by

$$P' = \begin{vmatrix} l & m & n \\ r & s & t \end{vmatrix}.$$

Three types of correlates representing the genotypic frequency of pairs of sibs with parents represented by the product P′ can be treated separately according as we take boy pairs, girl pairs, or mixed pairs.

For boy pairs the correlate is

$$C_{\delta\delta} = \begin{vmatrix} (l+r)+\frac{1}{4}(m+s) & 0 & \frac{1}{4}(m+s) \\ 0 & 0 & 0 \\ \frac{1}{4}(m+s) & 0 & \frac{1}{4}(m+s)+(n+t) \end{vmatrix}.$$

For girl pairs the correlate is

$$C_{\female\female} = \begin{vmatrix} l+\frac{1}{4}m & \frac{1}{4}m & 0 \\ \frac{1}{4}m & n+r+\frac{1}{4}(m+s) & \frac{1}{4}s \\ 0 & \frac{1}{4}s & t+\frac{1}{4}s \end{vmatrix}.$$

For mixed pairs the symmetrical correlate (corresponding to a contingency table made by double entry) is

$$C_{♂♀} = \begin{vmatrix} l+\frac{1}{4}m & \frac{1}{8}(m+s)+\frac{1}{2}r & \frac{1}{8}(m+s) \\ \frac{1}{8}(m+s)+\frac{1}{2}r & 0 & \frac{1}{8}(m+s)+\frac{1}{2}n \\ \frac{1}{8}(m+s) & \frac{1}{8}(m+s)+\frac{1}{2}n & l+\frac{1}{4}r \end{vmatrix}$$

When dealing with sex-linked transmission, four types of cousinship may be considered (fig. 1):—

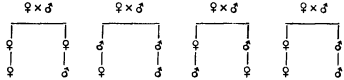
(*a*) When mothers are sisters (maternal cousins).

(*b*) When fathers are brothers (paternal cousins).

(*c*) When the mother of the male is the sister of the father of the female (mixed cousins, Type "A").

(*d*) When the father of the male is the brother of the mother of the female (mixed cousins, Type "B").

FIG. 1.



(*a*) Maternal Cousins. (*b*) Paternal Cousins. (*c*) Mixed, Type "A." (*d*) Mixed, Type "B"

The products which represent the marriages of cousins of types (*b*) and (*d*) in a system of random mating when transmission is sex linked are identical with the product for the system as a whole.

For all types of cousinship, in the proportions in which they occur when mating is random, taken together

$$P' = [E'(S_y \times Z_R)]^2$$

or

$$\begin{vmatrix} \frac{p^2}{32}(27p+5) & \frac{pq}{32}(54p+5) & \frac{27}{32}pq^2 \\ \frac{27}{32}p^2q & \frac{pq}{32}(54q+5) & \frac{q^2}{32}(27q+5) \end{vmatrix}$$

For maternal cousins (*b*) the mating system is given by

$$y[E'(S_y)_0 \times {}_0(Z_R)]^2$$

$$= \begin{vmatrix} \frac{p^2}{8}(5p+3) & \frac{pq}{8}(10p+3) & \frac{5}{8}pq^2 \\ \frac{5}{8}p^2q & \frac{pq}{8}(10q+3) & \frac{q^2}{8}(5q+3) \end{vmatrix}.$$

For mixed cousins, Type "A," the mating system is

$$y\{{}_0[\mathrm{E}(\mathrm{S}_y)_0 \times {}_0(\mathrm{Z}_\mathrm{R})] \times [\mathrm{E}_0(\mathrm{S}_y) \times (\mathrm{Z}_\mathrm{R})_0]_0\}$$

$$= \begin{vmatrix} \frac{p^2}{4}(3p+1) & \frac{pq}{4}(6p+1) & \frac{3}{4}pq^2 \\ \frac{3}{4}p^2q & \frac{pq}{4}(6q+1) & \frac{q^2}{4}(3q+1) \end{vmatrix}.$$

A correlation table corresponding to a correlate of any kind may be represented

$$\begin{array}{cccc} & 0 & h & 1 \\ 0 & \mathrm{L} & \mathrm{M} & \mathrm{N} \\ h & \mathrm{O} & \mathrm{P} & \mathrm{Q} \\ 1 & \mathrm{R} & \mathrm{S} & \mathrm{T} \end{array}$$

In such a table L, M, N, etc., have the same values as terms in the corresponding locus of the correlate-matrix. The unit of measurement is the difference between the genotypes R and D. The origin is the representative measurement of R, and h is a fraction greater than a half and less than one corresponding to the measurement of the genotype H. If the table is symmetrical (*i.e.* M = O, N = R, and Q = S), the value of r is then

$$\frac{\mathrm{P}h^2 + 2\mathrm{Q}h + \mathrm{T} - m^2}{h^2(\mathrm{M}+\mathrm{P}+\mathrm{Q}) + (\mathrm{N}+\mathrm{Q}+\mathrm{T}) - m^2}.$$

In this formula m, the mean of the population, is

$$h(\mathrm{M}+\mathrm{P}+\mathrm{Q}) + \mathrm{N}+\mathrm{Q}+\mathrm{T}.$$

For calculating the variance ratio as defined below (§ 4), the P-matrices representing the genotypic frequencies of the mating systems given in the preceding directions are necessary. For autosomal gene substitutions these are:

(*a*) General population. (*b*) Offspring of cousin marriages.

$$\mathrm{Z}_\mathrm{R} = \begin{vmatrix} p^2 & p^2 \\ 2pq & 2pq \\ q^2 & q^2 \end{vmatrix} \qquad \mathrm{Z}_\mathrm{O} = \begin{vmatrix} \frac{p}{16}(1+15p) & \frac{p}{16}(1+15p) \\ \frac{15}{8}pq & \frac{15}{8}pq \\ \frac{q}{16}(1+16q) & \frac{q}{16}(1+16q) \end{vmatrix}.$$

For sex-linked gene substitutions the relevant P-matrices are as follows:—

(*a*) General population, or offspring of paternal cousins and mixed cousins, Type "B"	(*b*) Offspring of all types of cousin marriage	(*c*) Offspring of maternal cousins	(*d*) Offspring of mixed cousins, Type "A"
$\begin{vmatrix} p^2 & p \\ 2pq & 0 \\ q^2 & q \end{vmatrix}$	$\begin{vmatrix} \frac{p}{64}(5+59p) & p \\ \frac{118}{64}pq & 0 \\ \frac{q}{64}(5+59q) & q \end{vmatrix}$	$\begin{vmatrix} \frac{p}{16}(3+13p) & p \\ \frac{13}{8}pq & 0 \\ \frac{q}{16}(3+13q) & q \end{vmatrix}$	$\begin{vmatrix} \frac{p}{8}(1+7p) & p \\ \frac{7}{4}pq & 0 \\ \frac{q}{8}(1+7q) & q \end{vmatrix}$

3 CORRELATIONS OF OFFSPRING OF CONSANGUINEOUS UNIONS

The ensuing analysis being based upon the matrix notation of the preceding section holds good on two assumptions, viz that mating occurs at random in the population as a whole, and that the sex ratio is unity For single autosomal gene substitutions the correlation between parent and child as well as between sibs will be given For single sex-linked gene substitutions the correlations for sisters and mixed sibs (double entry) will be given for (*a*) offspring of all types of cousinship taken together, (*b*) offspring of maternal cousins, and (*c*) offspring of mixed cousins, Type "A" For male pairs, when transmission is sex linked the contingency table is always the same as that calculated on the assumption that all matings occur with the same frequency as in the population at large. Likewise correlations calculated on the assumption that transmission is sex linked for sisters and mixed sibs whose parents are paternal cousins and mixed cousins of Type "B" do not differ from those of sisters and mixed sibs of parents taken at random.

(*a*) *Parent-child relation* (*autosomal case*)

The table is:

$$\begin{array}{cccc}
 & R & H & D \\
R & \frac{p^2}{8}(7p+1) & \frac{pq}{32}(28p+1) & 0 \\
H & \frac{pq}{32}(28p+1) & pq & \frac{pq}{32}(28q+1) \\
D & 0 & \frac{pq}{32}(28q+1) & \frac{q^2}{8}(7q+1)
\end{array}$$

The value of r is:

$$\frac{64\{hp(16h+28q+1)+14q^2+2q\}-q[62hp+31q+1]^2}{32\{62h^2p+31q+1\}-q[62hp+31q+1]^2}.$$

When $h=\frac{1}{2}$,

$$r=\tfrac{6}{11}=0\cdot55.$$

When dominance is complete (*i.e.* $h=1$) and $p=\frac{1}{2}=q$,

$$r=\tfrac{81}{209}=0\cdot39.$$

(*b*) *Fraternal relation* (*autosomal case*).

The table is:

	R	H	D
R	$\frac{p}{64}(12p^3+32p^2+19p+1)$	$\frac{pq}{32}(12p^2+16p+1)$	$\frac{pq}{64}(1+12pq)$
H	$\frac{pq}{32}(12p^2+16p+1)$	$\frac{3pq}{16}(5+4pq)$	$\frac{pq}{32}(12q^2+16q+1)$
D	$\frac{pq}{64}(1+12pq)$	$\frac{pq}{32}(12q^2+16q+1)$	$\frac{q}{64}(12q^3+32q^2+19q+1)$

The value of r is:

$$\frac{16ph\{15h+12pqh+12q^2+16q+1\}+48q^3+128q^2+76q+4-q(30ph+15q+1)^2}{16(30ph^2+15q+1)-q(30ph+15q+1)^2}.$$

When $h=\frac{1}{2}$,

$$r=\tfrac{9}{17}=0\cdot53.$$

When dominance is complete (*i.e.* $h=1$) and $p=\frac{1}{2}=q$,

$$r=\tfrac{351}{799}=0\cdot44$$

When transmission is autosomal the correlations between parent and child and between brethren for first-cousin unions and unions in general are collected in Table I.

TABLE I.—AUTOSOMAL CASE.

	Genotypic ($h=\frac{1}{2}$).		Somatic ($h=1$ and $p=\frac{1}{2}=q$).	
	Offspring of First Cousins	Offspring of Unrelated Parents.	Offspring of First Cousins	Offspring of Unrelated Parents.
Parent-child . . .	0·55	0·50	0·39	0·33
Fraternal . . .	0·53	0·50	0·44	0·42

The fraternal correlations for offspring of first cousins when transmission is sex linked will now be given. The first two tables for sisters and mixed sibs whose parents are first cousins of any type are based on the assumption that the several types occur in such proportions as would exist in a pure system of random mating.

(*c*) *Sibs whose parents are first cousins of all types* (*sex-linked case*).

(i) Sisters.

The table is:

	R	H	D
R	$\frac{p}{128}(54p^2+69p+5)$	$\frac{pq}{128}(54p+5)$	0
H	$\frac{pq}{128}(54p+5)$	$\frac{172}{128}pq$	$\frac{pq}{128}(54q+5)$
D	0	$\frac{pq}{128}(54q+5)$	$\frac{q}{128}(54q^2+69q+5)$

The value of r is:

$$\frac{32(172h^2p+108hpq+10hp+54q^2+69q+5)-q(118hp+59q+5)^2}{64(118h^2p+59q+5)-q(118hp+59q+5)^2}.$$

When $h=\frac{1}{2}$,

$$r=\frac{53}{69} \quad \text{or} \quad 0{\cdot}77.$$

When dominance is complete (*i e.* $h=1$) and $p=\frac{1}{2}=q$,

$$r=\frac{8807}{12903} \quad \text{or} \quad 0{\cdot}68.$$

(ii) Mixed sibs.

The table is:

	R	H	D
R	$\frac{p}{128}(54p^2+69p+5)$	$\frac{pq}{64}(27p+16)$	$\frac{pq}{4}$
H	$\frac{pq}{64}(27p+16)$	0	$\frac{pq}{64}(27q+16)$
D	$\frac{pq}{4}$	$\frac{pq}{64}(27q+16)$	$\frac{q}{128}(54q^2+69q+5)$

The value of r is:

$$\frac{128(108hpq+64hp+54q^2+69q+5)-q(118hp+59q+69)^2}{128(118h^2p+59q+69)-q(118hp+59q+69)^2}.$$

When $h=\frac{1}{2}$,

$$r=\frac{74}{197}=0{\cdot}38.$$

When dominance is complete (*i.e.* $h=1$) and $p=\frac{1}{2}=q$,

$$r=\frac{2209}{8865}=0{\cdot}25.$$

(*d*) *Offspring of maternal cousins (sex-linked case).*

(i) Sisters.

The table is:

$$\begin{array}{cccc} & R & H & D \\ R & \frac{p}{32}(10p^2+19p+3) & \frac{pq}{32}(10p+3) & 0 \\ H & \frac{pq}{32}(10p+3) & \frac{5}{8}pq & \frac{pq}{32}(10q+3) \\ D & 0 & \frac{pq}{32}(10q+3) & \frac{q}{32}(10q^2+19q+3) \end{array}$$

The value of r is:

$$\frac{8(36h^2p+20hpq+6hp+10q^2+19q+3)-q(26hp+13q+3)^2}{16(26h^2p+13q+3)-q(26hp+13q+3)^2}.$$

When $h=\frac{1}{2}$,

$$r=\tfrac{15}{19}=0·79.$$

When dominance is complete (*i.e* $h=1$) and $p=\frac{1}{2}=q$,

$$r=[illegible]=0·70$$

(ii) Mixed sibs.

The table is:

$$\begin{array}{cccc} & R & H & D \\ R & \frac{p}{32}(10p^2+19p+3) & \frac{pq}{16}(4+5p) & \frac{pq}{4} \\ H & \frac{pq}{16}(4+5p) & 0 & \frac{pq}{16}(4+5q) \\ D & \frac{pq}{4} & \frac{pq}{16}(4+5q) & \frac{q}{32}(10q^2+19q+3) \end{array}$$

The value of r is:

$$\frac{32(16hp+20hpq+10q^2+19q+3)-q(26hp+13q+19)^2}{32(26h^2p+13q+19)-q(26hp+13q+19)^2}.$$

When $h=\frac{1}{2}$,

$$r=\tfrac{22}{51}\quad\text{or}\quad 0·43$$

When dominance is complete (*i.e.* $h=1$) and $p=\frac{1}{2}=q$,

$$r=\tfrac{56}{177}\quad\text{or}\quad 0·32$$

(*e*) *Offspring of unions of which the male parent is the husband of his mother's brother's daughter (sex-linked case).*

(i) Sisters.

The table is:

$$\begin{array}{cccc} & R & H & D \\ R & \frac{p}{16}(6p^2+9p+1) & \frac{pq}{16}(6p+1) & 0 \\ H & \frac{pq}{16}(6p+1) & \frac{5}{4}pq & \frac{pq}{16}(6q+1) \\ D & 0 & \frac{pq}{16}(6q+1) & \frac{q}{16}(6q^2+9q+1) \end{array}$$

The value of r is:

$$\frac{4(20h^2p+12hpq+2hp+6q^2+9q+1)-q(14hp+7q+1)^2}{8(14h^2p+7q+1)-q(14hp+7q+1)^2}.$$

When $h=\frac{1}{2}$,

$$r=\tfrac{7}{9} \quad \text{or} \quad 0{\cdot}78.$$

When dominance is complete (*i.e.* $h=1$) and $p=\frac{1}{2}=q$,

$$r=\tfrac{143}{207} \quad \text{or} \quad 0{\cdot}69.$$

(ii) Mixed sibs.

The table is:

$$\begin{array}{cccc}
 & \text{R} & \text{H} & \text{D} \\
\text{R} & \frac{p}{16}(6p^2+9p+1) & \frac{pq}{8}(3p+2) & \frac{pq}{4} \\
\text{H} & \frac{pq}{8}(3p+2) & 0 & \frac{pq}{8}(3q+2) \\
\text{D} & \frac{pq}{4} & \frac{pq}{8}(3q+2) & \frac{q}{16}(6q^2+9q+1)
\end{array}$$

The value of r is:

$$\frac{16(12hpq+8hp+6q^2+9q+1)-q(14hp+7q+9)^2}{16(14h^2p+7q+9)-q(14hp+7q+9)^2}.$$

When $h=\frac{1}{2}$,

$$r=\tfrac{2}{5} \quad \text{or} \quad 0{\cdot}40.$$

When dominance is complete (*i.e.* $h=1$) and $p=\frac{1}{2}=q$,

$$r=\tfrac{271}{975} \quad \text{or} \quad 0{\cdot}28.$$

All the foregoing results, calculated on the assumption that transmission is sex-linked, are summarised in Table II.

TABLE II.—SEX-LINKED CASE

	Genotypic ($h=\frac{1}{2}$).		Somatic ($h=1$ and $p=\frac{1}{2}=q$)	
	Offspring of First Cousins	Offspring of Unrelated Parents	Offspring of First Cousins.	Offspring of Unrelated Parents.
A. Sister's	..	0·75	..	0·67
All types of cousinship	0·77	..	0·68	.
Maternal cousins	0·79	.	0·70	
Mixed cousins, Type "A"	0·78	.	0·69	.
B. Mixed sibs		0·33	..	0·20
All types of cousinship	0·38	..	0·25	.
Maternal cousins	0·43	.	0·32	
Mixed cousins, Type "A"	0·40	..	0·28	..

The noteworthy feature which emerges from this table is that the effect of first-cousin percentage is sufficient to raise the genotypic correlation for *mixed sibs*, whose parents are maternal cousins, by about 30 per cent. when transmission is sex linked. On the supposition that transmission is autosomal, the fraternal genotypic correlation is only raised by about 6 per cent when the parents are first cousins.

4. THE VARIANCE RATIO.

The *Variance Ratio*, which will now be dealt with, is defined as the ratio of the variance (mean square deviation) of the offspring of first-cousin marriages within a system of random mating to the variance of the offspring of unrelated parents within the same system of mating. Its value depends upon the frequency and number of the genes which contribute to the observed variance, and also upon the degree of dominance which each gene exhibits in the simplex state

(*a*) *Autosomal Genes*.—The most general case which will be dealt with is a multiple factor system of n gene substitutions, the frequency of each recessive gene being identical, and the representative measurements due to each gene substitution being the same. The frequency of all genotypes having the rth recessive gene in duplex is denoted x_r, the frequency of all genotypes with the same gene in the simplex state y_r, and the frequency of all genotypes in which the rth recessive gene is absent z_r. The frequency of all possible genotypes * is given by the successive terms of the trinomial:

$$(x_r + y_r + z_r)^n.$$

If we make the unit of measurement the difference between the n-fold dominant and the n-fold recessive, the representative measurement of any genotype with the class frequency $Cx^a y^b z^c$ is denoted

$$\frac{1}{n}(bh + c).$$

In this expression h is the measurement of the n-fold heterozygous class. The variance of such a population is easily shown to be

$$\frac{1}{n}\{h^2 y(1 - y) - z(y \,.\, \overline{2h - 1} - x)\}.$$

The variance ratio, as defined above, is evidently independent of n under the restrictions subject to which this formula is derived. In a system of random mating the genotypic proportions of unrelated marriages will not differ appreciably from the genotypic proportions in the population as a whole. Thus for offspring of cousins x is $\frac{1}{16}(p + 15p^2)$, and for offspring of unrelated parents x is p^2 when transmission is autosomal. Hence the variance ratio is

---

* Since $x_a = x_b = x_r$ . . . etc., by definition.

$$V_1 = \frac{60h^2(8 - 15pq) + (1 + 15q)(15q - 60hq + 16)}{256\{2h^2(1 - 2pq) - q(4hq - q - 1)\}}.$$

When there is no dominance h is $\frac{1}{2}$ and V is independent of p and q, having the numerical value $\frac{17}{16}$.

The values of V_1 are given for different values of h and of p below:

p.	$h = 0·5$.	$h = 0·6$.	$h = 0·7$	$h = 0·8$	$h = 0·9$	$h = 1$
0·5	1·062	1·061	1·058	1·052	1·046	1·04
0·1	1·062	1·133	1·195	1·322	1·476	1·55
0·01	1·062	1·131	1·273	1·647	3·057	7·18
0·001	1·062	1·133	1·284	1·711	3·921	63·43

(*b*) *Sex-linked Genes.*—In this case the variance ratio of male offspring of all first-cousin unions, or female offspring of paternal cousins, and of mixed cousins, Type "B," is always unity The variance ratios for female offspring will be treated under the same three headings as the correlations for sisters and mixed sibs, viz (i) all types of first-cousin parents, (ii) parents maternal cousins, (iii) parents mixed cousins, Type "A." The reasoning of the previous section applies equally.

(i) The variance ratio for offspring of all types of first cousins, when transmission is sex linked, is given by

$$V_2 = \frac{236h^2(32 - 59pq) - (5 + 59q)(236hq - 59q - 64)}{4096\{2h^2(1 - 2pq) - q(4hq - q - 1)\}}.$$

When $h = \frac{1}{2}$, this becomes $\frac{69}{64}$ or 1·08.

The values of V are given in tabular form for different values of h and p below:

p.	$h = 0·5$.	$h = 0·6$.	$h = 0·7$.	$h = 0·8$.	$h = 0·9$.	$h = 1·0$.
0·5	1·08	1·08	1·07	1·065	1 06	1·05
0·1	1·08	1·14	1·26	1·40	1·59	1·69
0·01	1·08	1·16	1·34	1·81	3·58	8 73
0·001	1·08	1·17	1·35	1·89	4·55	79·04

(ii) The variance ratio for female offspring of maternal cousins when transmission is sex linked is

$$V_3 = \frac{52h^2(8 - 13pq) - (3 + 13q)(52hq - 13q - 16)}{256\{2h^2(1 - 2pq) - q(4hq - q - 1)\}}.$$

When $h = \frac{1}{2}$, this becomes $\frac{19}{16}$ or 1·19

The values of V_3 are given in tabular form for different values of h and p below:

p.	$h = 0·5$.	$h = 0·6$.	$h = 0·7$.	$h = 0·8$.	$h = 0·9$.	$h = 1·0$.
0·5	1·19	1·18	1·17	1·15	1·13	1·11
0·1	1·19	1·34	1·59	1·96	2·42	2·64
0·01	1·19	1·39	1·82	2·93	7·15	19·5
0·001	1·19	1·40	1·85	3·13	9·74	188·28

(iii) For female offspring whose parents are first cousins of the mixed Type "A," when transmission is sex linked, the variance ratio is given by:

$$V_4 = \frac{28h^2(4-7pq)-(1+7q)(28hq-7q-8)}{64\{2h^2(1-2pq)-q(4hq-q-1)\}}.$$

When $h=\frac{1}{2}$, this becomes $\frac{9}{8}$ or 1·125

The values of V_4 are given in tabular form for different values of h and p below:

p.	$h=0·5$.	$h=0·6$.	$h=0·7$	$h=0·8$.	$h=0·9$.	$h=1·0$.
0·5	1·125	1·12	1·11	1·10	1·09	1·08
0·1	1·125	1·23	1·39	1·64	1·95	2·10
0·01	1·125	1·26	1·55	2·29	5·10	13·4
0·001	1·125	1·26	1·57	2·42	6·83	125·86

The most striking fact which emerges from the consideration of the variance ratio is that, in all cases discussed above, its value diverges very slowly with increasing rarity of the recessive gene even when its allelomorph displays a fairly high degree of dominance. It diverges rapidly when dominance is complete or nearly complete. At any given low frequency of the recessive gene the variance ratio diverges comparatively little for different degrees of dominance, so long as h does not exceed 0·9. Between $h=0·9$ and 1·0 it diverges rapidly.

5. SUMMARY.

(1) First-cousin parentage does not raise the fraternal correlation when transmission is autosomal to an extent which would be recognisable unless a very large sample of sibs whose parents are first cousins and a similar control group of sibs with unrelated parents is available.

(2) On the other hand, the correlation for mixed sibs whose mothers are sisters is appreciably higher than that for sibs of unrelated parents, when transmission is sex linked.

(3) The variance of offspring of first-cousin unions is very large compared with that of offspring of unrelated parents, when (*a*) the frequency of the recessive genes which contribute to the observed variability is low, (*b*) and the degree of dominance is very high.

Acknowledgment is made to Miss Pearl Moshinsky, B.Sc., and to Mrs Nada Roberts, M Sc, for invaluable assistance in checking the calculations in this communication

REFERENCES TO LITERATURE.

DAHLBERG, 1930. "Inbreeding in Man," *Genetics*, vol xxx.

HALDANE and MOSHINSKY, 1933 "Partial Inbreeding in Mendelian Populations, with Special Reference to Human Cousin Marriage," *Journ. Gen.* (In the press.)

HOGBEN, 1933. "A Matrix Notation for Mendelian Populations," *Proc Roy. Soc. Edin.*, vol. liii. pp. 7–25.

(*Issued separately September* 1, 1933)

XVIII.—The **Fæcal Pellets** of *Hippa asiatica*. By **Hilary B. Moore,** Ph.D. *Communicated by* Professor J. GRAHAM KERR, F.R S

(MS. received May 15, 1933 Read July 3, 1933)

IN a recent paper on the fæcal pellets of the Anomura,* pellets were described from a series of Galatheidea, Paguridea and Thalassinidea, but at that time no Hippidea were available. Since then, through Professor J. H. Orton, I have obtained specimens of *Hippa asiatica* from Madras and have prepared sections of their pellets. As the latter form an interesting addition to the series of curious pellets, apparently peculiar to the Anomura, a special description of their structure seems to be justified

Eight individuals were examined, having a carapace length of about 3 centimetres They had been preserved in alcohol, and their guts were full and in fairly good condition. Portions of the gut were removed and sectioned, this method in other Anomura having been found to give samples of the fæces in good condition. Although double embedded in collodion and paraffin wax, the material proved very difficult to cut, and the photograph shown is not of a thin section, but of the cut end

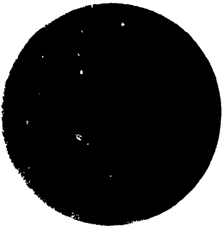


FIG A

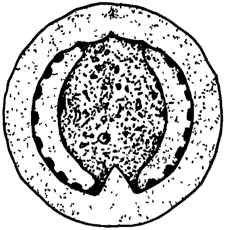


FIG B

---

* Moore, H. B, 1932 "The Fæcal Pellets of the Anomura," *Proc. Roy. Soc. Edin*, vol lii, pt. iii, No. 14.

INDEX SLIP.

PROC. R S.E., VOL. LIII, 1932–33, PART IV

GRESSON, R. A. R.—Studies on the Gametogenesis of *Stenophylax stellatus*, Curt. (Trichoptera). Oogenesis.
Proc. Roy. Soc. Edin., vol. liii, 1933, pp. 322–346.

Stenophylax stellatus, Curt.—Studies on the Gametogenesis of. Oogenesis
R. A. R GRESSON.
Proc. Roy. Soc. Edin , vol liii, 1933, pp. 322–346

Oogenesis: Studies on the Gametogenesis of *Stenophylax stellatus*, Curt.
R. A. R. GRESSON.
Proc Roy. Soc. Edin , vol liii, 1933, pp 322–346

SANDEMAN, IAN.—The Mathematical Representation of the Energy Levels of the Secondary Spectrum of Hydrogen.
Proc Roy. Soc. Edin , vol. liii, 1933, pp. 347–353.

Hydrogen, Mathematical Representation of the Energy Levels of the Secondary Spectrum of.
IAN SANDEMAN.
Proc. Roy. Soc. Edin., vol liii, 1933, pp. 347–353.

Spectrum of Hydrogen, Mathematical Representation of the Energy Levels of.
IAN SANDEMAN.
Proc. Roy. Soc. Edin., vol. liii, 1933, pp 347–353.

OBITUARY NOTICES:—Francis Ernest Bradley; William Inglis Clark; Sir Dugald Clerk; William Grant Craib; John Cuthbertson; William Hope Fowler; Joseph Robert Fraser; Frederick Gardiner; John Glaister; John Henderson; William Jacob Holland; Theophilus Bulkeley Hyslop; Robert T. A. Innes, Robert Jardine; John Edward Marr; Spencer Mort; Ralph Richardson; Johannes Schmidt; John Stephenson; John W. Tait; James Edward Talmage; Sir J. Arthur Thomson; John Millar Thomson; and William Arthur Williams.
Proc. Roy. Soc. Edin., vol. liii, 1932–33, pp. 354–384.

BRADLEY, F. E., Obituary Notice of.
Proc. Roy. Soc. Edin , vol. liii, 1932–33, p. 383.

CLARK, W. I., Obituary Notice of.
Proc. Roy. Soc Edin., vol. liii, 1932–33, p. 354.

CLERK, Sir D., Obituary Notice of.
Proc. Roy. Soc. Edin., vol. liii, 1932–33, pp. 355–356.

CRAIB, W. G., Obituary Notice of.
Proc. Roy. Soc. Edin., vol. liii, 1932–33, pp. 357–358.

Cuthbertson, J., Obituary Notice of
Proc. Roy. Soc Edin., vol. liii, 1932–33, p. 359.

Fowler, W. H., Obituary Notice of.
Proc. Roy. Soc. Edin., vol. liii, 1932–33, pp. 360–361.

Fraser, J. R., Obituary Notice of.
Proc. Roy. Soc. Edin., vol. liii, 1932–33, p. 383.

Gardiner, F., Obituary Notice of.
Proc Roy. Soc. Edin, vol. liii, 1932–33, p. 362.

Glaister, J., Obituary Notice of.
Proc. Roy. Soc. Edin., vol. liii, 1932–33, pp. 363–364.

Henderson, J., Obituary Notice of
Proc. Roy. Soc. Edin., vol. liii, 1932–33, p. 383.

Holland, W J, Obituary Notice of.
Proc Roy. Soc. Edin, vol. liii, 1932–33, pp 365–366

Hyslop, T. B, Obituary Notice of.
Proc. Roy. Soc. Edin., vol. liii, 1932–33, p. 383.

Innes, R. T. A, Obituary Notice of.
Proc. Roy. Soc. Edin., vol. liii, 1932–33, p 367.

Jardine, R., Obituary Notice of.
Proc. Roy. Soc. Edin., vol. liii, 1932–33, p. 383.

Marr, J. E., Obituary Notice of.
Proc. Roy. Soc. Edin., vol. liii, 1932–33, p. 368.

Mort, S., Obituary Notice of.
Proc Roy. Soc. Edin., vol. liii, 1932–33, p. 384.

Richardson, R., Obituary Notice of.
Proc. Roy. Soc. Edin., vol. liii, 1932–33, p. 369.

Schmidt, J., Obituary Notice of.
Proc. Roy. Soc. Edin., vol liii, 1932–33, pp. 370–373.

Stephenson, J., Obituary Notice of.
Proc. Roy. Soc. Edin., vol. liii, 1932–33, pp. 374–378

Tait, J. W., Obituary Notice of.
Proc. Roy. Soc. Edin., vol. liii, 1932–33, p. 384.

Talmage, J. E., Obituary Notice of.
Proc. Roy. Soc. Edin., vol. liii, 1932–33, p. 384.

Thomson, Sir J. A., Obituary Notice of.
Proc. Roy. Soc. Edin., vol. liii, 1932–33, pp. 379–380.

Thomson, J. M., Obituary Notice of.
Proc. Roy. Soc. Edin., vol. liii, 1932–33, p. 384.

Williams, W. A., Obituary Notice of.
Proc. Roy. Soc. Edin., vol. liii, 1932–33, p. 381.

of an embedded pellet. Although incomplete on the right-hand side, the photograph in question (fig. A) was chosen because of the clearness with which it shows all the structures described. The diagram (fig. B) was prepared from drawings of a number of pellets.

The pellets are cylindrical in shape, with an average diameter of about 1·5 mm. In transverse section they show an outer layer of fine-grade material about 0·2 mm. in thickness. This is of uniform thickness throughout except for an intruded wedge at one point, and for two pairs of slightly thickened ridges at the opposite side, the inner pair being the wider. The orientation of the pellet in the gut of the animal has not been determined, but an examination of the stomach shows that one of these structures will be dorsal and the other ventral.

Within the outer layer there is, on either side, a curved strip of material of the same fine composition, reaching almost to the wedge below, and to the beginning of the inner pair of ridges above. The outer side of these strips is divided only by a narrow crack from the inner wall of the outer case, except where the outer surface of the strip is indented at regular intervals by a series of about eight or nine longitudinal canals, semicircular in section. In the photograph the strip on the right-hand side has been damaged, but that on the left clearly shows these canals. The inner surface of the strip is more or less smooth, and is sharply marked off from the central area of the pellet which is composed of much coarser grade material, and often contains comparatively large fragments of shell and sand.

Comparison of this pellet with those found in other Anomura brings out several points of interest. The presence of longitudinal canals is common to all the deposit or detritus-eating Anomura which have so far been described Further, their arrangement in an arc on either side is comparable with the outer arc of the Galatheidea, and with the outer ring of some of the Thalassinidea. But there are two striking differences between the pellets of *Hippa* and those of the rest of the Anomura. In the first place the coarse material forms only about 25 per cent. of the pellet in *Hippa*. In *Galathea squamifera* it forms about 80 per cent., and even more in the other Galatheidea examined; while in the Thalassinidea the entire pellet is composed of what appears to be homologous with the coarse region of the Galatheidea. In the second place, in the Galatheidea and Thalassinidea the longitudinal canals are all situated in the coarse material, the fine material, when present, being structureless. In *Hippa* it is the fine material of the lateral strips which contains the longitudinal canals.

Very little is known of the processes taking place in the Anomuran stomach, but it has been shown, *loc. cit.*, that the canals in the pellets are produced by the flowing of semi-liquid food material round hair-covered longitudinal projections of the stomach wall, and by the withdrawal of these from the food mass as it passes backwards along the gut. The process is accompanied by withdrawal of water from the food mass, so that material which was at first liquid enough to flow round the stomach processes, leaves them in a sufficiently solid condition to retain their impress. If these projections of the stomach wall, and the canals which they produce, are instrumental in withdrawing water from the food mass, it is possible that the process is one of removal of liquefied food produced by the admixture of digestive fluid with the food mass in order to pass this to the hepato-pancreas. It is significant that the drainage canals in each group of the Anomura are found to infiltrate that area of the pellet in which is situated the particular grade of food material which forms the major part of the diet of the animal in question.

(*Issued separately September* 1, 1933)

XIX.—The Diffusion Coefficients of Bromine-Argon, Bromine-Methane, Bromine-Hydrogen Chloride, Bromine-Nitrous Oxide. By John E. Mackenzie, D.Sc., and Harry W. Melville, Ph.D.

(MS received June 19, 1933. Read July 3, 1933.)

In a previous communication (*Proc. Roy. Soc. Edin.*, vol. lii, 1932, p. 337) a visual method of measuring the diffusion coefficients of $Br_2 . H_2$, $Br_2 : N_2$, $Br_2 : O_2$, and $Br_2 : CO_2$ was described. The investigation has been extended to include similar measurements for bromine diffusing into argon, nitrous oxide, methane, and hydrogen chloride. Preliminary experiments in which a photoelectric cell replaces the human eye have also been made and confirmatory results have been obtained.

EXPERIMENTAL

The apparatus (*cf.* fig. 1, *loc. cit.*, p. 339) previously used was modified by sealing on to the upper end of the diffusion tube a T-tube provided with two glass taps in place of the rubber stopper and glass tap. This arrangement allowed of the diffusion tube being alternately evacuated and filled with gas (argon, methane, etc.). The repetition of the process two or three times ensured the gas in the tube being free from air.

The procedure was also modified in that after the diffusion tube had been filled with gas, a small gas-wash bottle containing just enough concentrated sulphuric acid to cover the end of the outlet tube was attached to one limb of the T-tube. The pressure exerted by the bromine and later by the bromine vapour was relieved by gas escaping through the sulphuric acid and the diffusion went on at atmospheric pressure. Thus the calculation of the mean value of the pressure of the bromine vapour as required in the earlier experiments (*loc. cit.*, p. 342) became unnecessary.

The argon and nitrous oxide were obtained from the British Oxygen Company and from Elliot & Company. The argon was stated to contain from 0·5 to 1 per cent. of nitrogen and the nitrous oxide to have not more than 0·2 per cent. impurity. Methane was prepared by the sodium acetate-soda lime method. It was collected over water and purified by repeatedly passing it through concentrated sulphuric acid, bromine, and potassium hydroxide solution to free it from unsaturated hydrocarbons, carbon dioxide, etc. Water vapour was removed by phosphorus pentoxide.

Hydrogen chloride was generated in a Kipp's apparatus from ammonium chloride and sulphuric acid and dried by concentrated sulphuric acid and phosphorus pentoxide. The gas was passed into and through the diffusion tube until on issuing from the tube it showed complete absorption by water.

VISUAL METHOD.

In Table I and in fig. 1 the complete results are given for a few typical experiments. The calculation of the diffusion coefficient was carried out as before (*loc cit*) and the results are given in Table II. Two experiments with CO_2 and with H_2 are also included for comparison with those in the first paper These experiments were made by different observers, and it will be noticed that the results agree very well with those obtained previously. A comparison of the rates of diffusion of bromine into nitrous oxide and into carbon dioxide was also made, but no difference could be detected. This is to be expected, as the diffusion coefficients are almost identical, $D_{Br_2\text{-}N_2O} = 0{\cdot}110$ and $D_{Br_2\text{-}CO_2} = 0{\cdot}107$. The calculated values of D are again all consistently higher than the observed figures.

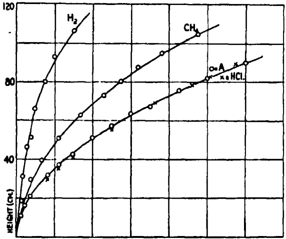


FIG. 1.

PHOTOELECTRIC METHOD.

It was thought necessary, therefore, in spite of the concordance of these results, to obtain independent confirmation by another method. Experiments were performed in which a photoelectric cell was employed

TABLE I

Br_2–A.		Br_2–CH_4		Br_2–HCl.		Br_2–H_2.		Br_2–CO_2.	
Time in mins	Height in cm. (x).	Time.	x.	Time.	x.	Time	x.	Time.	x.
1	8·5	1	9·0	2	11·0	1	18·0	1	8·0
2	10·5	3	18·0	5	18·0	2	25·0	2	11·0
4	15·5	7	29·5	8	21·5	3	31·0	7	20·5
7	21·0	13	39·5	16	30·0	4	40·0	10	25·0
11	27·0	22	51·0	22	35·5	5	46·0	20	36·0
16	32·0	34	63·0	30	41·0	7	51·5	30	44·5
22	37·0	46	73·0	42	47·0	10	66·0	40	51·0
30	43·0	55	80·0	50	55·0	15	80·0	60	63·0
40	51·0	64	87·5	74	69·0	20	93·0	75	70·0
50	57·0	77	95·0	92	78·0	30	107·0	100	80·0
60	63·0	95	105·0	102	83·0			120	88·0
70	67·5			114	89·0				
85	75·0								
100	81·5								
120	89·5								
Atmos. press.	742 mm	744		769		755		755	
Temp . .	16° C	16°		15°		16·5°		16·5°	
V.P. (liquid bromine).	144 mm.	144		137·5		147		147	
σ_A . .	$1{\cdot}66 \times 10^{-8}$ cm	σ_{CH_4} $1{\cdot}53 \times 10^{-8}$		σ_{HCl} $1{\cdot}47 \times 10^{-8}$		..		..	
Date . .	2.2.33	10.2.33		13.2.33		3.2.33		3.2.33	

TABLE II

Gas Mixture.	x (cm.).	Time (min.)	D cm^2 sec^{-1}	D_{760}	D_{calc}
Br_2–A	36	20	0·0862	0·0846	0·108
	62·5.	60	0·0861	0·0848	
	89·5	120	0·0897	0·0876	
Br_2–CH_4	30	7·5	0·1596	0·1563	0·175
	60	30·5	0·1571	0·1538	
	90	69·5	0·1586	0·1553	
Br_2–HCl	55	50	0·0820	0·0830	0·125
	67	70	0·0853	0·0863	
	83	102	0·0898	0·0908	
Br_2–H_2	46	5	0·563	0·560	0·598
	66	10	0·580	0·575	
	102	25	0·554	0·550	
Br_2–CO_2	46	20.	0·0860	0·0854	0·107
	66	70	0·0879	0·0873	
	102	120	0·0859	0·0853	

to measure the concentration of bromine vapour. Fig. 2 presents a plan and elevation of the apparatus adjoining the diffusion tube, and fig. 3 is a diagram of the electric circuit.

The diffusion tube (D) used in the above experiments was attached to a system of pulleys and gear wheels to permit of its being moved vertically. The photoelectric cell (C), slits (S_1 and S_2), and source of illumination (L) were mounted on a fixed platform (F), through which the diffusion tube could be raised or lowered. The widths of S_1 and S_2 were respectively 0·5 and 1·5 mm. A helium-filled potassium cell proved to be most suitable, as it is sensitive to the light absorbed by bromine. The photoelectric current was amplified by a valve (V) (Osram L 210). In the anode circuit there was a sensitive milliammeter (G), connected in parallel with a potentiometer system to balance out the anode current when the cell was not illuminated. If the valve is worked on the straight portion of the anode current/grid volts curve without running into grid current, and if the photoelectric current in the cell is proportional to the intensity of the light falling upon it, the reading of the milliammeter will be directly proportional to the intensity of illumination. In the present case these conditions were maintained as nearly as possible.

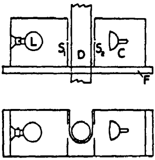


FIG. 2.

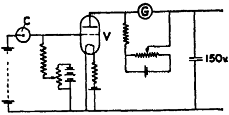


FIG 3

The results were calculated in the following way:—

Let I_0 and I be the intensities of the emergent beam from the empty tube and from the tube containing bromine vapour at a pressure p, then

$$I = I_0 e^{-kp}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where k is a constant depending only on the extinction coefficient of Br_2 and on the dimensions of the tube. When the deflection of the anode milliammeter is proportional to the intensity of the light entering the cell,

$$ln\, i/i_0 = -kp, \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where i = current at Br_2 pressure p.

i_0 = " " " nil.

Now

$$ln\, p_1/p_2 = x^2/2Dt, \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where p_1 and p_2 are the pressures of Br_2 at the surface of the liquid bromine and at a height x cm. above it, after t sec. If the corresponding currents are i_1 and i_2, then combining equations (2) and (3)

$$ln\left(\frac{ln\, i_0/i_1}{ln\, i_0/i_2}\right)=\frac{x^2}{2Dt}. \qquad (4)$$

Table III gives the results for Br_2–CO_2

TABLE III

Temp 20° C

Height of Slit above Liquid Bromine (cm).	Time of Diffusion (min)	i_2	i_1.	i_0	D_{760}
23·6	20	1·030	0·080	1·225	0·083
33·6	23	1·190	0·080	1·225	0·084
18·6	34	0·470	0·080	1·200	0·080
23·6	36	0·700	0·080	1·200	0·081

The values of D_{760} agree fairly well with those obtained by the visual method (*cf.* Table II). Further detailed results have not been given as these experiments are intended only to verify the previous results. It is hoped, however, by making a number of improvements in the photoelectric cell apparatus and in the accurate thermostating of the diffusion tube, to study in detail the variation of D with the composition of the mixture and with the total pressure.

The authors thank Messrs Brockie, Wood, Gray, and Thompson for assistance in some of the experiments.

SUMMARY.

The diffusion coefficient of Br_2–A, Br_2–CH_4, Br_2–HCl, and Br_2–N_2O have been determined by a visual method.

Preliminary experiments, in which photoelectric measurements replace those made visually, are described. The results thus obtained confirm those got by the visual method

(*Issued separately September* 1, 1933.)

XX.—On the Theory of Statistical Regression. By **M. S. Bartlett,** B.A., B.Sc. (Queens' College, Cambridge). *Communicated by* J. WISHART, M.A., D Sc.

(MS received May 12, 1933. Read June 5, 1933)

1. The product moment distribution in the general case of p normal variates, obtained in 1928 (1), and again in 1933 (2), has been awaiting further analysis. Some indication has already been given (Wishart, 1928) that new results might be expected from it; in the particular case of two variates obtained previously by Fisher (3), it has been used to deduce the distributions of the correlation coefficient (3), co-variance (4), and regression coefficient (5). In the general case, it has been used by Wilks (6) to furnish a proof of Fisher's distribution of the multiple correlation coefficient (7), and also in connection with his idea of a generalized variance (8) Further analysis appears to be most fruitful in studying statistical regression in general. It is shown in Part I of this paper that the product moment distribution can be split up into a chain of independent factors. Most of the known distributions related to regression or partial correlation are simply obtained, in a manner which clearly indicates the relations they bear to one another; the distribution of a partial regression coefficient of any order is also readily derived.

In Part II it is pointed out that the assumption of a normal system is not altogether necessary for some of the distributions to hold. A distribution which may be regarded as a further generalization of the product moment distribution, being the generalized partial product moment distribution, is obtained *ab initio*, in order to show what are the minimum assumptions about normality necessary for the various distributions obtained in I

A note should be made here on the notation used. This follows Yule (9) with regard to the symbols for partial correlations, partial variates, etc. Further, the convention that Greek and English letters are to be used for true and estimated values respectively is employed as far as possible not only for parameters but also for variates. Thus if ξ denotes the value of a variate measured from its true mean, the estimate of ξ given by $\xi - \Sigma\xi/n$ (where Σ, unless otherwise specified, will always denote summation over the n observations in a sample) will be denoted provisionally by x (usually this has been called $x - \bar{x}$). We may write further, for interdependent variates ξ_1, ξ_2, ξ_3, assuming the regressions to be linear,

$$\begin{cases} \xi_{2\cdot 1} = \xi_2 - \beta_{21}\xi_1, \\ \xi_{3\cdot 21} = \xi_3 - \beta_{32\cdot 1}\xi_2 - \beta_{31\cdot 2}\xi_1, \end{cases}$$

where β_{21} is the true simple regression coefficient of ξ_2 on ξ_1, $\beta_{32\cdot 1}$ the true partial regression coefficient of ξ_3 on ξ_2 for constant ξ_1, etc. Similarly

$$\begin{cases} x_{2\cdot 1} = x_2 - b_{21}x_1, \\ x_{3\cdot 21} = x_3 - b_{32\cdot 1}x_2 - b_{31\cdot 2}x_1, \end{cases}$$

where b_{21}, $b_{32\cdot 1}$, etc., are the corresponding estimated regression coefficients from the sample.

The product sums $c_{\mu\nu}$ obtained from the sample are defined by the equation

$$c_{\mu\nu} = \Sigma(x_\mu x_\nu), \quad . \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

and correspondingly we write

$$\gamma_{\mu\nu} = \Sigma(\xi_\mu \xi_\nu). \quad . \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

As an extension of this notation, we write further

$$\begin{cases} c_{\mu\nu\cdot 1} = \Sigma(x_{\mu\cdot 1}x_{\nu\cdot 1}), \\ \gamma_{\mu\nu\cdot 1} = \Sigma(\xi_{\mu\cdot 1}\xi_{\nu\cdot 1}), \text{ etc.} \end{cases}$$

I. AN ANALYSIS OF THE PRODUCT MOMENT DISTRIBUTION.

2. The distribution of p normal variates will be written

$$\mathrm{U}_p(\xi_\mu) \equiv \pi^{-\frac{1}{2}p} \mid \mathrm{A} \mid^{\frac{1}{2}} e^{-\mathrm{A}(\xi,\, \xi)} \prod_{\mu=1}^{p} d\xi_\mu, \quad . \quad . \quad . \qquad (3)$$

where $\mathrm{A}(\xi, \xi)$ is the positive definite quadratic form of the ξ_μ, with matrix $\mathrm{A} \equiv (a_\mu) = (\frac{1}{2}\Delta_{\mu\nu}/\sigma_\mu\sigma_\nu\Delta)$, Δ being the determinant of correlations $\rho_{\mu\nu}$, and $\Delta_{\mu\nu}$ the co-factor of $\rho_{\mu\nu}$ in Δ. The determinant of A is $\mid \mathrm{A} \mid$. Similarly we write $\mathrm{C} \equiv (c_{\mu\nu})$, $\mid \mathrm{C} \mid \equiv \mid c_{\mu\nu} \mid$

The distribution of product moments is more conveniently for the present purpose regarded as a distribution of product sums $c_{\mu\nu}$. It has been shown to be independent of the distribution of the means, and the complete distribution may be written

$$\mathrm{U}_p(u_\mu)\mathrm{V}_p(c_{\mu\nu},\, n-1), \quad . \quad . \quad . \quad . \quad . \qquad (4)$$

where $u_\mu = \Sigma\xi_\mu/\sqrt{n}$, and

$$\mathrm{V}_p(c_{\mu\nu},\, n-1) \equiv \frac{\mid \mathrm{A} \mid^{\frac{1}{2}(n-1)} \mid \mathrm{C} \mid^{\frac{1}{2}(n-p-2)} \exp\left(-\sum\limits_{\mu,\,\nu=1}^{p} a_{\mu\nu}c_{\mu\nu}\right)}{\pi^{\frac{1}{4}p(p-1)} \prod\limits_{r=1}^{p} \Gamma\frac{1}{2}(n-r)} \prod_{1 \leqslant \mu \leqslant \nu \leqslant p} dc_{\nu\mu}. \qquad (5)$$

We write the distribution (5) $V_p(c_{\mu\nu}, n-1)$ to show that one degree of freedom has been lost by using the variates x_μ in place of ξ_μ, the distribution of $\gamma_{\mu\nu}$ being given by $V_p(\gamma_{\mu\nu}, n)$.

The complete distribution (4), obtained originally (1) by geometrical methods, has recently been obtained by another method (2), as the result of a certain multiple integral. The fact that this integral was evaluated (10) as a repeated integral has suggested that $V_p(c_{\mu\nu})$ can be split up into a product of independent distributions This analysis of (5) is carried out in the case of two and three variates, and in the general case of p variates.

3. *Two Variates.*—In the case of two variates, we have

$$V_2(c_{\mu\nu}, n-1) \equiv \frac{|C|^{\frac{1}{2}(n-4)} e^{-\frac{1}{2(1-\rho_{12}^2)}\left(\frac{c_{11}}{\sigma_1^2} - \frac{2\rho_{12}c_{12}}{\sigma_1\sigma_2} + \frac{c_{22}}{\sigma_2^2}\right)}}{\pi^{\frac{1}{2}}\left[2\sigma_1\sigma_2\sqrt{\left(1-\rho_{12}^2\right)}\right]^{n-1}\Gamma\frac{1}{2}(n-1)\Gamma\frac{1}{2}(n-2)} dc_{11}dc_{12}dc_{22}. \tag{6}$$

Change the variates c_{11}, c_{12}, c_{22} to new variates c_{11}, $c_{22\cdot1}$, $u_{2\cdot1}$, where

$$c_{22\cdot1} = \Sigma x_{2\cdot1}^2 = \Sigma(x_2 - b_{21}x_1)^2 = |C|/c_{11},$$

since

$$b_{21} = \Sigma(x_2x_1)/\Sigma x_1^2,$$

and

$$u_{2\cdot1} = \sqrt{c_{11}}(b_{21} - \beta_{21}).$$

Then

$$dc_{11}dc_{22\cdot1}du_{2\cdot1} = dc_{11}dc_{12}dc_{22}/\sqrt{c_{11}},$$

and

$$\frac{1}{\left(1-\rho_{12}^2\right)}\left(\frac{c_{11}}{\sigma_1^2} - \frac{2\rho_{12}c_{12}}{\sigma_1\sigma_2} + \frac{c_{22}}{\sigma_2^2}\right) = \frac{c_{11}}{\sigma_1^2} + \frac{\Sigma(x_2 - \beta_{21}x_1)^2}{\sigma_2^2\left(1-\rho_{12}^2\right)}$$

$$= \frac{c_{11}}{\sigma_1^2} + \frac{c_{22\cdot1}}{\sigma_{2\cdot1}^2} + \frac{u_{2\cdot1}^2}{\sigma_{2\cdot1}^2},$$

where $\sigma_{2\cdot1}$ is the standard deviation of $\xi_{2\cdot1}$. Hence

$$V_2(c_{\mu\nu}, n-1) = \left[\frac{c_{11}^{\frac{1}{2}(n-3)} e^{-\frac{1}{2}c_{11}/\sigma_1^2} dc_{11}}{(2\sigma_1^2)^{\frac{1}{2}(n-1)}\Gamma\frac{1}{2}(n-1)}\right]\left[\frac{c_{22\cdot1}^{\frac{1}{2}(n-4)} e^{-\frac{1}{2}c_{22\cdot1}/\sigma_{2\cdot1}^2} dc_{22\cdot1}}{\left(2\sigma_{2\cdot1}^2\right)^{\frac{1}{2}(n-2)}\Gamma\frac{1}{2}(n-2)}\right]\left[\frac{e^{-\frac{1}{2}u_{2\cdot1}^2/\sigma_{2\cdot1}^2} du_{2\cdot1}}{\left(2\pi\sigma_{2\cdot1}^2\right)^{\frac{1}{2}}}\right], \tag{7}$$

$$= V_1(c_{11}, n-1)V_1(c_{22\cdot1}, n-2)U_1(u_{2\cdot1}).$$

We thus obtain the following results.

(i) $V_2(c_{\mu\nu})$ has been split up into three independent distributions. The particular function $u_{2\cdot1} = \sqrt{c_{11}}(b_{21} - \beta_{21})$ is normally distributed with standard deviation $\sigma_{2\cdot1}$. Further, while c_{11} is distributed like γ_{11}, but with one less degree of freedom (see, for example (2), p. 4), $c_{22\cdot1}$ is distributed like $\gamma_{22\cdot1}$, but with two less degrees of freedom. We may in fact consider

$$V_1(c_{22\cdot1}, n-2)U_1(u_{2\cdot1}),$$

write $c_{22\cdot 1}+u_{2\cdot 1}^2=y$, $u_{2\cdot 1}^2=yz$, say, and integrate for z. We then have the distribution of $y=\Sigma(x_2-\beta_{21}x_1)^2$ given by

$$\mathrm{V}_1(y,\ n-1)\int_0^1 \frac{\Gamma\frac{1}{2}(n-1)}{\pi^{\frac{1}{2}}\Gamma\frac{1}{2}(n-2)}(1-z)^{\frac{1}{2}(n-4)}z^{-\frac{1}{2}}dz=\mathrm{V}_1(y,\ n-1),$$

showing that the only effect of the substitution of b_{21} for β_{21} is that the distribution loses one degree of freedom. Our estimate of $\sigma_{2\cdot 1}^2$ is thus

$$s_{2\cdot 1}^2=c_{22\cdot 1}/(n-2).$$

(ii) If we consider

$$\mathrm{V}_1(c_{11},\ n-1)\mathrm{U}_1(u_{2\cdot 1}),$$

write $u_{2\cdot 1}=\lambda\sqrt{c_{11}}$, and integrate out for c_{11}, we have $f(\lambda)d\lambda$, say,

$$=\int_0^\infty \frac{c_{11}^{\frac{1}{2}(n-2)}e^{-\frac{1}{2}c_{11}\left(\frac{1}{\sigma_1^2}+\frac{\lambda^2}{\sigma_{2\cdot 1}^2}\right)}d\lambda dc_{11}}{(2\sigma_1^2)^{\frac{1}{2}(n-1)}\left(2\pi\sigma_{2\cdot 1}^2\right)^{\frac{1}{2}}\Gamma\frac{1}{2}(n-1)}$$

$$=\frac{\Gamma\frac{1}{2}n}{\pi^{\frac{1}{2}}\Gamma\frac{1}{2}(n-1)}\ \frac{\sigma_1}{\sigma_{2\cdot 1}}\left(1+\frac{\lambda^2\sigma_1^2}{\sigma_{2\cdot 1}^2}\right)^{-\frac{1}{2}n}d\lambda, \qquad . \qquad (8)$$

or writing $\lambda=b_{21}-\beta_{21}$, and $\sigma_{2\cdot 1}^2=\sigma_2^2(1-\rho_{12}^2)$, we have finally for the distribution of the simple regression coefficient b_{21},

$$f(b_{21})db_{21}=\frac{\left(1-\rho_{12}^2\right)^{\frac{1}{2}(n-1)}\Gamma\frac{1}{2}n}{\pi^{\frac{1}{2}}\Gamma\frac{1}{2}(n-1)}\ \frac{\sigma_1}{\sigma_2}\left(1-2\rho_{12}\frac{\sigma_1}{\sigma_2}b_{21}+\frac{\sigma_1^2}{\sigma_2^2}b_{21}^2\right)^{-\frac{1}{2}n}db_{21} \quad . \qquad (9)$$

This distribution was first obtained independently by Romanovsky and Pearson (5).

(iii) If instead we again consider

$$\mathrm{V}_1(c_{22\cdot 1},\ n-2)\mathrm{U}_1(u_{2\cdot 1}),$$

this being likely to lead to a more useful result, since $c_{22\cdot 1}$ and $u_{2\cdot 1}$ are both related to the same variate $\xi_{2\cdot 1}$, we have a normal variate $u_{2\cdot 1}$ with standard deviation $\sigma_{2\cdot 1}$, of which our independent estimate is $s_{2\cdot 1}=\sqrt{\{c_{22\cdot 1}/(n-2)\}}$.

It at once follows that the distribution of

$$t=u_{2\cdot 1}/s_{2\cdot 1}$$

is given by the well-known "t distribution," with $n-2$ degrees of freedom. This enables us to test the significance of an estimate b_{21} from any hypothetical value β_{21}. This result should be compared with that obtained by Fisher, who has shown (11) that exactly the same test is applicable whatever the distribution of ξ_1, provided ξ_2 is normal for each ξ_1, and we can suppose the set of values ξ_1 in the sample fixed from sample to sample. Any function of ξ_1 and ξ_2, provided it is a linear function of

ξ_2, is then, of course, normally distributed—for example, the regression coefficient b_{21} itself. In II the question of assumptions is reconsidered, and these two apparently distinct problems are shown to be special cases, the only condition necessary for the test to hold being the normality of $\xi_{2\cdot 1}$.

If we consider the particular case when β_{21} or ρ_{21} is zero, we have

$$t = r_{12}\sqrt{(n-2)}/\sqrt{\left(1-r_{12}^2\right)},$$

where r_{12} is our estimate of ρ_{12}. This explains why this particular function of r_{12} is distributed in a "t distribution," the test of significance of a correlation from zero being more fundamentally the test of significance of a regression from zero

4. *Three Variates.*—We have in the case of three variates

$$V_3(c_{\mu\nu}, n-1) \equiv \frac{|C|^{\frac{1}{2}(n-5)} \exp\left\{-\frac{1}{2\Delta}\sum_{\mu,\nu=1}^{3}\frac{c_{\mu\nu}\Delta_{\mu\nu}}{\sigma_\mu\sigma_\nu}\right\} dc_{11}dc_{12}\,.\,.\,.\,dc_{33}}{\pi^{\frac{3}{2}}(2^3\sigma_1{}^2\sigma_2{}^2\sigma_3{}^2\Delta)^{\frac{1}{2}(n-1)}\Gamma\tfrac{1}{2}(n-1)\Gamma\tfrac{1}{2}(n-2)\Gamma\tfrac{1}{2}(n-3)}. \qquad (10)$$

First write

$$\begin{cases} c_{22\cdot 1} = \Sigma x_{2\cdot 1}^2 = \Sigma(x_2 - b_{21}x_1)^2 = C_{33}/c_{11}, \\ c_{23\cdot 1} = \Sigma(x_{2\cdot 1}x_{3\cdot 1}) = \Sigma(x_2 - b_{21}x_1)(x_3 - b_{31}x_1) = -C_{23}/c_{11}, \\ c_{33\cdot 1} = \Sigma x_{3\cdot 1}^2 = \Sigma(x_3 - b_{31}x_1)^2 = C_{22}/c_{11}, \end{cases}$$

and

$$u_{2\cdot 1} = \sqrt{c_{11}}(b_{21} - \beta_{21}), \qquad u_{3\cdot 1} = \sqrt{c_{11}}(b_{31} - \beta_{31})$$

Then

$$dc_{11}dc_{12}\,.\,.\,.\,dc_{33} = c_{11}du_{2\cdot 1}du_{3\cdot 1}dc_{11}dc_{22\cdot 1}dc_{23\cdot 1}dc_{33\cdot 1},$$

and

$$\sum_{\mu,\nu=1}^{3}\frac{c_{\mu\nu}\Delta_{\mu\nu}}{\sigma_\mu\sigma_\nu\Delta} = \frac{c_{11}}{\sigma_1{}^2} + \sum_{\mu,\nu=2}^{3}\left(\frac{\Delta_{\mu\nu}}{\sigma_\mu\sigma_\nu\Delta}\Sigma(x_\mu - \beta_{\mu 1}x_1)(x_\nu - \beta_{\nu 1}x_1)\right)$$

$$= \frac{c_{11}}{\sigma_1{}^2} + \sum_{\mu,\nu=2}^{3}\left(\frac{\Delta_{\mu\nu}}{\sigma_\mu\sigma_\nu\Delta}[c_{\mu\nu\cdot 1} + u_{\mu\cdot 1}u_{\nu\cdot 1}]\right).$$

For this identity depends on the coefficients of c_{12}, c_{13}, and c_{11} being respectively equal on each side. For the first two we must have

$$\Delta_{12} + \rho_{12}\Delta_{22} + \rho_{13}\Delta_{23} = 0,$$
$$\Delta_{13} + \rho_{13}\Delta_{33} + \rho_{12}\Delta_{23} = 0,$$

which (since $\rho_{11} = 1$) are obviously true. Multiply the first of these by ρ_{12}, the second by ρ_{13}, and add; we get

$$\Delta - \Delta_{11} + \rho_{12}^2\Delta_{22} + 2\rho_{12}\rho_{13}\Delta_{23} + \rho_{13}^2\Delta_{33} = 0,$$

which is the identity required for the coefficient of c_{11}. We have also *

$$|C| = c_{11}\begin{vmatrix} c_{22\cdot 1} & c_{23\cdot 1} \\ c_{23\cdot 1} & c_{33\cdot 1} \end{vmatrix}.$$

---

* *Cf.* Ingham (10), p. 5

It readily follows that

$$V_3(c_{\mu\nu},\ n-1)=V_1(c_{11},\ n-1)V_2(c_{\mu\nu\cdot1},\ n-2)U_2(u_{\mu\cdot1}), \quad . \quad . \quad (11)$$

where μ, ν on the right-hand side can take the values 2 and 3.

Since $-\Delta_{32}/\sqrt{(\Delta_{22}\Delta_{33})}=\rho_{32\cdot1}$—the correlation between $\xi_{3\cdot1}$ and $\xi_{2\cdot1}$, or the partial correlation between ξ_3 and ξ_2—and we have also $\Delta=(1-\rho^2_{32\cdot1})\Delta_{22}\Delta_{33}$, we may write

$$V_2(c_{\mu\nu\cdot1},\ n-2)=\frac{|\,c_{\mu\nu\cdot1}\,|^{\frac{1}{2}(n-5)}e^{-\frac{1}{2(1-\rho^2_{32\cdot1})}\left(\frac{c_{22\cdot1}}{\sigma^2_{2\cdot1}}-\frac{2\rho_{32\cdot1}c_{32\cdot1}}{\sigma_{2\cdot1}\sigma_{3\cdot1}}+\frac{c_{33\cdot1}}{\sigma^2_{3\cdot1}}\right)}}{\pi^{\frac{1}{2}}\left[2\sigma_{2\cdot1}\sigma_{3\cdot1}\sqrt{\left(1-\rho^2_{32\cdot1}\right)}\right]^{n-2}\Gamma\frac{1}{2}(n-2)\Gamma\frac{1}{2}(n-3)}dc_{22\cdot1}dc_{32\cdot1}dc_{33\cdot1}. \quad (12)$$

If we compare this result with (6), we see that the quantities $c_{22\cdot1}$, $c_{32\cdot1}$, $c_{33\cdot1}$ are jointly distributed exactly like c_{22}, c_{23}, c_{33}, but with one less degree of freedom. We may thus define partial variances and co-variance by the equations

$$\begin{cases} v_{22\cdot1}=s^2_{2\cdot1}=\Sigma x^2_{2\cdot1}/(n-2) \\ v_{32\cdot1}=r_{32\cdot1}s_{2\cdot1}s_{3\cdot1}=\Sigma(x_{3\cdot1}x_{2\cdot1})/(n-2) \\ v_{33\cdot1}=s^2_{3\cdot1}=\Sigma x^2_{3\cdot1}/(n-2). \end{cases}$$

We thus have the further results that not only, as we saw in the case of two variates, is $v_{22\cdot1}$ distributed in an "s^2 distribution" with $n-2$ degrees of freedom, but also

(iv) the partial co-variance $v_{32\cdot1}$ will be distributed in the Bessel function distribution (4) and (12) obtained for v_{32} or v_{21}, and the partial correlation coefficient $r_{32\cdot1}$ will be distributed exactly like a simple correlation coefficient r_{32} or r_{21}, each, however, with one less degree of freedom Fisher has shown this for the important case of the partial correlation coefficient by means of a geometrical argument (13).

We should notice that $r_{32\cdot1}$ is defined above as

$$r_{32\cdot1}=\Sigma(x_{3\cdot1}x_{2\cdot1})/\sqrt{\left[\Sigma\left(x^2_{2\cdot1}\right)\Sigma\left(x^2_{3\cdot1}\right)\right]},$$

but this may be written

$$r_{32\cdot1}=(r_{32}-r_{31}r_{21})/\sqrt{\left[\left(1-r^2_{31}\right)\left(1-r^2_{21}\right)\right]}.$$

To reduce (11) further, we now treat $V_2(c_{\mu\nu\cdot1},\ n-2)$ exactly as we did $V_2(c_{\mu\nu},\ n-1)$ in the case of two variates. That is, write

$$\begin{aligned} c_{33\cdot12}&=(c_{33\cdot1}c_{22\cdot1}-c^2_{23\cdot1})/c_{22\cdot1}=|\,C\,|/C_{33} \\ &=\Sigma x^2_{3\cdot12}=\Sigma(x_3-b_{32\cdot1}x_2-b_{31\cdot2}x_1)^2, \\ u_{3\cdot21}&=\sqrt{c_{22\cdot1}}(b_{32\cdot1}-\beta_{32\cdot1}). \end{aligned}$$

We then have

$$\begin{aligned} V_3(c_{\mu\nu},\ n-1)=V_1(c_{11},\ n-1)&V_1(c_{22\cdot1},\ n-2)V_1(c_{33\cdot12},\ n-3) \\ &\times U_2(u_{2\cdot1},\ u_{3\cdot1})U_1(u_{3\cdot21}). \quad . \quad . \quad . \quad . \quad . \quad (13) \end{aligned}$$

Besides furnishing again all the results obtained for two variates, the case of three variates now gives the following.

(v) The quantity $c_{33\cdot 12}$ is distributed like $\gamma_{33\cdot 12}$, but with three less degrees of freedom. We may thus write $c_{33\cdot 12}=(n-3)v_{33\cdot 12}$, where $v_{33\cdot 12}$ is our estimate of the variance of $\xi_{3\cdot 12}$, that is, of ξ_3 from the regression plane

$$\xi_3-\beta_{32\cdot 1}\xi_2-\beta_{31\cdot 2}\xi_1=0.$$

Further, the quantities $u_{2\cdot 1}$, $u_{3\cdot 1}$, $u_{3\cdot 21}$ are normally distributed. Notice that since the correlation between $u_{2\cdot 1}$ and $u_{3\cdot 1}$ is $\rho_{32\cdot 1}$, we can write $w_1=u_{3\cdot 1}-\beta_{32\cdot 1}u_{2\cdot 1}=\sqrt{c_{11}}(b_{31}-\beta_{32\cdot 1}b_{21}-\beta_{31\cdot 2})$, $w_2=u_{3\cdot 21}$, and obtain three independent normal variates $u_{2\cdot 1}$, w_1, and w_2; moreover, w_1 and w_2 have the same standard deviation $\sigma_{3\cdot 21}$.

(vi) If we consider

$$\mathrm{V}_1(c_{22\cdot 1},\ n-2)\mathrm{U}_1(u_{3\cdot 21}),$$

we may obtain the distribution of the partial regression coefficient $b_{32\cdot 1}$, exactly as we found the distribution of the simple regression coefficient b_{21} by considering

$$\mathrm{V}_1(c_{11},\ n-1)\mathrm{U}_1(u_{2\cdot 1})$$

The only point to notice is that one degree of freedom has been lost, so that if we write $u_{3\cdot 21}=\lambda\sqrt{c_{22\cdot 1}}$, and integrate out for $c_{22\cdot 1}$, we shall have

$$f(\lambda)d\lambda=\frac{\Gamma\frac{1}{2}(n-1)}{\pi\Gamma\frac{1}{2}(n-2)}\frac{\sigma_{2\cdot 1}}{\sigma_{3\cdot 21}}\left(1+\frac{\lambda^2\sigma^2_{2\cdot 1}}{\sigma^2_{3\cdot 21}}\right)^{-\frac{1}{2}(n-1)}d\lambda \quad . \tag{14}$$

analogously to (8), where here $\lambda=b_{32\cdot 1}-\beta_{32\cdot 1}$ If we write $\sigma^2_{3\cdot 21}=\sigma^2_{3\cdot 1}(1-\rho^2_{32\cdot 1})$, we have the result analogous to (9),

$$f(b_{32\cdot 1})db_{32\cdot 1}=\frac{\left(1-\rho^2_{32\cdot 1}\right)^{\frac{1}{2}(n-2)}\Gamma\frac{1}{2}(n-1)}{\pi^{\frac{1}{2}}\Gamma\frac{1}{2}(n-2)}\frac{\sigma_{2\cdot 1}}{\sigma_{3\cdot 1}}\left(1-2\rho_{32\cdot 1}\frac{\sigma_{2\cdot 1}}{\sigma_{3\cdot 1}}b_{32\cdot 1}+\frac{\sigma^2_{2\cdot 1}}{\sigma^2_{3\cdot 1}}b^2_{32\cdot 1}\right)^{(n-1)}db_{32\cdot 1} \tag{15}$$

Similarly we obtain the distribution of $b_{31\cdot 2}$ by interchanging the suffixes 1 and 2, but the two distributions will not, of course, be independent.

(vii) Like the distribution of b_{21}, this distribution is of little value for testing the significance of regression coefficients, since it contains unknowns besides $\beta_{32\cdot 1}$ It is therefore more useful to consider

$$\mathrm{V}_1(c_{33\cdot 12},\ n-3)\mathrm{U}_1(u_{3\cdot 21}),$$

since now we have simply the normal variate $u_{3\cdot 21}$ with standard deviation $\sigma_{3\cdot 21}$, of which our independent estimate $s_{3\cdot 21}$ is given by

$$s^2_{3\cdot 21}=c_{33\cdot 12}/(n-3).$$

We may thus write

$$t=u_{3\cdot 21}/s_{3\cdot 21},$$

where t follows the "t distribution" with $n-3$ degrees of freedom. This enables us to test the significance of $b_{32\cdot 1}$ from any hypothetical value $\beta_{32\cdot 1}$, since $\beta_{32\cdot 1}$ is the only unknown that occurs.

When $\beta_{32\cdot 1}$ or $\rho_{32\cdot 1}$ is supposed zero, we have

$$t=r_{32\cdot 1}\sqrt{(n-3)}/\sqrt{\left(1-r^2_{32\cdot 1}\right)},$$

and we can test the significance of a partial correlation from zero.

(viii) We had not only $w_2=u_{3\cdot 21}$, but also $w_1=u_{3\cdot 1}-\beta_{32\cdot 1}u_{2\cdot 1}$ normally and independently distributed with standard deviation $\sigma_{3\cdot 21}$, both being further independent of the distribution of $s^2_{3\cdot 21}$. It follows that if we write

$$\begin{aligned}2s^2&=w_1{}^2+w_2{}^2\\&=(b_{31\cdot 2}-2\beta_{31\cdot 2})c_{13}+(b_{32\cdot 1}-2\beta_{32\cdot 1})c_{32}\\&\quad+\beta^2_{31\cdot 2}c_{11}+2\beta_{32\cdot 1}\beta_{31\cdot 2}c_{12}+\beta^2_{32\cdot 1}c_{22},\qquad . \quad . \quad . \quad (16)\end{aligned}$$

then $z=\frac{1}{2}\log(s^2/s^2_{3\cdot 21})$ is distributed in Fisher's "z distribution" with $s^2_{3\cdot 21}$ having $n-3$ degrees of freedom, and s^2 two degrees of freedom. This enables us to make a single test for the significance of $b_{32\cdot 1}$ and $b_{31\cdot 2}$ from hypothetical values $\beta_{32\cdot 1}$ and $\beta_{31\cdot 2}$ It is clearly a problem in the analysis of variance (see Fisher (14)).

If we suppose $\beta_{32\cdot 1}=\beta_{31\cdot 2}=0$, we have simply

$$2s^2=b_{32\cdot 1}c_{32}+b_{31\cdot 2}c_{31},$$

or since R^2, the estimated multiple correlation coefficient of ξ_3 with ξ_1 and ξ_2 is given by

$$R^2=(b_{32\cdot 1}c_{32}+b_{31\cdot 2}c_{31})/c_{33},$$

we have

$$2s^2/(n-3)s^2_{3\cdot 21}=R^2/(1-R^2)$$

Thus, just as the test of significance of a partial regression coefficient $b_{32\cdot 1}$ from zero becomes identical with the test of significance of the corresponding partial correlation coefficient $r_{32\cdot 1}$ from zero, so the joint test of significance of $b_{32\cdot 1}$ and $b_{31\cdot 2}$ from zero must become identical with the test whether there is any significant multiple correlation

5. *The General Case of* p *Variates.*—To obtain the various distributions above in their general form, we consider finally

$$V_p(c_{\mu 1},\ n-1).$$

Write

$$\begin{cases}c_{\mu\nu\cdot 1}=\Sigma(x_{\mu\cdot 1}x_{\nu\cdot 1})=c_{\mu\nu}-c_{\mu 1}c_{\nu 1}/c_{11},\\ u_{\mu\cdot 1}=\sqrt{c_{11}}(b_{\mu 1}-\beta_{\mu 1}),\end{cases}$$

where the convention is adopted that μ, ν can take all values 1 to p except those occurring explicitly—*e g.* μ cannot take the value 1 in $c_{\mu\nu\cdot 1}$.

Then

$$\Pi\, dc_{\mu\nu} = c_{11}{}^{\frac{1}{2}p}\,\Pi\, dc_{\mu\nu\,1}\,\Pi\, du_{\mu\cdot 1},$$

and

$$\sum_{\mu,\,\nu=1}^{p}\frac{c_{\mu\nu}\Delta_{\mu\nu}}{\sigma_\mu\sigma_\nu\Delta} = \frac{c_{11}}{\sigma_1{}^2} + \sum_{\mu,\,\nu=2}^{p}\left(\frac{\Delta_{\mu\nu}}{\sigma_\mu\sigma_\nu\Delta}\Sigma(x_\mu - \beta_{\mu 1}x_1)(x_\nu - \beta_{\cdot 1}x_1)\right)$$

$$= \frac{c_{11}}{\sigma_1{}^2} + \sum_{\mu,\,\nu=2}^{p}\left(\frac{\Delta_{\mu\nu}}{\sigma_\mu\sigma_\nu\Delta}[c_{\mu\nu\,1} + u_{\mu\cdot 1}u_{\nu\,1}]\right);$$

for similarly to the case of three variates, we have

$$\sum_{r=1}^{p}\rho_{1r}\Delta_{sr} = 0 \quad (s \neq 1),$$

and multiplying this equation by ρ_{1s}, and adding for all $s \neq 1$, we have

$$\Delta - \Delta_{11} + \sum_{r,\,s=2}^{p}\rho_{1r}\rho_{1s}\Delta_{sr} = 0,$$

and these identities are the equations required for the coefficients of $c_{1\mu}$ and c_{11}. We have also

$$|\,C\,| = c_{11}\,|\,c_{\mu\nu\,1}\,|.$$

It follows that

$$V_p(c_{\mu\nu},\, n-1) = V_1(c_{11},\, n-1)V_{p-1}(c_{\mu\nu\,1},\, n-2)U_{p-1}(u_{\mu\cdot 1}).$$

Similarly

$$V_{p-1}(c_{\mu\nu\cdot 1},\, n-2) = V_1(c_{22\,1},\, n-2)V_{p-2}(c_{\mu\nu\cdot 12},\, n-3)U_{p-2}(u_{\mu\,21}),$$

$$\vdots$$

$$V_3(c_{\mu\nu\,1\,\ldots\,p-3},\, n-p+2) = V_1(c_{p-2,\,p-2\cdot 1\,\ldots\,p-3},\, n-p+2)V_2(c_{\mu\nu\cdot 1\,\ldots\,p-2},\, n-p+1) \times U_2(u_{\mu\cdot p-2,\,\ldots\,1}).$$

We may deduce from the distribution

$$V_2(c_{\mu\nu\cdot 1\,\ldots\,p-2},\, n-p+1) \quad . \quad . \quad . \quad . \quad (17)$$

results corresponding to those obtained from (12) in the case of three variates. That is, we may write

$$v_{\mu\nu\cdot 1\,\ldots\,p-2} = c_{\mu\nu\cdot 1\,\ldots\,p-2}/(n-p+1),$$

and obtain that

(ix) the distribution of the partial co-variance $v_{p,\,p-1\cdot 1\,\ldots\,p-2}$ is like that of a simple co-variance, and the distribution of the partial correlation coefficient $r_{p,\,p-1\cdot 1\,\ldots\,p-2}$ like that of a simple correlation coefficient, each distribution, however, with only $n-p+1$ degrees of freedom.

We now write further

$$V_2(c_{\mu\nu\cdot 1\,\ldots\,p-2},\, n-p+1) = V_1(c_{p-1,\,p-1\cdot 1\,\ldots\,p-2},\, n-p+1)V_1(c_{pp\cdot 1\,\ldots\,p-1},\, n-p) \times U_1(u_{p\cdot p-1,\,\ldots\,1}),$$

and hence have

$$V_p(c_{\mu\nu},\, n-1) = V_1(c_{11},\, n-1)\prod_{r=1}^{p-1}\{V_1(c_{\mu\nu\cdot 1\,\ldots\,r},\, n-r-1)U_{p-r}(u_{\mu\cdot r\,\ldots\,1})\}. \quad (18)$$

We may therefore deduce in the general case the following results.

(x) The quantity $c_{pp\cdot 1 \dots p-1}$ is distributed like $\gamma_{pp\cdot 1 \dots p-1}$, but with p less degrees of freedom. We thus write

$$v_{pp\cdot 1 \dots p-1} = c_{pp\cdot 1 \dots p-1}/(n-p)$$

as our estimate of the variance of ξ_p from its regression "plane."

Further, the quantities $u_{\mu r \dots 1}$ are all normally distributed. We may, moreover, write

$$\begin{cases} w_1 = u_{p\cdot 1} - \beta_{p2\,13 \dots p-1} u_{2\,1} \cdots\cdots - \beta_{p,\,p-1\,1 \dots p-2} u_{p-1\cdot 1} \\ w_2 = u_{p\,21} - \beta_{p3\cdot 124 \dots p-1} u_{3\cdot 21} \cdots\cdots - \beta_{p,\,p-1\cdot 1 \dots p-2} u_{p-1\,21} \\ \vdots \\ w_{p-1} = u_{p\cdot p-1, \dots 1}, \end{cases}$$

and obtain $p-1$ independent normal variates each with the same standard deviation. They may alternatively be written

$$\begin{cases} w_1 = \sqrt{c_{11}}(b_{p1} - \beta_{p,\,p-1\cdot 1 \dots p-2} b_{p-1,\,1} \cdots\cdots - \beta_{p1\,2 \dots p-1}) \\ w_2 = \sqrt{c_{22\,1}}(b_{p2\cdot 1} - \beta_{p,\,p-1\cdot 1 \dots p-2} b_{p-1,\,2\cdot 1} \cdots\cdots - \beta_{p2\,13 \dots p-1}) \\ \vdots \\ w_{p-1} = \sqrt{c_{p-1,\,p-1\cdot 1 \dots p-2}}(b_{p,\,p-1\cdot 1 \dots p-2} - \beta_{p,\,p-1\cdot 1 \dots p-2}), \end{cases} \qquad (19)$$

showing that they contain only the unknowns $\beta_{p1\cdot 2 \dots p-1}, \quad \dots, \quad \beta_{p,\,p-1\,1 \dots p-2}$.

(xi) The distribution of the partial regression coefficient

$$b \equiv b_{p,\,p-1\cdot 1 \dots p-2}$$

is obtained exactly as that of b_{21}, $b_{32\cdot 1}$, etc We consider

$$V_1(c_{p-1,\,p-1\cdot 1 \dots p-2},\ n-p+1)U_1(u_{p\,p-1, \dots 1}),$$

and obtain

$$f(\lambda)d\lambda = \frac{\Gamma\frac{1}{2}(n-p+2)}{\pi^{\frac{1}{2}}\Gamma\frac{1}{2}(n-p+1)} \frac{\sigma_{p-1\cdot 1 \dots p-2}}{\sigma_{p\,1 \dots p-1}} \left(1 + \frac{\lambda^2 \sigma^2_{p\,11\,.\,p-2}}{\sigma^2_{p\cdot 1\,.\,p-1}}\right)^{-\frac{1}{2}(n-p+2)} d\lambda, \qquad (20)$$

where

$$\lambda = b - \beta_{p,p-1\cdot 1 \dots p-2},$$

or

$$f(b)db = \frac{(1-\rho^2)^{\frac{1}{2}(n-p+1)}\Gamma\frac{1}{2}(n-p+2)}{\pi^{\frac{1}{2}}\Gamma\frac{1}{2}(n-p+1)} \frac{\sigma'}{\sigma}\left(1 - 2\rho\frac{\sigma'}{\sigma}b + \left[\frac{\sigma'}{\sigma}b\right]^2\right)^{-\frac{1}{2}(n-p+2)} db, \qquad (21)$$

where $\rho = \rho_{p,\,p-1\cdot 1 \dots p-2}$, $\sigma = \sigma_{p\cdot 1 \dots p-2}$, and $\sigma' = \sigma_{p-1\,1 \dots p-2}$.

(xii) As before, if we consider instead

$$V_1(c_{pp\cdot 1 \dots p-1},\ n-p)U_1(u_{p\cdot p-1, \dots 1}),$$

we may write

$$t = u_{p\cdot p-1, \dots 1}/s_{p\cdot p-1, \dots 1},$$

where $s^2_{p\cdot p-1, \dots 1} = v_{pp\cdot p-1, \dots 1} = c_{pp\cdot 1 \dots p-1}/(n-p)$, and $u_{p\cdot p-1, \dots 1}$ ($\equiv w_{p-1}$) is given in (19).

This is therefore the test in the general case for the significance of the partial regression coefficient b from any hypothetical value β. When we put $\beta=0$, we have

$$t=r\sqrt{(n-p)}/\sqrt{(1-r^2)},$$

where r is the estimated partial correlation coefficient between ξ_p and ξ_{p-1} when $\xi_1 \ldots \xi_{p-2}$ are eliminated, and we have therefore a test of significance of r from zero.

(xiii) If we write

$$\begin{aligned}(p-1)s^2 &= w_1^2+w_2^2+\ \ldots\ w_{p-1}^2\\ &= \sum_{r=1}^{p-1}(b_{pr\,1\ldots r-1,\,r+1,\ldots p-1}-2\beta_{pr\,1\ldots r-1,\,r+1,\ldots p-1})c_{pr}\\ &\qquad+\sum_{r,\,s=1}^{p-1}\beta_{pr\cdot1\ldots r-1,\,r+1,\ldots p-1}\beta_{ps\,1\ldots s-1,\,s+1,\ldots p-1}c_{rs},\end{aligned}\tag{22}$$

then $z=\frac{1}{2}\log(s^2/s_{p\,1\ldots p-1}^2)$ is distributed in Fisher's z distribution with $s_{p\cdot1\ldots p-1}^2$ having $n-p$ degrees of freedom, and s^2 $p-1$ degrees of freedom. This gives, therefore, a single test for the significance of $b_{p1\cdot2\ldots p-1},\ldots,b_{p,\,p-1\cdot1\ldots p-2}$ from hypothetical values $\beta_{p1\,2\ldots p-1},\ldots,\beta_{p,\,p-1\,1\ldots p-2}$.

If we suppose these values all zero, we have simply

$$\begin{aligned}(p-1)s^2 &= \sum_{r=1}^{p-1}b_{pr\,1\ldots r-1,\,r+1,\ldots p-1}c_{pr}\\ &= c_{pp}\mathrm{R}^2,\end{aligned}$$

where R^2 is the estimated multiple correlation coefficient of ξ_p with $\xi_1,\ldots\xi_{p-1}$; and

$$(p-1)s^2/(n-p)s_{p\cdot1\ldots p-1}^2=\mathrm{R}^2/(1-\mathrm{R}^2)=v,$$

say.

The distribution of $v=n_1s_1^2/n_2s_2^2$, where s_1^2 and s_2^2 are independent estimates of a variance σ^2, with n_1 and n_2 degrees of freedom respectively, has been shown by Fisher to be

$$f(v)dv=\frac{\Gamma\frac{1}{2}(n_1+n_2)}{\Gamma\frac{1}{2}n_1\Gamma\frac{1}{2}n_2}\frac{v^{\frac{1}{2}n_1-1}}{(1+v)^{\frac{1}{2}(n_1+n_2)}}dv,$$

from which the z distribution is derived.

Write $n_1=p-1$, $n_2=n-p$, and $v=\mathrm{R}^2/(1-\mathrm{R}^2)$, and we obtain the well-known distribution (15) of R^2 when there is no real multiple correlation,

$$g(\mathrm{R}^2)d(\mathrm{R}^2)=\frac{\Gamma\frac{1}{2}(n-1)}{\Gamma\frac{1}{2}(p-1)\Gamma\frac{1}{2}(n-p)}(\mathrm{R}^2)^{\frac{1}{2}(p-3)}(1-\mathrm{R}^2)^{\frac{1}{2}(n-p-2)}d(\mathrm{R}^2).\quad .\tag{23}$$

The joint test of significance of the coefficients $b_{p1\cdot 2\ldots p-1}, \ldots, b_{p,\,p-1\cdot 1\ldots p-2}$ from zero thus becomes identical with the test of the multiple correlation being significant.

II. On the Assumption of Normality.

6. So far the notation has been used that ξ denotes the deviation of a variate from its true mean, and x the deviation from its estimated mean. This was convenient, since it allowed the estimate of $\xi_{2\cdot 1}$ to be written $x_{2\cdot 1}$, etc.

It may now be remarked, however, that if x is given its usual interpretation as the value of an observation, *e g.*

$$x_1 = \xi_1 + m_1,$$

where m_1 is the true mean of x_1, the notation can be made quite complete by defining a quantity $x_0 \equiv 1$, and writing now $x_{1\cdot 0}$ as our estimate of ξ_1 ($\xi_{1\cdot 0} \equiv \xi_1$, since ξ_1 is already measured from its true mean). For we had previously the relations

$$\begin{cases} x_{\mu\cdot\nu} &= x_\mu - x_\nu \Sigma(x_\mu x_\nu)/\Sigma x_\nu^2 \\ c_{\mu\mu\cdot\nu} &= c_{\mu\mu} - c_{\mu\nu}^2/c_{\nu\nu} \\ u_{\mu\cdot\nu} &= \sqrt{c_{\nu\nu}}(b_{\mu\nu} - \beta_{\mu\nu}), \end{cases}$$

and analogously we now have

$$\begin{cases} x_{1\cdot 0} = x_1 - \Sigma x_1/n = x_1 - \bar{x}_1 \\ c_{11\cdot 0} = c_{11} - (\Sigma x_1)^2/n = \Sigma(x_1 - \bar{x}_1)^2 \\ u_{1\cdot 0} = \sqrt{n}(\bar{x}_1 - m_1), \end{cases}$$

since

$$\Sigma(x_1 x_0) = \Sigma x_1 = n\bar{x}_1, \qquad \Sigma x_0^2 = n.$$

We may now write $c_{22\cdot 01}$ instead of $c_{22\cdot 1}$, etc , and in these expressions we shall now have the number of digits referring to variates eliminated giving the number of degrees of freedom lost. The joint distribution of the means and the product sums given by equation (4) will now be written

$$W_{p,\,0} \equiv \dot{U}_p(u_{\mu\cdot 0})V_p(c_{\mu\nu\cdot 0},\, n-1). \quad . \quad . \quad . \quad (24)$$

7. In the first part of this paper, the distributions arising out of the problem of regression were conveniently derived from the distribution $W_{p,\,0}$, which holds only on the assumption that the system of variates ξ_μ is normal. We have seen, however (using now the complete distribution $W_{p,\,0}$ given by (24), which includes the distribution of the means), that in the case of two variates we may write

$$W_{2,0} = \{V_1(c_{11\cdot 0},\, n-1)U_1(u_{1\cdot 0})\}\{V_1(c_{22\cdot 01},\, n-2)U_1(u_{2\cdot 0} - \beta_{21}u_{1\cdot 0})U_1(u_{2\cdot 10})\}, \quad (25)$$

where the variance of $u_{1\cdot 0}$ is σ_1^2, estimated by $c_{11\cdot 0}/(n-1)$, and the variance of $u_{2\cdot 0} - \beta_{21}u_{1\cdot 0}$ or $u_{2\cdot 10}$ is $\sigma_{2\cdot 1}^2$, estimated by $c_{22\cdot 01}/(n-2)$.

The original normal distribution $U_2(\xi_\mu)$ can of course be written

$$U_2(\xi_\mu) = U_1(\xi_1)U_1(\xi_2 - \beta_{21}\xi_1),$$

and equation (25) suggests that $W_{2,0}$, which depends on the normality of ξ_1 and ξ_2, can also be split up into two factors which depend respectively on the normality of ξ_1 and $\xi_{2\cdot 1}$. If we assume for the moment this to be true, we may observe that any distribution or test derived from both factors, such as the distribution of the regression coefficient, of the correlation coefficient, or of the co-variance, will therefore depend on the normality of ξ_1 and $\xi_{2\cdot 1}$, *i.e.* of ξ_1 and ξ_2. On the other hand, any distribution or test *derived from the second factor alone*, such as the distribution of the partial variance, or the test of significance of the regression coefficient, will depend simply on the normality of the partial or residual variate $\xi_{2\cdot 1}$

For three variates we may write

$$W_{3,0} = \{V_1(c_{11\cdot 0}, n-1)U_1(v_{1\cdot 0})\}\{V_2(c_{\mu\nu\cdot 01}, n-2)U_2(u_{\mu\cdot 0} - \beta_{\mu 1}u_{1\cdot 0})U_2(u_{\mu\cdot 10})\}, \quad (26)$$

where we may expect the first factor to depend on ξ_1 being normal, the second on $\xi_{2\cdot 1}$, $\xi_{3\cdot 1}$ being normal, and then for the second factor we may write further

$$\begin{gathered}\{V_1(c_{22\cdot 01}, n-2)U_1(u_{2\cdot 0} - \beta_{21}u_{1\cdot 0})U_1(u_{2\cdot 1})\} \\ \times\{V_1(c_{33\cdot 012}, n-3)U_1(u_{3\cdot 0} - \beta_{31\cdot 2}u_{1\cdot 0} - \beta_{32\cdot 1}u_{2\cdot 0})U_1(u_{3\cdot 10} - \beta_{32\cdot 1}u_{2\cdot 10})U_1(u_{3\cdot 210})\}, \quad (27)\end{gathered}$$

where in this expression we may expect the last complete factor to depend simply on $\xi_{3\cdot 21}$ being normal.

In order to give a formal proof of these results, which makes no assumptions that are not necessary, it is proposed to establish *ab initio* a general distribution which is an extension of the distribution $W_{p,0}$ obtained by Wishart, being the corresponding partial distribution when κ of the variates are eliminated. It will be written

$$W_{p,\kappa}.$$

The method is to find the moment-generating function of the quantities whose distribution we wish to find. This method was used in a recent paper (2) to obtain the distribution $W_{p,0}$, and it is there more fully explained. The work below gives perhaps the most general result obtainable in this way.

8 First, however, it is necessary to give a very brief discussion of moment-generating functions and their properties. The most fundamental definition of the moment-generating function of a quantity ϕ, which is some function of the pn observations in a sample in p variates ξ_μ, is perhaps

$$M(t) = E(e^{t\phi}), \quad . \quad . \quad . \quad . \quad . \quad (28)$$

where E denotes mathematical expectation—that is, $e^{t\phi}$ is to be averaged for all possible values of ϕ. Thus the moment of the rth order, which may be written $E(\phi^r)$, is the coefficient of $(it)^r/r!$ in the expansion of $M(t)$. We write it so that $M(t)$ is finite for all real t

If $\phi = x_1 + x_2$, where x_1 and x_2 are any two independent quantities, we have

$$\begin{aligned} E(e^{it(x_1+x_2)}) &= E(e^{itx_1} . e^{itx_2}) \\ &= E(e^{itx_1})E(e^{itx_2}) \end{aligned}$$

or

$$M_{x_1+x_2} = M_{x_1} . M_{x_2}.$$

For n independent quantities $x_1 \dots x_n$, we have similarly

$$M_{\Sigma x} = \Pi M_{x_r}. \qquad . \qquad . \quad . \quad . \quad (29)$$

In particular, if $x_1 \dots x_n$ are observations referring to the same variate x, we have

$$M_{\Sigma x} = M_x{}^n.$$

These results hold whether $x_1 \dots x_n$ are continuous variates or not, and the definition (28) thus renders obvious in all cases the property of moment-generating functions given in (29), which, since this may be written

$$K_{\Sigma x} = \log M_{\Sigma x} = \Sigma K_x,$$

is sometimes called the additive property of semi-invariants.

If $\phi = x$, one of the observations, and x is a continuous variate with distribution $f(x)$, we have

$$E(e^{itx}) = \int_{-\infty}^{\infty} e^{itx} f(x)dx,$$

and similarly we have in general, when ϕ is continuous,

$$M(t) = E(e^{it\phi}) = \int_{-\infty}^{\infty} e^{it\phi} F(\phi)d\phi;$$

this equation is important for determining $F(\phi)$, the distribution of ϕ, when we know $M(t)$. In order to find $M(t)$, we must, however, average $e^{it\phi}$ in terms of the original observations, since its value is clearly the same whether we average for values of ϕ or for values of the observations of which ϕ is a function, and while we do not yet know the chance of a particular ϕ arising, given by $F(\phi)d\phi$, we do know the chance of particular values of the observations arising.

Notice, if $\phi = \phi(x_1, x_2)$, say,

$$E(e^{it\phi}) = E_{x_1}E_{x_2}(e^{it\phi}),$$

or symbolically

$$E = E_{x_1}E_{x_2},$$

where E_{x_1} denotes averaging with respect to x_1, etc , and if in particular $E_{x_2}(e^{it\phi})$ were independent of x_1, we should have simply

$$E(e^{it\phi}) = E_{x_2}(e^{it\phi}).$$

The above discussion deals only with the moment-generating function of a single quantity ϕ. For the present purpose we require the joint moment-generating function of q quantities ϕ_r, but this may be defined in exactly the same way by

$$M(t_r) = E\left(\exp \sum_{r=1}^{q} it_r\phi_r\right), \quad . \quad . \quad . \quad . \quad (30)$$

and all the corresponding properties hold.

9. We suppose we have two sets of variates $\xi_1 \ldots \xi_\kappa, \eta_{\kappa+1} \ldots \eta_p$, where the variates $\eta_\mu (\mu = \kappa + 1 \ldots p)$ are independent of the variates $\xi_m (m = 1 \ldots \kappa)$, and constitute a normal system

$$U_p(\eta_\mu) = \pi^{-\frac{1}{2}p} \mid A \mid^{\frac{1}{2}} e^{-A(\eta, \eta)} \prod_{\mu=\kappa+1}^{p} d\eta_\mu \quad . \quad . \quad . \quad (31)$$

The sample contains n sets of values of the p variates ξ_m, η_μ. The n sets of values of η_μ are of course assumed independent. No assumption is made, however, about the variates ξ_m.

The estimated variates $x_{1\cdot 0}$, etc., corresponding to ξ_1, etc., are as before. We consider the quantities

$$w_{\mu,m} = \Sigma\lambda_m\eta_\mu,$$

where

$$\lambda_m = \begin{cases} x_0/\sqrt{\Sigma x_0^2}, & (m = 0) \\ x_{m\,0\ldots m-1}/\sqrt{\Sigma x^2_{m\,0\ldots m-1}}, & (m = 1 \ldots \kappa); \end{cases}$$

and

$$\Sigma_{\mu v} = \Sigma\eta_\mu\eta_v, \quad (\mu, v = \kappa + 1 \ldots p).$$

The quantity x_0 we may subsequently put equal to unity.

The joint moment-generating function of $w_{\mu,m}, \Sigma_{\mu v}$ will be evaluated by averaging first with respect to the η, and then with respect to the ξ. Since the quantities $w_{\mu,m}, \Sigma_{\mu v}$ are each sums of n quantities $\lambda_m\eta_\mu$, $\eta_\mu\eta_v$ independent with respect to the η, we may write

$$M(2t_{\mu,m},\ t_{\mu v}) = E_\xi \Pi E_\eta\left(\exp\left\{2i\sum_{\mu=\kappa+1}^{p}\tau_\mu\eta_\mu + iT(\eta, \eta)\right\}\right), \quad . \quad (32)$$

where Π denotes a product of n factors corresponding to the n sets of observations in the sample, $T \equiv (t_{\mu v})$, and

$$\tau_\mu = \sum_{m=0}^{\kappa} \lambda_m t_{\mu,m}.$$

Now we have from (31),

$$\mathrm{E}_\eta\left(\exp\left\{2i\sum_{\mu=\kappa+1}^{p}\tau_\mu\eta_\mu+i\mathrm{T}(\eta,\eta)\right\}\right)$$

$$=\int\cdots\int_{-\infty}^{\infty}\pi^{-\frac{1}{2}p}\,|\,\mathrm{A}\,|^{\frac{1}{2}}\exp\left\{2i\sum_{\mu=\kappa+1}^{p}\tau_\mu\eta_\mu-(\mathrm{A}-i\mathrm{T})(\eta,\eta)\right\}\prod_{\mu=\kappa+1}^{p}d\eta_\mu.$$

Integrating * this expression, we therefore have from (32)

$$\mathrm{M}=\mathrm{E}_\xi\Pi(\{|\,\mathrm{A}\,|/\,|\,\mathrm{A}-i\mathrm{T}\,|\}^{\frac{1}{2}}e^{-\mathrm{B}(\tau,\tau)}),$$

where $\mathrm{B}\equiv(b_{\mu\nu})$ is the reciprocal of $(\mathrm{A}-i\mathrm{T})$; that is,

$$\mathrm{M}=\{|\,\mathrm{A}\,|/|\,\mathrm{A}-i\mathrm{T}\,|\}^{\frac{1}{2}n}\mathrm{E}_\xi(e^{-\Sigma\mathrm{B}(\tau,\tau)}).$$

Now, since †

$$\Sigma x_{m\cdot 0\ldots m-1}x_{n\cdot 0\ldots n-1}=0$$

if

$$n\neq m,$$

$$\Sigma\lambda_m\lambda_n=0$$

if

$$n\neq m,$$

and also obviously from the definition of λ_m,

$$\Sigma\lambda_m^2=1;$$

hence we have

$$\begin{aligned}\Sigma\mathrm{B}(\tau,\tau)&=\sum_{\mu,\nu=\kappa+1}^{p}b_{\mu\nu}(\Sigma\tau_\mu\tau_\nu)\\&=\sum_{\mu,\nu=\kappa+1}^{p}b_{\mu\nu}\left(\sum_{m,n=0}^{\kappa}t_{\mu,m}t_{\nu,n}\Sigma\lambda_m\lambda_n\right)\\&=\sum_{m=0}^{\kappa}\mathrm{B}(t_m,t_m)\end{aligned}$$

Now this expression is independent of the ξ; hence averaging with respect to ξ does not affect the result, and we have finally

$$\mathrm{M}=\{|\,\mathrm{A}\,|\,/\,|\,\mathrm{A}-i\mathrm{T}\,|\}^{\frac{1}{2}n}\exp\left\{-\sum_{m=0}^{\kappa}\mathrm{B}(t_m,t_m)\right\}.\qquad\text{(33)}$$

Now since, if $\mathrm{F}(w_{\mu,m},\Sigma_{\mu\nu})$ represents the joint distribution of $w_{\mu,m}$, $\Sigma_{\mu\nu}$, we may write

$$\mathrm{M}=\int\cdots\int_{-\infty}^{\infty}\exp\left\{2i\sum_{\mu=\kappa+1}^{p}\sum_{m=0}^{\kappa}t_{\mu,m}w_{\mu,m}+i\sum_{\mu,\nu=\kappa+1}^{p}t_{\mu\nu}\Sigma_{\mu\nu}\right\}\mathrm{F}\,dw\,d\Sigma,$$

where

$$dw\equiv\prod_{\mu=\kappa+1}^{p}\prod_{m=0}^{\kappa}dw_{\mu,m},\qquad d\Sigma\equiv\prod_{\kappa+1\leqslant\mu\leqslant\nu\leqslant p}d\Sigma_{\mu\nu},$$

we have by a generalized form of Fourier's Integral Theorem (see (2)),

$$\mathrm{F}=(2\pi)^{-(p-\kappa)}\pi^{-\frac{1}{2}(p-\kappa)(p+\kappa+1)}\int\cdots\int_{-\infty}^{\infty}\exp\left\{-2i\sum_{\mu=\kappa+1}^{p}\sum_{m=0}^{\kappa}t_{\mu,m}w_{\mu,m}-i\sum_{\mu,\nu=\kappa+1}^{p}t_{\mu\nu}\Sigma_{\mu\nu}\right\}$$

$$\times\,\mathrm{M}\prod_{m=0}^{\kappa}dt_m\,dt,$$

---

* *Cf.* Wishart and Bartlett (2), p. 2, equation (5)

† See Yule (9), p. 183.

where

$$dt_m \equiv \prod_{\mu=\kappa+1}^{p} dt_{\mu,m}, \qquad dt \equiv \prod_{\kappa+1 \leqslant \mu \leqslant \nu \leqslant p} dt_{\mu\nu},$$

and M is given by (33). Integrate * successively with respect to the t_m ($m=0, 1, \ldots \kappa$), and we have

$$F = \int \cdots \int_{-\infty}^{\infty} \frac{|A|^{\frac{1}{2}n} \exp\left\{-i \sum_{\mu,\nu=\kappa+1}^{p} t_{\mu\nu}\Sigma_{\mu\nu} + \sum_{m=0}^{\kappa} B^{-1}(w_m, w_m)\right\}}{(2\pi)^{-(p-\kappa)}\pi^{-\frac{1}{4}p(p-\kappa)} |A - iT|^{\frac{1}{2}n} |B|^{\frac{1}{2}\kappa}} dt,$$

or since

$$B^{-1}(w_m, w_m) = A(w_m, w_m) - iT(w_m, w_m),$$

and

$$|B|^{-\frac{1}{2}\kappa} = |A - iT|^{\frac{1}{2}\kappa},$$

we have

$$F = \prod_{m=0}^{\kappa} \{\pi^{-\frac{1}{2}(p-\kappa)} |A|^{\frac{1}{2}} e^{-A(w_m, w_m)}\} . \int \cdots \int_{-\infty}^{\infty} \frac{|A|^{\frac{1}{2}(n-\kappa-1)} \exp\left\{-i \sum_{\mu,\nu=\kappa+1}^{p} t_{\mu\nu}\Sigma_{\mu\nu}'\right\}}{(2\pi)^{(p-\kappa)}\pi^{\frac{1}{4}(p-\kappa)(p-\kappa-1)} |A - iT|^{\frac{1}{2}(n-\kappa-1)}} dt,$$

where

$$\Sigma_{\mu\nu}' = \Sigma_{\mu\nu} - \sum_{m=0}^{\kappa} w_{\mu,m} w_{\nu,m}$$

The integral remaining is exactly similar to the integral obtained in the paper quoted (**2**) to obtain $V_p(c_{\mu\nu\cdot 0}, n-1)$, but with $n-k-1$ instead of $n-1$. It has been evaluated by Ingham (**10**), and using his result, changing the variates from $w_{\mu,m}$, $\Sigma_{\mu\nu}$ to $w_{\mu,m}$, $\Sigma_{\mu\nu}'$, and inserting the differentials, we have

$$F dw d\Sigma' = \prod_{m=0}^{\kappa} \{U_{p-\kappa}(w_{\mu,m})\} V_{p-\kappa}(\Sigma_{\mu\nu}', n-\kappa-1).$$

To interpret this result, we simply write

$$\eta_\mu = \xi_{\mu 1 \ldots \kappa},$$

so that in a set of variates $\xi_1 \ldots \xi_p$ we have assumed only that the $p-\kappa$ partial variates $\xi_{\mu 1 \ldots \kappa}$ are normal.

The quantities $w_{\mu,m}$ are readily expressible in terms of the observations $x_1 \ldots x_p$. They are in fact analogous to the $w_1 \ldots w_{p+1}$ of equation (19), except for $w_{\mu,0}$, which are functions of the means—which were not considered in I. They may be written

$$\begin{cases} w_{\mu,0} = \sqrt{n}(\bar{x}_\mu - m_\mu - \beta_{\mu\kappa 1 \ldots \kappa-1}[\bar{x}_\kappa - m_\kappa] \ldots - \beta_{\mu 1 2 \ldots \kappa}[\bar{x}_1 - m_1]) \\ w_{\mu,1} = \sqrt{c_{11\cdot 0}}(b_{\mu 1} - \beta_{\mu\kappa 1 \ldots \kappa-1} b_{\kappa 1} \ldots - \beta_{\mu 1 2 \ldots \kappa}) \\ w_{\mu,2} = \sqrt{c_{22\cdot 01}}(b_{\mu 2\cdot 1} - \beta_{\mu\kappa\cdot 1 \ldots \kappa-1} b_{\kappa 2\cdot 1} \ldots - \beta_{\mu 2\cdot 13 \ldots \kappa}) \\ \vdots \\ w_{\mu,\kappa} = \sqrt{c_{\kappa\kappa\cdot 0 \ldots \kappa-1}}(b_{\mu\kappa 1 \ldots \kappa-1} - \beta_{\mu\kappa\cdot 1 \ldots \kappa-1}) \end{cases} \qquad (34)$$

---

* *Cf.* Wishart and Bartlett, *loc. cit.*

Just as $x_{\mu\cdot 0}$, $b_{\mu 1}$, etc. refer to the variates $\xi_1 \ldots \xi_p$, let $x_{\mu\cdot 0}'$, $b_{\mu 1}'$, etc. refer to the variates $\xi_1 \ldots \xi_{\kappa+1}, \eta_\kappa \ldots \eta_p$. Then we have

$$
\begin{aligned}
\Sigma\eta_\mu\eta_\nu - \sum_{m=0}^{\kappa} w_{\mu,m}w_{\nu,m} &= \Sigma x_{\mu\,0}'x_{\nu\cdot 0}' - \sum_{m=1}^{\kappa} w_{\mu,m}w_{\nu,} \\
&= \Sigma(x_{\mu 0}' - b_{\mu 1}'x_{1\cdot 0})(x_{\nu\cdot 0}' - b_{\nu 1}'x_{1\cdot 0}) - \sum_{m=2}^{\kappa} w_{\mu,m}w_{\nu,m} \\
&= \Sigma(x_{\mu\cdot 01}'x_{\nu\,01}') - \sum_{m=2}^{\kappa} w_{\mu,m}w_{\nu,n} \\
&= \ldots \\
&= \Sigma x_{\mu\cdot 01}'{}_{\ldots\kappa}x_{\nu\cdot 01}'{}_{\ldots\kappa}.
\end{aligned}
$$

But

$$
\begin{aligned}
x_{\mu\cdot 0}'{}_{\ldots\kappa} &= x_{\mu\cdot 0\ldots\kappa} - \beta_{\mu\kappa\cdot 1\ldots\kappa-1}x_{\kappa\cdot 0\ldots\kappa} \ldots \quad - \beta_{\mu 1\cdot 2\ldots\kappa}x_{1\cdot 0\ldots\kappa} \\
&= x_{\mu\,0\ldots\kappa},
\end{aligned}
$$

since by definition $x_{\kappa\cdot 0\ldots\kappa}$, etc. $=0$. Hence

$$\Sigma_{\mu\nu}' = c_{\mu\nu\,0\ldots\kappa}.$$

Thus we may write the distribution finally

$$W_{p,\kappa} \equiv \prod_{m=0}^{\kappa}\{U_{p-\kappa}(w_{\mu,m})\}V_{p-\kappa}(c_{\mu\nu\,0\ldots\kappa}, n-\kappa-1). \qquad (35)$$

It is suggested that the set of distributions represented by $W_{p,\kappa}$ is the more fundamental, at any rate for the study of regression or partial correlation, for though we saw in I that it may be deduced from Wishart's distribution, it is now seen to be true under more general conditions—namely, that the $p-\kappa$ variates $\xi_{\mu 1\ldots\kappa}$ are normal—and Wishart's distribution may alternatively be obtained from (35) by writing $\kappa=0$. For the present purpose, the most important cases are given by $\kappa=p-2$, and $p-1$.

10. Thus, for $p=2$, $\kappa=0$ gives Fisher's distribution for two variates. When $\kappa=1$, we have

$$W_{2,1} = U_1(w_0)U_1(w_1)V_1(c_{22\,01}, n-2), \qquad \ldots \qquad (36)$$

where

$$\begin{cases} w_0 \equiv w_{2,0} = \sqrt{n}(\bar{x}_2 - m_2 - \beta_{21}[\bar{x}_1 - m_1]) \\ w_1 \equiv w_{2,1} = \sqrt{c_{11\,0}}(b_{21} - \beta_{21}). \end{cases}$$

This requires only that $\xi_{2\cdot 1}$ exists and is normal (this condition implies that the regression of ξ_2 on ξ_1 is linear, and that the variance of ξ_2 for each ξ_1 is constant).

Thus the quantity w_1 is normally distributed whatever the distribution of ξ_1, whether the observations ξ_1 can be regarded as fixed from sample to sample or not, or whether they are specially selected. The reason for this is that w_1, regarded as a function of $\xi_{2\cdot 1}$, is so weighted that the variance is independent of ξ_1. Since the infinite population of ξ_1

and $\xi_{2\cdot 1}$ can be split up into an infinite number of sub-populations for which the ξ_1 are fixed while $\xi_{2\cdot 1}$ vary (for $\xi_{2\cdot 1}$ and ξ_1 are assumed independent), the distribution of w_1 will be the sum of an infinite number of distributions each of which is normal, with the same variance and mean. Consequently, the distribution of w_1 will be similarly distributed, however ξ_1 varies.

In the case where ξ_1 is fixed from sample to sample the problem is simplified, since it is not so essential to consider the particular function w_1. Any linear function of $\xi_{2\cdot 1}$ will then be normal, but for any other function the variance will of course depend on ξ_1. There is no doubt that the condition that the set ξ_1 may be supposed fixed is commonly met with in practice, and this case was the one considered by Fisher (**11**), although he seems to suggest in conclusion that his test holds under somewhat wider conditions than he assumed.

It is important to notice that the test of significance of the mean $\bar{x}_2$ obtainable from $W_{2,1}$ is *only* valid when the set ξ_1 is fixed, for then $\bar{x}_1 \equiv m_1$, and w_0 becomes $\sqrt{n}(\bar{x}_2 - m_2)$ Otherwise, as for a random sample in two normal variates, we should have to consider $W_{1,0}$ for the variate ξ_2, given by $U_1(\sqrt{n}[\bar{x}_2 - m_2])V_1(c_{22\cdot 0}, n-1)$, with the corresponding assumption that ξ_2 is normal. The "t test" of significance of b_{21} from β_{21} is, however, valid, with no restrictions on ξ_1. Moreover, the special case of this, the test of significance of b_{21} or r_{21} from zero, is clearly to be regarded as a special case of the test of regression, not of correlation, for fewer assumptions are involved in the test of regression than in the test of correlation. Thus if we regarded the distribution of the correlation coefficient r_{21} when ρ_{21} is zero as a special case of the general distribution when $\rho_{21} \neq 0$, we might have supposed that ξ_2, ξ_1 must both be normal.

The simplest way to consider this distribution is to use Fisher's geometrical methods (**3**), and consider the chance of the two *radii vectores* representing in n dimensions the sample of ξ_1 and ξ_2 making an angle θ with each other when they are independent of each other—since $r_{21} = \cos\theta$ Clearly, however restricted the *radius vector* of ξ_1, say, is, if any direction of the *radius vector* of ξ_2 is equally likely, then the angle θ will be perfectly random, and the distribution of r_{21} when ρ_{21} is zero follows. The condition that any direction of the radius vector is equally likely is the condition of the normal law (compare Maxwell's proof of the normal law in the dynamical theory of gases). Hence the only condition for the distribution to hold is seen to be that ξ_2, say, is a normal variate (and the n observations ξ_2, of course, independent of each other).

In the case of three variates, put $p=3$, $\kappa=1$, and we have

$$W_{3,1} = U_2(w_{\mu,0})U_2(w_{\mu,1})V_2(c_{\mu\nu\cdot 01}, n-2). \qquad (37)$$

From $V_2(c_{\mu\nu\cdot 01}, n-2)$ is obtained the distribution of the partial correlation coefficient $r_{32\cdot 1}$, and this therefore depends only on $\xi_{3\cdot 1}$ and $\xi_{2\cdot 1}$ being normal, as has been pointed out by Fisher ((14), p. 163).

If instead we put $\kappa = 2$, we have

$$W_{3,2} = U_1(w_0)U_1(w_1)U_1(w_2)V_1(c_{33\cdot 012},\ n-3), \quad . \quad . \quad (38)$$

where

$$\begin{cases} w_0 \equiv w_{3,0} = \sqrt{n}(\bar{x}_3 - m_3 - \beta_{32\cdot 1}[\bar{x}_2 - m_2] - \beta_{31\cdot 2}[\bar{x}_1 - m_1]) \\ w_1 \equiv w_{3,1} = \sqrt{c_{11\cdot 0}}(b_{31} - \beta_{32\cdot 1}b_{21} - \beta_{31\cdot 2}) \\ w_2 = w_{3,2} = \sqrt{c_{22\cdot 01}}(b_{32\cdot 1} - \beta_{32\cdot 1}), \end{cases}$$

provided only that $\xi_{3\cdot 21}$ exists and is normal.

The test of the partial regression coefficient $b_{32\cdot 1}$ from $\beta_{32\cdot 1}$ thus depends only on this condition. Similarly for that of $b_{31\cdot 2}$ from $\beta_{31\cdot 2}$. This applies also to the joint test of regression using the "z distribution," and to the special case of this latter test when $\beta_{31\cdot 2}$ and $\beta_{32\cdot 1}$ are put equal to zero, when it coincides with the test of a significant multiple correlation.

The condition that $\xi_{3\cdot 21}$ exists implies that the partial regressions with ξ_2 and ξ_1 are linear, since

$$\xi_{3\cdot 21} = \xi_3 - \beta_{32\cdot 1}\xi_2 - \beta_{31\cdot 2}\xi_1,$$

but since we have seen that the distributions of ξ_2 and ξ_1 are immaterial, we may suppose them to be any required functions of the original observations, or what is more usual in practice if we have no prior knowledge of these functions, may consider the partial regressions with ξ_1, ξ_1^2, etc. We may, for example, by putting $\xi_2 \equiv \xi_1^2$, interpret the above equation as representing the parabolic regression of a variate ξ_3 on ξ_1. Thus curvilinear regression and curvilinear partial regression provide no further theoretical difficulties. When a curved regression line is being fitted, it is often convenient to consider the partial regressions not with ξ_1, ξ_1^2, . . ., but with some orthogonal set such as $\xi_{1\cdot 0}$, $\xi_{2\cdot 01}$, . . ., where $\xi_2 \equiv \xi_1^2$, etc.; the regression line may then be fitted term by term. This procedure is practically most important when the values of ξ_1 are separated by equidistant intervals.

11. The test of significance of $\bar{x}_1 - m_1$ has been extended by Fisher to include the significance of a difference $\bar{x}_1 - \bar{x}_1'$, where $\bar{x}_1$ and $\bar{x}_1'$ are the means from two independent samples S and S′. We may now regard the variate

$$w_{1,0} = \sqrt{n}(\bar{x}_1 - m_1)$$

as a special case of the variates $w_{1,0}$, $w_{2,1}$, etc.; one, moreover, which is especially simple, as it is a function of x_1 and x_0, where the variate x_0 is not only fixed from sample to sample, but can take only the value unity. Thus since x_0 is fixed, it is not necessary to eliminate it to eliminate m_1, and we can not only consider

$$w_{1,0} - w_{1,0}' = \sqrt{n}(\bar{x}_1 - \bar{x}_1') \quad . \quad . \quad . \quad . \quad (39)$$

when the samples are equal in size, but

$$w_{1,0}/\sqrt{n} - w_{1,0}'/\sqrt{n'} = (\bar{x}_1 - \bar{x}_1'), \quad . \quad . \quad . \quad (40)$$

say, if they are unequal.

In the general case of the significance of any $w \sim w'$, the condition that the "independent variates" should be fixed from sample to sample for the test to be strictly applicable immediately becomes apparent.

Thus in the case of two variates, we only have

$$u \equiv w_{2,1} - w_{2,1}' = \sqrt{c_{11\cdot0}}(b_{21} - b_{21}') \quad . \quad . \quad . \quad (41)$$

provided that $\sqrt{c_{11\cdot0}'} = \sqrt{c_{11\cdot0}}$, that is, the samples are equal in size, and the same set of ξ_1 is contained in each sample. We may then consider the normal variate in (41), with its corresponding estimated variance

$$v = \left(c^2_{22\cdot01} + (c_{22\cdot01}')^2\right)/2(n-2),$$

and test the significance of the difference between b_{21} and b_{21}' by means of $t = u/\sqrt{v}$, which will follow the "t distribution" with $2(n-2)$ degrees of freedom.

If the samples are unequal in size, but the ξ_1 are fixed, we can weight $w_{2,1}$ and $w_{2,1}'$ in order to eliminate β_{21}, similarly to (40).

For three variates, when ξ_1 and ξ_2 are supposed fixed from sample to sample, we have similarly to (41) for two equal samples,

$$u_2 \equiv w_{3,2} - w_{3,2}' = \sqrt{c_{22\cdot01}}(b_{32\cdot1} - b_{32\cdot1}'). \quad . \quad . \quad (42)$$

Further, under the same conditions,

$$u_1 \equiv w_{3,1} - w_{3,1}' = \sqrt{c_{11\cdot0}}(b_{31} - b_{31}'), \quad . \quad (43)$$

and we have not only from (42) a normal variate by which we can test the significance of the difference between $b_{32\cdot1}$ and $b_{32\cdot1}'$, but from (43) a more sensitive test of significance between b_{31} and b_{31}' if ξ_2 appreciably affects ξ_3 and ξ_1. Again, since

$$2v_1 \equiv u_1^2 + u_2^2 = (c_{32\cdot0} - c_{32\cdot0}')(b_{32\cdot1} - b_{32\cdot1}') + (c_{31\cdot0} - c_{31\cdot0}')(b_{31\cdot2} - b_{31\cdot2}'), \quad . \quad (44)$$

we may test the joint significance between $b_{32\cdot1}$, $b_{31\cdot2}$, and $b_{32\cdot1}'$, $b_{31\cdot2}'$ respectively, by means of the "z distribution."

Analogous tests hold for three variates if the samples are unequal, provided ξ_1 and ξ_2 are still fixed for each sample; similarly for any number of variates.

12. To sum up the results of II, the following distributions depend simply on the existence and normality of the two partial or residual variates $\xi_{p\cdot1\ldots p-2}$, $\xi_{p-1\cdot1\ldots p-2}$:

(*a*) The distribution of the partial correlation coefficient $r_{p,\, p-1\,1 \ldots p-2}$.
(*b*) The distribution of the partial co-variance $v_{p,\, p-1\cdot 1 \ldots p-2}$.
(*c*) The distribution of the partial regression coefficient $b_{p,\, p-1\cdot 1 \ldots p-2}$.

On the other hand, the following distributions and tests of regression depend only on the existence and normality of the one residual, $\xi_{p\cdot 1 \ldots p-1}$:

(*d*) The distribution of the partial variance $v_{pp\cdot 1 \ldots p-1}$.

(*e*) The distribution of the normal and independent variates (defined in (34)), $w_{p,\, 0} \ldots w_{p,\, p-1}$.

(*f*) The test of significance of $b_{p,\, p-1\cdot 1 \ldots p-2} \sim \beta_{p,\, p-1\,1 \ldots p-2}$—including the particular case when $\rho_{p,\, p-1\cdot 1 \ldots p-2}$ assumed zero.

(*g*) The joint test of significance of $b_{pr.1 \ldots r-1,\, r+1 \ldots p-1} \sim \beta_{pr\,1 \ldots r-1,\, r+1 \ldots p-1} (r = 1 \ldots p-1)$—including the particular case when no real multiple correlation is assumed.

(*h*) In the problem of two samples, provided we can regard the set of variates $\xi_1, \ldots \xi_{p-1}$ as fixed, the test of significance of $b_{p,\, p-1\cdot 1 \ldots p-2} \sim b_p{}'_{,\, p-1\cdot 1 \ldots p-2}$, or the joint test of significance of $b_{pr\,1 \ldots r-1,\, r+1 \ldots p-1} \sim b_{pr\cdot 1}{}'_{\ldots r-1,\, r+1 \ldots p-1} (r = 1 \ldots p-1)$.

Regression is seen to be a wider concept than correlation; for fewer assumptions are involved in the more important distributions and tests, which are moreover simpler.

Though all correlation tests—when we are concerned only with finding whether there is any correlation at all—reduce to regression tests, a distinction should obviously be made between problems of regression in general, and those where the idea of correlation may be usefully employed. In the case of two variates, for example, since the distribution of the correlation coefficient when ρ_{21} is not zero requires a random sample of two normal variates ξ_2 and ξ_1, a correlation coefficient has most meaning in a sample of this kind.

No mention has so far been made of the test of goodness of fit of regression lines (see Fisher (**16**)), since this depends on our having an array of values of ξ_2 for each value of ξ_1. The test is, however, simply another application of the analysis of variance, the "*z* distribution" being used to test, say, whether we have satisfactorily obtained a normal variate $\xi_{2\cdot 1}$ by assuming $\xi_{2\cdot 1} = \xi_2 - \beta_{21}\xi_1$ and fitting a straight line. The variation within arrays is compared with the variation of the means of the arrays from the fitted regression line. The test is thus strictly possible only when several values of ξ_2 correspond to each ξ_1, but in practice when this condition is not fulfilled we can sometimes group the values of ξ_2 provided the grouping is sufficiently fine. Alternatively, we could fit the regression line first, and afterwards group the deviations from this line; we should then, however, as an approximation have to neglect the number of degrees of freedom lost in fitting, and assume the fitted regression line to be the true (linear) regression line.

An exact theoretical test for the goodness of fit of a straight line would be to fit the regression line, and then group coarsely, making an estimate of $\sigma_{2\cdot1}^2$ by finding $c_{22\cdot01}$ *for each group.* That is, if there were a groups, finding

$$(n-2a)v_1 = \sum_{r=1}^{a} (c_{22\cdot01})_r,$$

$$2(a-1)v_2 = c_{22\cdot01} - \sum_{r=1}^{a} (c_{22\cdot01})_r,$$

and comparing the independent estimates of $\sigma_{2\cdot1}^2$ given by v_1 and v_2. Even if we were testing the fit of a curved regression line, it would probably be sufficient to assume the regression in any group to be linear. But the awkwardness of the above test makes its practical importance limited; especially as the test of goodness of fit is supplementary to the specific tests of significance of regression coefficients—tests which are usually more sensitive.

Thus it might happen that a regression coefficient b_{21} is significant, although the goodness of fit seemed adequate when β_{21} was assumed zero; this is because in the test of significance of b_{21} the variance due to the linear regression is isolated as a single square. This particular criticism of the test of goodness of fit is general, of course, and applies also to the case where there are arrays of values of ξ_2 for each ξ_1

In conclusion, we may perhaps recall that the calculation of all regression coefficients is a problem in the theory of least squares, and consequently their accuracy depends only on the assumptions necessary for this theory. We had, for example, the normal variate

$$w_{2,1} = \sqrt{c_{11\cdot0}}(b_{21}-\beta_{21}) = \Sigma\lambda_1\xi_{2\cdot1},$$

where

$$\lambda_1 = x_{1\cdot0}/\sqrt{\Sigma x_{1\cdot0}^2},$$

whether the values of ξ_1 are fixed or not, provided $\xi_{2\cdot1}$ is normal. It is clear, however, that $w_{2,1}$ is approximately normal whatever the distribution of $\xi_{2\cdot1}$, provided no values of $\xi_{2\cdot1}$ (or λ_1) are abnormal, since a linear function of n variates goes to normality as n becomes large.

Hence it is likely that

$$\sqrt{c_{11\cdot0}}\,|\,b_{21}-\beta_{21}\,| < 2\sigma_{2\cdot1},$$

and for a reasonably sized sample, so that $\sqrt{c_{11\cdot0}}$ is reasonably large, we may conclude that the discrepancy between b_{21} and β_{21} is likely to be small. But the exact test of significance of $b_{21} - \beta_{21}$ must, for any finite sample, involve the assumption that $\xi_{2\cdot 1}$ is normal.

I should like to express my thanks to Dr J Wishart, to whom I owe the suggestion that a more systematic and complete derivation might be possible of the various distributions and tests associated with regression than has perhaps hitherto been given. I am also indebted to Dr Wishart for advice and criticism while this paper was being prepared.

REFERENCES TO LITERATURE

1. WISHART, J., 1928. *Biometrika*, vol. xx, A, pp. 32–52.
2. WISHART, J., and BARTLETT, M. S , 1933 *Proc Camb Phil Soc*, vol xxix, pp. 260–270.
3. FISHER, R. A., 1915. *Biometrika*, vol. x, pp 507-521.
4. PEARSON, K., JEFFERY, G. B., and ELDERTON, E. M , 1929 *Biometrika*, vol. xxi, pp 164–193.
5. ROMANOVSKY, V., 1926. *Bulletin de l'Academie des Sciences de l'U R.S S.*, p. 646

   PEARSON, K., 1926 *Proc. Roy. Soc.*, A, vol. cxii, pp 1–14.
6. WILKS, S. S., 1932. *Annals Math Statistics*, pp. 196–203.
7. FISHER, R. A., 1928. *Proc. Roy. Soc*, A, vol. cxxi, pp. 654–673
8. WILKS, S. S , 1932. *Biometrika*, vol xxiv, pp 471–494
9. YULE, G. U., 1907. *Proc. Roy. Soc.*, A, vol. lxxix, pp. 182–193.
10. INGHAM, A E , 1933. *Proc. Camb. Phil Soc*, vol. xxix, pp. 271–276.
11. FISHER, R. A., 1925. *Metron*, vol. v, No 3, pp. 90–104.
12. WISHART, J , and BARTLETT, M. S , 1932 *Proc. Camb. Phil. Soc*, vol. xxviii, pp. 455–459.
13. FISHER, R. A , 1924. *Metron*, vol iii, Nos. 3 4, pp. 329–333
14. FISHER, R. A., 1930. *Statistical Methods for Research Workers* (3rd ed.).
15. FISHER, R. A., 1924. *Phil. Trans.*, B, vol ccxiii, pp. 89–142
16. FISHER, R. A., 1922. *Journ. Roy. Stat Soc*, vol. lxxxv, pp. 597–612

(*Issued separately September* 29, 1933.)

XXI.—Geology and Petrology of the Dolerites of Spitsbergen.

By **G. W. Tyrrell**, D.Sc., F.G.S. (University of Glasgow), and **K. S. Sandford**, M.A., Ph.D. (University of Oxford). (With a Plate and Nine Text-figs.)

(MS. received May 20, 1932. Revised MS received May 20, 1933
Read February 1, 1932)

CONTENTS.

PAGE
I. INTRODUCTION 285
- 1. General 285
- 2. Synopsis of the Geological History of Spitsbergen . 285
- 3. History of Investigation 287

II. GEOLOGY OF THE DOLERITES 289
- 1 Geographical Distribution 289
- 2. Field Observations . . 290
  - (i) Hinlopen Strait and North-east Land 290
  - (ii) Stor Fjord Region . . 292
    - (1) Region South of Negri Glacier . . 293
      - (*a*) Cape Dufferin . 293
      - (*b*) Agardh Bay to the Negri Glacier 293
    - (2) Region North-east of Negri Glacier 294
    - (3) Barents Island 295
    - (4) Edge Island 297
  - (iii) Ice Fjord and Bell Sound . 299
    - (1) Region between Ekman Bay and Dickson Bay 299
    - (2) Region between Dickson Bay and Klaas Billen Bay . 300
    - (3) Northern Side of Sassen Bay 300
    - (4) Southern Side of Sassen Bay 301
    - (5) Region about Bell Sound, stretching Northward to Ice Fjord 301
  - (iv) Wyches Islands (King Charles Land) . . 302
  - (v) Franz Josef Land 302

III PETROLOGY . . 303
- 1 History of Investigation . . 303
- 2 Petrography . 304
- 3 Chemistry 308
- 4. Comparisons with Other Regions of Mesozoic Igneous Activity . . 313

IV. AGE AND GEOLOGICAL RELATIONS 314
- 1 Age of Dolerite Intrusions . . . 314
- 2. Morphology and Modes of Intrusion . . . 315
- 3. Tectonic Relations 316

V. SUMMARY 317

VI REFERENCES TO LITERATURE 319

I. INTRODUCTION.

1. *General*

WITHIN the group of islands which together comprise Spitsbergen outcrops of dolerite may be traced over an area of about 50,000 square miles. The area is doubled if the neighbouring dolerites and basalts of Franz Josef Land are included. Even so, this is but a part of a network widely spread over the north-western part of the Euro-Asiatic continental platform, and over adjoining circumpolar lands. The sill-swarms of Spitsbergen alone rival in extent any of approximately similar age in other parts of the world, such as those of South Africa, South America, India, Antarctica, or North Britain. They merit a great deal more attention than it has yet been possible for geologists to devote to them.

On the basis of their work in Spitsbergen during the field seasons of 1919, 1920, and 1924, the authors consider that, along with previously published information, there is now a sufficiency of data on which to found a general account of the geology and petrology of the dolerites. Dr Tyrrell traversed all but four of the numerous outcrops of dolerite in the Stor Fjord region, and in addition visited the Anser I , Gips Hook, and C Diabas exposures within Ice Fjord. Dr Sandford explored the great Alk Range intrusion and other masses in the central parts of Hinlopen Strait and Wahlenberg Bay; and his colleagues collected information regarding the exposures on the southern coasts of North-east Land. Some of the intrusions were mapped from air photographs or from study at a distance, but only after their continuity with visited exposures had been satisfactorily demonstrated. For the sake of completeness earlier geological and petrological work on Spitsbergen dolerites has been summarised and included in the text. Where such derived information is included it is always associated with the name of the investigator concerned, the remaining data being original.

2. *Synopsis of the Geological History of Spitsbergen* *

It seems desirable to introduce at this stage a brief history of geological events in Spitsbergen to serve as a frame of reference for the ensuing discussion of the dolerites. Spitsbergen may be regarded as the geological outpost of Europe in Arctic regions Through its wide eastern and central plateaus, consisting of comparatively undisturbed Late Palæozoic, Mesozoic, and Kainozoic formations, it is connected with the Russian Platform; and through its great western fold-mountain zone it is connected with the long Caledonian belt of crustal weakness which runs through Western Scandinavia into Scotland, and in the other direction into the north of Greenland. Its geological formations and structures thus have affinities with both Eastern and Western Europe.

---

* An excellent modern summary of the geological structure and history of Spitsbergen is to be found in H Knothe (49, pp. 30-48). The numbers within brackets refer to the bibliography at the end of this paper. Other valuable summaries are given by O. Nordenskiöld (30), G. de Geer (27), and A G. Nathorst (19)

The oldest known formation in Spitsbergen is the Hecla Hoek System, consisting of dolomites, slates, and quartzites, of Early Palæozoic (probably Cambrian to Silurian) age.* Certain metamorphic rocks, schists, and gneisses which, along with the Hecla Hoek, are cut by later granites, were formerly regarded as Archæan, but are now thought to represent mainly Hecla Hoek rocks which have been highly metamorphosed by pressure aided by magmatic injection during the Caledonian orogeny that ensued towards the end of Silurian times. The folding of this period, which took place along N.N.W. to S S.E. lines mainly in the west of Spitsbergen, becomes less intense towards the east. On the worn-down surfaces of this ancient folded basement were laid down almost undisturbed strata of Downtonian and Devonian ages. These are continental deposits of estuarine and fluviatile origin consisting largely of red and green sandstones, conglomerates, and shales, with a total thickness of about 30,000 feet, and are closely comparable with the Old Red Sandstone of Scotland and Norway.

The massifs thus formed were now cleft by gigantic faults into great graben or horst blocks of widely divergent inclination and elevation. In certain areas the Devonian strata were completely swept away to the new sea-level. The faulting died out in the succeeding period of the Culm, during which strata of typical "Coal Measures" aspect were laid down to a depth of 3000 feet The Carboniferous sea then transgressed widely over the region with the deposition of thick, massive, highly fossiliferous limestones with intercalations of gypsum. There followed limestones and cherts assigned to the Permo-Carboniferous, and finally 750 feet of Permian sandstones and shales.

Mesozoic formations attain a very wide distribution in Spitsbergen, mainly in the south-eastern region. The Trias consists of shales, limestones, and sandstones, with a maximum thickness of about 1800 feet. There is often a considerable stratigraphical gap between the uppermost beds of the Trias and the succeeding Jurassic rocks. The latter are composed of shales, thin limestones, marls, and plant-bearing sandstones, partly of shallow marine, partly of continental origin, with a total thickness approaching 2000 feet. In several localities Cretaceous rocks are found to rest conformably upon the Jurassic, and are of lithologically similar types. In Early Cretaceous times basaltic eruptions occurred over wide areas in Spitsbergen and Franz Josef Land, and the dolerites which are the subject of the present paper undoubtedly represent the underground manifestations of this period of igneous activity. The only tectonic event of any importance in the Mesozoic era was the growth of an extensive downfold or flexure extending north-west to south-east along the line of Ice Fjord. The possible connection between the igneous and tectonic events is discussed later in this paper.

---

* For the geological distribution of this and other stratigraphical groups see the geological map, Plate

Unconformably upon the Cretaceous there rests a series of conglomerates, sandstones, and shales of Kainozoic age (Palæocene to Eocene), which reaches a thickness of 4600 feet. These rocks, together with all older deposits, were then compressed by an eastward thrust of the old Caledonian fold-mountain block in a renewal of orogenic movement, which led to violent folding immediately east of the block, but left the strata unaffected a few miles farther east The final tectonic phase was a reversion to epeirogenic movement during which there were notable block subsidences within the ancient graben and horsts, along much the same lines as their bounding faults. In addition, narrow belts were lowered within the blocks both along the lines already mentioned and across them. A fjord system thus came into existence, within which considerable changes of level took place in Glacial times.

3. *History of Investigation.**

Apparently the first mention of the dolerites and their recognition as igneous rocks were made by the geologists of the French Expedition of 1838–1840 (E. Robert (2)), although an isolated reference to a dark "syenite" at Whales Point (Edge Island) had been made by B. M Keilhau in 1829 (1). Robert described "Une sélagite (syénite hypersthénique) de couleur plus ou moins vert noirâtre, la seule roche crystalline et d'origine supposée ignée que j'ai rencontrée a Bellsound." The most complete early accounts of the Spitsbergen dolerites, however, are given in two memoirs by A. E. Nordenskiöld, both of which have been translated into English. In his "Sketch of the Geology of Spitsbergen," (3), which is a translation of papers of 1862, 1864, and 1866 in the *Transactions of the Royal Swedish Academy of Science*, Nordenskiold devotes a section to "Hyperite," in which he emphasises the vast extent of this rock, its extreme uniformity of appearance and composition, and its columnar structure He particularly notes the very full development of the "Hyperite" in the region of Hinlopen Strait, and around the Stor Fjord, as well as in Bell Sound. Misled apparently by the gravelly mode of decomposition and the stratified aspect of these rocks, he adopted the view that they had been formed by the induration of volcanic ashes or of gravel from destroyed plutonic rocks. He showed the structures and stratigraphical relations of the dolerites in several instructive coloured sections engraved on the geological map which accompanies this memoir.

---

* The purely petrographical papers are dealt with at the beginning of Section III, PETROLOGY, see p. 303.

Nordenskiöld's second memoir, entitled "Utkast til Isfjordens och Belsunds geologi" (1874–75), was translated into English and published in the *Geological Magazine* (7) Many further details of the occurrence of dolerite in the above-mentioned regions are given, together with some instructive sections, and others which are rather puzzling. Although von Drasche (5) had already proved the normal igneous nature of the dolerites (see p. 303), Nordenskiöld stated that "it is scarcely possible to consider these beds purely eruptive formations, although I by no means deny that the veins of diabase, and probably also that which forms isolated rocks without admixture, have broken out in a state of fusion from the interior of the earth," and maintained his earlier view of the origin of the stratified diabase (see above)

The contemporaneous basaltic lavas of Franz Josef Land were described by Koettlitz (9) and by Newton and Teall in 1897 (8) and 1898 (10), and those of the Wyches Islands by A. Hamberg in 1899 (11) from material provided by A. G. Nathorst (13), but the next advance in knowledge of the Spitsbergen dolerites as a whole were made by the geologists of the Swedish and Russian Arc of Meridian Expeditions of 1898–1901. This work gave rise to a series of memoirs of which H. G. Backlund's paper on the diabases of Eastern Spitsbergen (15) is the most informative as regards geological occurrence, modes of intrusion, tectonic associations, and petrology. Backlund emphasises the connection of the dolerites with the formation of the fjords, and shows that the intrusions tend to occur at the points of intersection of the network of structural lines. In the Stor Fjord region their maximum development is toward the north about Helis Sound, where sills and especially dikes are most frequent. In a later paper Backlund (28) indicated the relations between the pendulum observations of Bornsdorff in East Spitsbergen and the occurrence of dolerite. The gravity determinations show a preponderance of mass beneath the northern Oldland, and a deficiency of mass beneath the more southerly region of unfolded Newer Rocks, where subsidence has undoubtedly taken place in Tertiary times, and where the most extensive intrusions of dolerite have occurred. The trend of the zero isogamms is along the tectonic lines of Hinlopen Strait and Stor Fjord.

In the meantime G. de Geer (17, 18, 27) had advanced the view that the dolerite accompanies a great flexure extending diagonally across the Ice Fjord region from N W to S E., which, in the critical region of Sassen Bay, bends to the N.E around the southern end of the northern Oldland towards the south coast of North-east Land.

A. G. Nathorst, in his classical memoir of 1910 (19) on the geology of Spitsbergen, Bear Island, and King Charles Land, published some instructive observations and sketches of the dolerite intrusions of the C. Thordsen area (Ice Fjord), which had been made as far back as 1870.

In 1910 A. Hoel and O. Holtedahl (20) discovered sheets of basaltic lava resting on the supposed Lower Cretaceous peneplain in the region between Wood Bay and Wijde Bay, which they regarded as the effusive equivalents of the dolerites. In 1914 Hoel published further observations upon these lavas and accompanying dikes and discussed their age (22).

Modern work on Spitsbergen dolerites, the final results of which are incorporated in the present paper, dates from the Scottish Spitsbergen Syndicate Expeditions of 1919 and 1920, and the Oxford University Expeditions of 1921, 1923, and 1924 (see 29, 31, 36, 38).

II. GEOLOGY OF THE DOLERITES.

1. *Geographical Distribution.**

Three principal regions of intrusion may be recognised within Spitsbergen itself:—

(i) Hinlopen Strait and North-east Land, including the intrusions on the mainland and North-east Land sides, as well as the islands within the strait, and occurrences on the south and east coasts of North-east Land.

(ii) Stor Fjord, comprising the intrusions on the mainland side and on Edge Island and Barents Island, up to and including those on either side of Helis Sound between Barents Island and the mainland.

(iii) Ice Fjord and Bell Sound, including four main areas: southern part of Andrée Land peninsula; southern part of Dickson Land peninsula (C. Thordsen); Sassen Bay, including Anser I. with the Gipshook peninsula, and the region on the south side of the bay between C Diabas and Mt. Marmier; the western mountains between Ice Fjord and the Bell Sound region.

* See map, Plate.

In addition must be noted the important dolerite and basalt regions of:—

(iv) The Wyches Islands (King Charles Land), south-east of North-east Land; and

(v) Franz Josef Land, the archipelago east-north-east of the Spitsbergen group.

2. *Field Observations.*

(1) *Hinlopen Strait and North-east Land.*

A few irregular narrow dikes occur sporadically within the region of Older Rocks at the northern end of Hinlopen Strait (*e.g.* head of Murchison Bay and near North Cape in North-east Land; Hecla Mount, Treurenberg Bay). The last-named is figured by Nathorst (19, p. 303).

In the central parts of Hinlopen Strait the Alk Range, south of C. Fanshawe, marks the position of what is probably the largest dolerite intrusion in Spitsbergen. The vertical cliff rises from sea-level at its southern end to a height of over 1000 feet in a distance of several miles, and a sounding of about 90 fathoms has been recorded near its foot (G. de Geer (33)). The whole mass underlies *Cyathophyllum* Limestone, and at its northern end, where it is thickest, the rock is a coarse quartz-gabbro at sea-level, and there is no trace of its base.

From the Alk Range cliffs a thick mass of dolerite tapers upward and northward to form C. Fanshawe (fig. 1). In common with two thick sills which run along the southern cliffs of Lomme Bay, it has forced its way through the *Cyathophyllum* Limestone (Middle to Upper Carboniferous). To the south the top of the Alk Range intrusion gradually descends to sea-level, and a sill makes its appearance at the top of the scarp a mile or two inland (section, fig. 1). This can be followed southward for about 20 miles, and then disappears, with all other rocks, beneath the land ice.

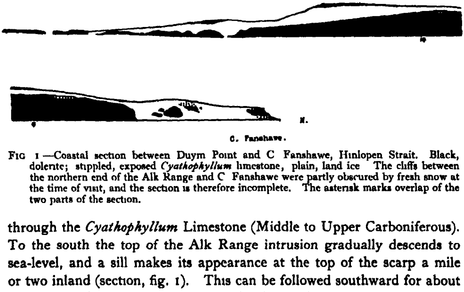


FIG 1 —Coastal section between Duym Point and C Fanshawe, Hinlopen Strait. Black, dolerite; stippled, exposed *Cyathophyllum* limestone, plain, land ice The cliffs between the northern end of the Alk Range and C Fanshawe were partly obscured by fresh snow at the time of visit, and the section is therefore incomplete. The asterisk marks overlap of the two parts of the section.

A line of dolerite islands across Hinlopen Strait (map, Plate) opposite the southern termination of the outcrop of the Alk Range intrusion seems to represent the continuation of the outcrop towards the east. The authors think it probable that the Alk Range intrusion represents the central part of a huge laccolith, possibly of cedar-tree type, which is intersected in the neighbourhood of its vertical feeder communicating with the depths from which the magma rose, and that sills originating from it penetrate the lateral and superincumbent strata.

On the opposite (eastern) side of Hinlopen Strait large masses of dolerite occur at Hyperite Point and the Gylden Islands at the mouth of Wahlenberg Bay. On the north side of Wahlenberg Bay a sill appears in the Permo-Carboniferous strata of Mt. Idun. What is probably the same sill occurs on the south side of the bay in the C. Selander peninsula, and continues east of Palander Bay for a distance of at least 40 miles, maintaining a practically constant level between horizontal strata, although it seems to split into two branches east of Ivory Gull Cliff, where it is 150 feet thick (36, p. 642). In 1924 erratic blocks of dolerite were seen in moraines on the east coast of North-east Land (Isis Point), due east of this sill, and about 50 miles from its last appearance.*

At the Ivory Gull Cliff exposure the underlying limestone is calcined and brecciated, the fragments being partially recemented by deposits of specular iron ore. Below this level the limestone has been silicified, and its fossils destroyed, preserved as casts, or preserved in perfect condition in silicified form, according to depth below the sill. In all the limestones are more or less altered to a depth of 30 feet below the sill, and at greater distances from its margin they are quite unaltered.

The lower part of the sill itself is likewise altered; the actual margin is marked by an inch-wide band of gypsum (secondary); above, the rock is ochreous, soft, and amorphous, but passes upward into a "white trap," a pseudomorph of the dolerite thoroughly crystalline and fairly hard, to a total thickness of 10 feet, passing into unaltered holocrystalline dolerite of excellent quality and of special character (see Section III, PETROLOGY), 150 feet in thickness. The overlying beds, where exposed from beneath the land ice, are reduced to loose blocks of white, silicified, and burnt limestone The main Wahlenberg Bay sill, on account of its remarkable length, uniform thickness, and petrographical interest, may be regarded as typical of the major sills in Spitsbergen.

---

* In a recent report C T. Dalgety (51, p. 132) has recorded on Great Island "outcrops stated to be dolerite sills," and C Leigh Smith is described as "closely resembling the cliffs at Vega Bay," and as "in all probability dolerite." Specimens, however, are not available, and the observations need confirmation.

South of Wahlenberg Bay sills are almost continuously exposed on both sides of Hinlopen Strait. They are especially prominent near Black Mountain and C. Torell on the eastern side, and C. Eremite and Thumb Point (with dike) on the western side. At Black Mountain and Wahlenberg Island the sills are in Permian strata, at Capes Torell and Eremite in Trias, at Thumb Point in Jurassic, with effusive basalt beneath unbaked Lower Cretaceous sediments (G. de Geer (33)).

Attention may finally be directed to two remarkable clusters of dolerite islands in the southern part of Hinlopen Strait. The first of these is found between Wahlberg Island and C Eremite on the mainland At the last-named locality a thick sill is given off from a mass of dolerite, of which the base is not seen The dolerite islands are surrounded by deep water, and are arranged in the form of an irregular incomplete ring (see map, Plate). Another and much more perfect ring of dolerite islands is formed by the Bastion Islands group south-east of Thumb Point The evidence that these islands may possibly be the relics of vertically-sided intrusions, and not irregularly-sunken sills, has been discussed by Sandford (36, p. 650). It may be suggested here that the igneous form they most closely resemble is the massive ring-dike which has been found to take such a prominent part in Hebridean igneous activity,* and which has now been recognised in other igneous fields.

A series of similar vertical or nearly vertical intrusions, rather than sills, form three main groups of dome-shaped hills between C. Torell and Ulve Bay on the south coast of North-east Land. The meagre information which is available regarding these recently identified dolerite masses will be found in the above-mentioned paper (pp. 649–650).

(ii) *Stor Fjord Region.*

The Stor Fjord region includes the coastal parts of the great bight which opens southward between the eastern side of the mainland of Spitsbergen on the one hand and Barents and Edge Islands on the other. Northward it narrows to Ginevra Bay and Helis Sound between the northern coast of Barents Island and the mainland. Dolerite sills and the accompanying dike-offshoots or feeders are exposed from C Dufferin (south of Agardh Bay) on the Spitsbergen mainland up to Helis Sound, and then far down on the eastern side of Stor Fjord to Negro Point, Edge Island, and the archipelago of the Thousand Islands south of Edge Island. On the mainland side dolerite has not been recorded south of C. Dufferin. The igneous rocks occur as wide-spreading sills with occasional dike-offshoots, including transgressive portions where the sills ascend or descend from one horizon to another. The massive vertical intrusions characteristic of the Hinlopen Strait region are not found in Stor Fjord unless the unexplored exposures in the Helis Sound area represent such masses. Basaltic lavas are likewise unrepresented in the Stor Fjord region.

---

* J. E. Richey, "Tertiary Ring Structures in Britain," *Trans. Geol. Soc. Glasgow*, vol. xix, pt. 1, 1932, pp. 42–140.

The most complete account of the dolerites of the Stor Fjord region is that by H. G Backlund (15), who distinguished three principal developments:—(1) Centering about Ginevra Bay; (2) centering about Freeman Strait; and (3) centering about Whales Point (Edge Island). With the discovery of connecting intrusions (see later) these dolerite areas are not so sharply marked off as Backlund supposed

The Stor Fjord dolerites will be described in clockwise order around the bight, beginning with the southernmost mainland occurrence at C. Dufferin.

(1) *Region South of Negri Glacier.*

(a) *Cape Dufferin.*—A thick dolerite sill forms the foreshore and coastal platform at C. Dufferin at the eastern foot of Rurik's Forberg (see map in G. de Geer (27), or T. H. Hagerman (35)). As the sea floor in the vicinity is hard and rocky, and affords very insecure anchorage, there is little doubt but that the sill extends eastward under the sea. In the neighbourhood of the cape the sill rises at least 100 feet above sea-level. It extends some miles northwards along the coast, but south of C. Dufferin it gradually disappears beneath sea-level Just south of the cape an excellent cliff section 150 feet in height shows dolerite passing under shale, which, apart from bleaching, has suffered remarkably little alteration. Thin irregular protrusions of the igneous rock are injected into the shales from the upper contact. Near the upper contact the dolerite passes into a whitish bleached rock resembling a "white trap," which weathers with a rusty crust. This altered zone extends downwards about a foot, and has a sharp boundary against the fresh interior rock of the sill which weathers with a smooth dark surface. The sill must be more than 100 feet thick, this figure representing the visible thickness The total thickness must be much greater, for in the lowest visible rock there is no sign of fine grain indicating proximity to the lower contact. K. Gripp gives a section (43, p 239) showing a thin tongue of diabase terminating in Jurassic shales on a mountain-scarp to the south of Agardh Bay. This may represent the northward dying-out of the Cape Dufferin sill.

(b) *Agardh Bay to the Negri Glacier.*—A thick sheet of diabase is mentioned by A. E. Nordenskiold (7, p. 211) as occurring in Agardh Mt.

K. Gripp shows an outcrop of dolerite in this region on his map (43, p. 240), and mentions the occurrence of a dolerite sill in his text (p. 238). In an unpublished report of his traverse from Sassen Bay to Agardh Bay in 1919, Mr A. Stevens, of the Geographical Department of Glasgow University, stated that dolerite sills occur in Agardh Mt and in the mountain to the west, and throughout the Agardh Bay and Fulmar Valley areas. He described a dolerite sill up to 300 feet thick at a uniform horizon of about 500 feet above sea-level intruded into the Daonella Shales of the Wallenberg massif, just south of the Rabot Glacier Traced eastward, the sill cut upward through the strata and appeared at a considerably higher horizon. Mr Stevens's observations thus indicate the probability of a linkage between the Stor Fjord and Ice Fjord dolerites along the line of the Sassen Valley. At the same time H. Philipp's observation in the coastal and inland region to the north of Agardh Bay (23, p. 14) that the diabase sheet of the area between Duner Bay and Mohn Bay can be followed far inland, *i e.* westward, serves again to link the coastal dolerite with that found by Stevens in the Wallenberg.

Near Duner Bay, as seen from the sea, a thin, broken, transgressive dolerite sill makes its appearance at about 500 feet above sea-level, rising northward to 1000 feet, when it caps the eastern and part of the northern edge of the plateau between Duner Bay and Mohn Bay, which is called Johannsen Mt. Its scarp ends off on the northern edge of Johannsen Mt at a height of 880 feet, and then bears south-west across the summit of the plateau. The profile of the eastern face of Johannsen Mt. shows the sill capping the northern portion of the plateau, and ending off at a small fault.

On the northern side of the Usher Glacier, which fills a small valley bordering the northern face of Johannsen Mt., rises the plateau of Krogh Mt According to K. Gripp (43, p. 242), a thick dolerite sill occurs at the foot of the southern peak of Krogh Mt. Gripp also records thin dolerite sills in the Hahn Mts. (Teist Mt. of some maps), the extensive plateau between the Hayes and Negri Glaciers. H. Philipp (23, p. 14) also records a terrace of diabase at the foot of Teist Mt., and that the overlying series of shales and bituminous limestones is cut by diabase dikes.

(2) *Region North-east of Negri Glacier.*

There are two main exposures of dolerite, Mts Edlund and Hellwald, in this region. H. G. Backlund (15) is the chief source of the scanty available information. A. E. Nordenskiöld's geological map and sections (3) show two sheets of diabase in Edlund Mt. which cut Triassic strata, one at the base of a cliff and forming a wide foreshore, the other capping the flat-topped mountain. According to Backlund (15, p. 4), the southern part of Edlund Mt. is capped by a thick sheet of diabase exhibiting columnar structure. Lower down, at a height of 250 metres, a thin sill traverses the whole front of the mountain. This sill divides into upper and lower branches, and at the north-eastern extremity of the plateau the branches reunite to form an oblique intrusion. The "Hyperite" peninsula at the foot of Edlund Mt. is regarded by Backlund as probably composed of the remnants of anastomosing dikes in connection with the diabase sheet at the summit.

The Hellwald Mt. plateau, according to Backlund, is capped by a diabase sheet 82 metres thick resting on Upper Jurassic strata. On the flank facing Ginevra Bay four sills of diabase are to be seen, the lowermost at a height of 116 metres above sea-level, the second (27 metres thick) at 255 metres, the third at 392 metres, and the fourth (presumably the above-mentioned summit sheet) at 525 metres On the eastern flank a dike is seen, the prolongation of which forms the second sill. At the base of the mountain a diabase rock rising 43 metres above sea-level projects from talus.

(3) *Barents Island.*

Practically nothing is known of the great mass of dolerite which forms the northernmost promontory of Barents Island on Helis Sound, and the island in the middle of the sound, except that exposures are recorded on the Russian Arc of Meridian Expedition map (Backlund (15)) Another outcrop occurs in the bay to the south-west.

Numerous exposures of dolerite are found in the peninsula which terminates westward in Förväxling Point. The last-named projection is formed by a large dolerite dike which runs nearly east and west. The dike is about 800 feet in width, it hades slightly to the south, and undisturbed Triassic strata are seen close to its northern margin. It can be seen to run inland for about three miles, its southern edge forming a black precipitous cliff falling straight into the sea The rock is a coarse, uniform dolerite, fine-grained towards the margins of the dike, but with little or no internal variation. Backlund (15) also mentions this dike, and states that it is joined by another which extends to the south-east, and appears to connect the Ginevra Bay exposures with those of the Freeman Strait region. But, as is shown below, there is a continuous series of sills connecting the two regions.

Many intrusions of dolerite are to be found along the western coast of Barents Island. On the northern side of Vossen Bay the relations are as shown in the sketch section (fig. 2). The dolerite caps the southern portion of the high plateau, being let down into this position by a fault with a downthrow to the south. From its highest elevation at 800 feet above sea-level it sinks in half a mile in one of its branches to sea-level. Traced southward it appears to split into three branches. The lowermost plunges steeply through the strata with a thin underlying loop (see fig. 2), and it can be seen on the shore cutting obliquely across their edges.

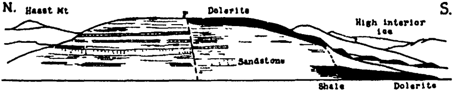


FIG 2—Sketch section along coast, north side of Vossen Bay, Barents Island. Length, about 3 miles

A second branch crosses the floor of the valley to the south and rises gradually until its scarp is seen on the middle slopes of the next valley to the south, where it gradually sinks again to sea-level. The third and highest branch can be seen at the summit of the plateau on the northern side of the valley to the south (not seen in fig. 2), and retains its parallelism with the strata.

On the hillside fringing the coast immediately north of the Duckwitz Glacier the occurrence of the dolerite is as shown in fig. 3. The dolerite

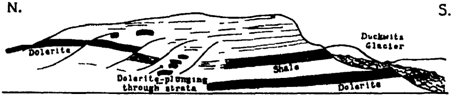


FIG 3—Sketch section along coast north of Duckwitz Glacier, Barents Island Length, about 3 miles

comes in from the north at a level of about 500 feet, and splits into two branches. The upper branch apparently fades out in the strata, but the main branch plunges down through the beds nearly to sea-level, and then slowly rises as a regular scarp conformable to the strata with a dip of about 30° to the E.N.E.

The Anderson Islands, which are now almost wholly covered with ice owing to the extraordinary recent seaward advance of the Duckwitz Glacier (31, p. 240), are known from the work of the Arc of Meridian Expedition to be composed of dolerite (12).

A section in a hillside extending east and west on the south side of the Duckwitz Glacier is shown in fig. 4. There are here apparently two dolerite horizons, the higher of which is brought down to the plateau top by a fault.

Backlund (15) mentions dikes which pass into sills in the valley south of the Duckwitz Glacier, and also a dike with fine columnar structure occurring half-way between the Duckwitz Glacier and Cape Barkham.

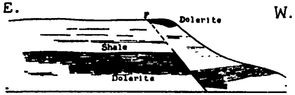


FIG 4 —Section in hillside south of Duckwitz Glacier, Barents Island Length, about 1 mile

At C. Barkham and in the adjacent islands there are wide sheets of dolerite at or about sea-level Vuyl Island, a mile or two west of C. Barkham, consists of a thick dolerite sill which appears to dip towards the north. A specimen from this sill has been selected for chemical analysis (see Section III, PETROGRAPHY).

Krefft Mt., the wide plateau which forms the hinterland of the south-western corner of Barents Island, is barred by two horizons of dolerite, as shown in the sketch section, fig. 5 The lower of these is at sea-level, and is probably the inland and eastern continuation of the C. Barkham mass. The upper dolerite shows remarkable dike-like transgressions across the strata, a feature which is common to all great sill-swarms.

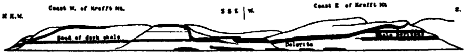


FIG. 5.—Sketch section of coastal scarps at S W end of Barents Island. Length, about 10 miles

(4) *Edge Island*

The northern coast of Edge Island, bordering the narrow Freeman Strait which separates it from Barents Island, is also barred by a dolerite sill (see sketch section, fig. 6). The dolerite occurs on the lip of a gorge N.W. of C. Lee at a height of 315 feet above sea-level. On the eastern

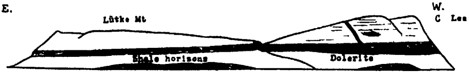


FIG. 6 —Sketch section along Freeman Strait, N.E of C. Lee, Edge Island Length, about 5 miles

side of the gorge it makes a sudden upward turn, cutting almost vertically through the strata, and emerging at the surface of the plateau. This transgressive jump takes place at a point almost exactly opposite a similar jump on the Barents Island side of Freeman Strait (see fig. 5), and therefore probably represents a change of horizon, possibly along a fault, which extends over a wide area. The sill is from 70-80 feet thick, but its base is not seen. About half a mile west of the gorge a dike-like offshoot from the sill strikes obliquely upward through the strata (fig. 6).

In the C Lee district the sill appears to keep to a prominent horizon of blue bituminous shales, although it sometimes behaves irregularly, and makes sudden plunges up and down in the strata These changes of horizon cause remarkably little disturbance of the beds.

At C. Lee a very thick sill of dolerite outcrops at the shore, and a thinner, less regular sill bars the hillside. Backlund (15) gives the thickness of this upper sill as about 10 metres. Opposite the anchorage (fig 7) the thick lower sill throws off branches to north and south, which appear to unite with the upper sill. A little north of the anchorage the upper sill makes a sudden upward jump through the shales and lifts a portion of them to a higher level, thus making a slight anticlinal structure. It then returns to its former horizon, itself forming a sharp pseudo-anticline. At the top of the arch it gives off a thin dike striking obliquely to the top of the hill, where it emerges just under the Russian cairn (fig. 7) Backlund (15) also mentions this dike. To the south the lower sill disappears below sea-level only to emerge again near C Blanck. The upper sill also continues southward, keeping to the horizon of bituminous shales, and again shows remarkable pseudo-anticlines (fig. 7). It comes to an abrupt end in the valley south of the Rosenberg Valley, where it turns up vertically and emerges at the surface as a dike. This disturbance is on the same north to south line as those on the shores of Freeman Strait mentioned above. All the sills appear to die out when traced up the valleys in an easterly direction.

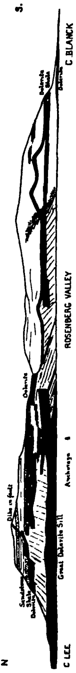


FIG 7 —Sketch section of coastal scarps south of C Lee, Edge Island Length, about 6 miles

The latter conclusion is confirmed by the work of N. L. Falcon (42, pp. 136–37), who has recently studied the extensive dolerite exposures on both sides of Deevie Bay at the south-western corner of Edge Island. The islands at the mouth of Deevie Bay, and the Thousand Isles to the south, are all composed of dolerite. Falcon states that the sills are usually about 100 feet thick, and when followed inland, *i.e.* eastward, transgress upward in the geological succession until they die out H. G. Backlund (15) has fully described and figured the extensive dolerite sills in the Whales Point region on the northern side of Deevie Bay. The exposures here present all the familiar features of sills jumping from horizon to horizon, with oblique connecting dike-like portions and vertical feeding-dikes. A. E. Nordenskiöld's geological map and sections (3) show three "diabase" sills at Whales Point, one forming the base of the cliff and a wide rocky foreshore, the others occurring some way up the cliff, but not capping the plateau. Backlund's plate depicting the mountain at Whales Point essentially confirms this interpretation.

(iii) *Ice Fjord and Bell Sound.*

Comparatively little is known of the dolerite intrusions in this extensive region. Five principal areas may be distinguished: (1) the southern termination of the peninsula known as Andrée Land, between Ekman Bay and Dickson Bay; (2) the southern termination of the Dickson Land peninsula (Middle Hook and C. Thordsen area) between Dickson Bay and Klaas Billen Bay; (3) the northern side of Sassen Bay, including the Anser Islands and the area between Anser Point and Gips Bay; (4) the southern side of Sassen Bay between C Diabas and Mt. Marmier; and (5) the western mountain range between Green Bay and Bell Sound, including also areas on the south side of Bell Sound. The distribution of the dolerites in the four Ice Fjord regions is best shown on the Geological Map of Central Spitsbergen by G. de Geer (18).

(1) *Region between Ekman Bay and Dickson Bay.*

G. de Geer's map (*op. cit.*) shows a single sill of "diabase" on the south-eastern side of the peninsula, which is intrusive into Carboniferous sediments. On the south-western side three large spreads of diabase are mapped on the low-lying rock platform N.W. of C. Woern.

A. E. Nordenskiöld's section (7, p. 68) of strata on the eastern side of Ekman Bay shows a single sheet of diabase resting upon Carboniferous strata, and covered by ice at the summit of the mountain.

G. de Geer's map and section of 1919 (27) shows a diabase sill topping Mt. Capitolium on the eastern side of Ekman Bay, and transgressing to a lower horizon toward the north-east. It cuts the Upper Carboniferous limestone and cherts. A. Hoel (22) mentions a sheet of diabase at a height of 500–600 metres on Mt. Colosseum on the western side of Ekman Bay.

(2) *Region between Dickson Bay and Klaas Billen Bay.*

Numerous outcrops are shown on G. de Geer's map (18) between Mt. Middle Hook on the west and Mt. Skansen on the eastern side of the Dickson Land peninsula. There seems to be two main horizons of dolerite —one near sea-level, and the other high up on the hillsides. They here cut Triassic strata.

A. E. Nordenskiold's map (3) and accompanying coloured sections show a profile indicating two sheets of diabase at C. Thordsen, and a thick sheet capping the Carboniferous mountains on both sides of Skans Bay, which descends southward to sea-level. Nathorst (19, p. 370) states, however, that Nordenskiold's "diabase" in Mt. Skansen is really the dark chert of the Permo-Carboniferous. A section in Nordenskiold's paper of 1876 (7, p. 126) gives a profile of strata between Cape Thordsen and Skans Bay, showing diabase capping the mountain top and also appearing disconnectedly at or near sea-level. A vertical feeder-dike is also shown. Von Drasche's section (5, p 192) of Tschermak's Berg (Middle Hook), on the western side of the peninsula, shows two sheets of diabase conformably interbedded with the Triassic strata.

The most complete account of the dolerites in the C. Thordsen region is that given by Nathorst (19, pp. 369–73), which is illustrated by many instructive sections. All the familiar features of dolerite intrusion are here displayed to perfection. Sills transgress obliquely across the strata in zig-zag fashion, and throw off numerous small apophyses. Nathorst figures a diabase dike 150 metres thick which terminates upward in thick lateral apophyses, and has arched the overlying strata in laccolith fashion.

(3) *Northern Side of Sassen Bay.*

De Geer's map (18) shows a wide spread of dolerite occupying the Anser Islands at the junction of Klaas Billen Bay and Sassen Bay, and practically the whole of the low-lying rock platform between C. Anser and Gips Bay on the mainland.

A landing on the Anser Islands and at Gips Hook by one of the authors (G. W. T.) was made in 1919. The dolerite has the relations shown in fig. 8. The sill is intercalated between the Upper Gypsum and the *Cyathophyllum* Limestone of the Carboniferous. Starting at mid-slope in Gips Valley it makes a monoclinal flexure which brings it down to sea-level at C. Anser, disappearing below sea-level in the Anser

Islands. The upper surfaces of the islands consist of fine-grained rock and are probably near the original upper contact of the sill, which, judging from the contraction fissures, must have dipped about 5° to the west. The sill ends very abruptly to the south-east at the mouth of Gips Valley. It must end equally abruptly to the north, for it cannot be traced far north of C. Anser. The sill doubtless ends off at a N.–S line of disturbance which is indicated on the more recent geological maps (map, Plate) and of which traces can be seen in the coastal cliffs of Klaas Billen Bay. Columnar structure is well developed in the Anser Islands.

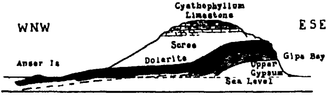


FIG 8.—Sketch section at C. Anser, Sassen Bay. Length, about 3 miles.

(4) *Southern Side of Sassen Bay.*

De Geer's map (18) shows numerous outcrops of diabase between C. Diabas and Mt. Marmier. At C. Diabas and to the west there is a wide spread of the igneous rock at sea-level At Mt. Triabas there are, as the barbarously-composed name indicates, three sills, one of which caps the mountain. Two sills are mapped east of De Geer Valley, but only one on Mt. Marmier. One of the authors (G. W. T.) has sketched the dolerite outcrops in the region about C. Diabas and De Geer Valley as seen from Anser Island (fig. 9). For the possible connection of the Sassen Bay dolerites with those of the Stor Fjord region see p. 294

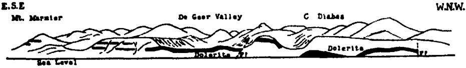


FIG 9.—Dolerite scarps on south shore of Sassen Bay, as seen from C. Anser Length, about 10 miles

(5) *Region about Bell Sound, stretching Northward to Ice Fjord.*

De Geer's map (18) shows a thin sill of diabase conformably interbedded with the steeply inclined Permian rocks on the western side of Green Bay, just south of Ice Fjord. A. E. Nordenskiöld's section of this locality (7, p. 67) shows three sills of diabase interbedded with

Carboniferous strata dipping steeply to the east. The diabase is stated to contain "pretty large feldspar crystals."

Nordenskiöld's geological section of 1867 (3) shows a sheet of diabase intercalated in nearly vertical Carboniferous strata in a section west of the Frithiof Glacier on the northern side of Bell Sound. His geological map shows in addition exposures of diabase at Middle Hook, the peninsula separating Van Mijens Bay and Van Keulen Bay, and also at the head of Recherche Bay on the south side of Bell Sound.

Nordenskiöld's later work (7, p. 67) gives a puzzling section along the northern shore of Van Mijens Bay east of the Frithiof Glacier, which shows an enormous sheet of diabase tilted in conformity with the steep eastern dip of the Carboniferous and Permo-Carboniferous strata, and, in a neighbouring mountain, transgressing almost horizontally across their upturned edges.

(iv) *Wyches Islands (King Charles Land).*

This group of islands lies about 45 miles south-east of C. Mohn at the south-eastern corner of North-east Land. It consists of three main islands—Swedish Foreland, King Charles Island (Jena Island), and Abel Island, besides a number of smaller ones. The principal sources of information on its geology are a paper by Nathorst (13), which is summarised in his general memoir of 1910 (19), and a paper by Hamberg (11). The geology of the islands is very simple; they consist of horizontal or gently inclined Jurassic and Neocomian strata, the latter interbedded with and capped by basaltic lavas. Dikes and sills of dolerite are also found cutting the underlying strata. The preservation of the islands is due to their extensive capping of basaltic lava which has protected the soft Mesozoic strata from severe erosion. True sills occur in the Nordenskiöld and Johnsen Mts. Nathorst gives a section of the former, showing a sill splitting into two thinner branches Dikes occur, at least partly, in connection with faulting. Nathorst states that the eruptions of basalt probably took place at the same time as those in Franz Josef Land, *i.e.* between the end of the Jurassic and the beginning of the Cretaceous.

(v) *Franz Josef Land.*

This archipelago, consisting of 19 large islands and numerous smaller ones, extends over an area of 20,000 square kilometres, and is situated about 100 miles to the E.N.E. of the Spitsbergen archipelago. Its geology (8, 9, 10, 30, 47, 48) is essentially the same as that of the Wyches Islands. It consists of a fragmented plateau land of horizontal Jurassic and Neocomian strata rising 400–600 feet above sea-level, and capped by 500–700 feet of basalt lavas. To the north the basalts thicken and a slight northerly dip finally brings them down to sea-level. Koettlitz (9, p. 633) states that the lavas occur in layers or tiers varying in thickness from 10 to 70 feet. Plant remains from sediments interbedded with the basalts, identified by Nathorst as of Upper Jurassic affinities, led Koettlitz to regard the flows as of Upper Jurassic age.* From the scanty evidence available there is some reason to suspect intrusive sills in the lower part of the sedimentary series.

III. PETROLOGY.

1. *History of Investigation.*

The dolerites of Spitsbergen were first described by the French investigators under E. Robert (2) as "sélagite ou syénite hypersthénique" A. E. Nordenskiold (3) substituted the term *hyperite* for selagite, and stated that the rock consisted of an "even and rather fine-grained, dark-brown mixture of greyish-green labrador, hypersthene, and specular iron or ilmenite" His erroneous notion as to the origin of this rock is mentioned on p. 288.

The microscopic petrography of Spitsbergen dolerites (diabases) and basalts began with R. von Drasche (5), but F. Zirkel (4) had already corrected the mistaken identification of the diabases as hyperite or hypersthenite. Von Drasche described diabases from Tschermak's Berg (Middle Hook) and Norwegian Valley on the C. Thordsen peninsula, the Anser Islands in Sassen Bay, the Frithiof Glacier in Bell Sound, and from C. Staratschin near Green Harbour in Ice Fjord. He also briefly described a diabase from Prince Charles Foreland, which is more probably one of the greenstones contemporaneous with the Hecla Hoek Series of that island (34, p. 465). Von Drasche noted the abundant occurrence of titaniferous iron ore in the diabases, and, because of the crystalline constitution and uniformity of the rocks, dismissed Nordenskiold's view that they represented altered beds of ash and tuff. Nordenskiold, however, still adhered to practically the same view in 1876 (7, p 125), although he had by then adopted the term *diabase* for the Spitsbergen rocks in deference to Zirkel's opinion. He nevertheless thought that *dolerite* was the more correct designation (7, p. 122). A. E Törnebohm (6) described some diabases from Bell Sound, Ice Fjord, and Treurenberg Bay.

---

* G. Horn (47, p. 9), however, states that the age of the highest sediments, with *Gingko polaris* Nathorst, is uncertain. In view of the unreliability of plants as indicators of geological age, of the established Lower Cretaceous age of the basalts and associated sediments in the Wyches Islands, and the occurrence of a plant-bearing Continental Series in the Lower Cretaceous of Spitsbergen itself, the authors venture to believe that the basalts of Franz Josef Land in all probability are of the same age.

No further work appears to have been done on the petrology of Spitsbergen dolerites until the twentieth century, but in the interval Sir J. J. H. Teall (8, 10) had described the Mesozoic basalts of Franz Josef Land, and A. Hamberg (11) the basalts and dolerites of King Charles Land (Wyches Islands)—valuable papers providing microscopical data and chemical analyses which show the affinities of these effusive rocks with their contemporary intrusive representatives in Spitsbergen.

In 1905 D. L. Bryant (14) in his thesis on Spitsbergen petrography described certain basic igneous rocks to which, on account of their fresh appearance and their general habit, he preferred to apply the term dolerite rather than diabase, but he omitted to mention the localities whence his specimens were derived.

In 1907 appeared H. G. Backlund's two most valuable memoirs—the first on the diabases of eastern Spitsbergen (15), describing the material collected on the Russian Arc of Meridian Expedition; the second describing further material from other localities in Spitsbergen, and basalts from King Charles Land and Franz Josef Land (16). Backlund made close mineralogical determinations of the plagioclases and pyroxenes, the latter being found mainly in two different forms, namely, the "enstatite-augite" of Wahl, and basaltic augite or titanaugite. To Backlund also belongs the credit of establishing the occurrence of olivine and quartz side by side in the Spitsbergen dolerites (15, p 25).

In 1911 A. Hoel and O. Holtedahl (20) dealt with the geology of the sheets of basaltic lava which were found lying upon a supposed Cretaceous peneplain between Wood Bay and Wijde Bay in the north of Spitsbergen. V. M. Goldschmidt (21) described the petrography of these rocks. A Hoel (22) offered some new observations on this lava field, with petrological determinations by V. M. Goldschmidt.

In his work on the basaltic rocks of Arctic regions Professor A. Holmes (25) described a dolerite from Sassen Valley and a basalt from Hooker Island, Franz Josef Land, both rocks being analysed by Dr. H. F. Harwood. Finally, further analyses and descriptions of Spitsbergen dolerites have been provided in a work on Spitsbergen soils by E. Blanck, A. Rieser, and A. Mortensen (40).

2. *Petrography.*

This section is based on the study of thirty-nine thin slices cut from specimens obtained from fourteen different localities, ranging from the Anser Islands to C. Lee, Edge Island.

For purposes of petrographical description the dolerites of Spitsbergen may be grouped in four main classes:

A. The normal medium- to fine-grained rocks which constitute the greater part of all the intrusions.

B. The coarse gabbroidal and pegmatitic types which occur in the interior parts of the larger sills and massive intrusions.

C. The dense, often slightly porphyritic, basaltoid, contact rocks.

D. The "white traps"—endomorphic modifications due to intrusion into carbonaceous and calcareous rocks.

A. *Normal Medium- to Fine-Grained Types.*—The following description is based mainly on thin slices of the analysed rock from Vuyl Island (Barents Island), but it is fully representative of the type which makes up the greater part of the sills and other intrusions. These are grey, medium- to fine-grained granular rocks, in which shining cleavage faces of plagioclase laths stand out prominently against a dark grey background of pyroxenes and iron ores. In thin section the rocks are seen to be built of a plexus of laths of plagioclase in ophitic relations with pyroxenes, interspersed with skeletal crystals and rods of titaniferous iron ore, and involving patches of imperfectly crystallised mesostasial matter. Very occasionally a group of large plagioclase laths offers a kind of phenocrystic development, but typically the rocks are non-porphyritic.

From extinction measurements on Albite and Carlsbad twinned crystals the larger feldspars appear to be bytownites of composition An 71–74, whereas the smaller laths which are in ophitic relations with the pyroxenes are more sodic, ranging about An 55 (acid labradorite) The pyroxene is a pale, purplish-brown variety occurring in irregular plates. It is of variable but on the whole rather low optic axial angle, and thus approximates to the enstatite-augite or pigeonite series of monoclinic pyroxenes. Many crystals show the peculiar radial structure described by Backlund, indicated by a wave-like extinction passing over the apparently homogeneous plate in the form of an arm rotating round a point, just as in a spherulite. Titaniferous magnetite, or perhaps ilmenite, in the form of skeletal aggregates involved with the pyroxenes in mutual growths, or in long rods which penetrate crystals and mesostasial areas alike, occurs in comparative abundance. A few irregular areas of red, non-pleochroic serpentine enclosed in the pyroxenes attest the former presence of olivine. The mesostasial matter is collected in rather large irregular areas, and is comparatively sparse in the intervening spaces. It consists of tiny plagioclase laths, small crystals of quartz, much chloritic material, grains of pyroxene altered with the formation of scraps of brown hornblende and biotite, many needles of apatite, and irregular fragments of iron ore, all imbedded in an ill-defined altered feldspathic base which is probably alkali-feldspar. In some rocks (*e.g.* C. Dufferin and Mt. Johannsen) the base is definitely glassy.

B. *Coarse Gabbroidal and Pegmatitic Varieties.*—The analysed rock from the Alk Range is coarser than that from Vuyl Island, and exhibits fresh olivine in conjunction with much micropegmatite. It is the first term in a series of rocks of increasingly coarse grain, which culminates in a pegmatitic type rich in quartz, hornblende, and biotite. The analysed rock is not itself pegmatite, but may be regarded as a transitional variety between the common type described above and the pegmatitic types. The coarse pegmatitic varieties, however, are abundantly represented in the Alk Range, and the following description is mainly drawn from the study of thin sections from this great exposure.

The rocks are medium- to coarse-grained, of greyish colour, and gabbroidal appearance. In thin section the chief differences from the above-described rocks consist in their coarser grain, the greater abundance of quartz in the form of micropegmatite, and the appearance of a second, nearly colourless, pyroxene, which, while often in optical continuity with the common pyroxene, is virtually uniaxial, and no doubt represents the extreme iron- and magnesium-rich end-member of the diopside-pigeonite series. The appearance of this pyroxene in rocks, which must be regarded as later differentiates of the dolerite magma, accords with Barth's observation * that the early-formed phenocrystic pyroxenes of basaltic (and doleritic) magmas are enriched in the lime or wollastonite molecule, whilst the late-formed groundmass pyroxenes are enriched in the iron (hypersthene) and magnesium (enstatite) molecules. A similar variation of composition has been noted in the pyroxenes of the Karroo dolerites of South Africa.†

The uniaxial augite differs from the common type of augite in its slightly higher refractive index, its relative freedom from cleavage, and in its liability to alteration. In the most acid rocks of all a diallagic pyroxene, in which a central parting and twinning parallel to 100 and a close basal striation produce a characteristic "herring-bone" structure, comes in abundantly. While differing in habit this pyroxene does not seem to differ notably in chemical composition and optical characters from the common pyroxene of the dolerites.

Large irregular crystals of fresh olivine enveloped in red serpentine occur in the comparatively fine-grained analysed rock from the Alk

---

* T. F. W. Barth, "Crystallisation of Pyroxenes from Basalts," *Amer. Mineralogist*, vol xvi, 1931, pp 195–208.

† R. A. Daly and T. F. W. Barth, *Geol. Mag.*, vol. lxvii, 1930, p. 105.

Range, but these completely disappear in the coarser gabbroidal and pegmatitic types. In this rock there is no clear evidence of the transformation of olivine into pyroxene, although in the common fine-grained types the serpentinised remains of olivine are almost invariably enclosed in the pyroxene plates. The olivine in the Alk Range rocks co-exists with abundant quartz, and we therefore have a perfect example of Bowen's reaction principle in operation. As Bowen shows, it is not necessary to prove the actual transformation of olivine into pyroxene in order to establish the reaction relation between them * Both minerals may appear as perfectly formed crystals, the magmatic liquid dissolving olivine at the appropriate stage and precipitating further pyroxene. The enrichment of the later pyroxenes in iron and magnesium may then be ascribed to the renewed availability of hypersthene and enstatite molecules in the magmatic liquid owing to the re-solution of olivine, and a relative deficiency of the lime molecule because of the early formation of diopsidic pyroxene and lime-soda feldspars. Some of the basic feldspars preceded the olivine in crystallisation, as is shown by the enclosure of plagioclase laths in one of the largest olivine crystals of the Alk Range rock.

The gabbroidal and pegmatitic types of the Alk Range contain large amounts of quartz in micropegmatitic intergrowth with alkali-feldspar. The micropegmatite occupies large interspaces between the feldspars and pyroxenes and is associated with chlorite, biotite, and brown, pale green, deep green, and bluish varieties of amphibole. Biotite is especially well developed in contact with skeletal masses of iron ore, but it is also formed, in conjunction with chlorite and the pale green variety of amphibole, by the alteration of pyroxene. A reddish biotite, and the deeply coloured green and blue varieties of amphibole, occur within and on the margins of the areas of micropegmatite, especially where these come into contact with pyroxenes and iron ores. It is well known that the late acid differentiates from quartz-dolerite magmas, as shown by the composition of the acid veins which are found traversing many of these rocks, are decidedly sodic.† It may be inferred that the deeply coloured green and blue amphiboles are the products of reaction between the sodic residual magmas and the earlier pyroxenes, whilst the red biotite is formed by similar reactions when the titaniferous iron ores are also present.

C. *Marginal Varieties*.—Towards the contacts of the sills and dikes the grain becomes progressively finer, the ophitic texture is gradually lost and is replaced by intergranular and finely intersertal texture, the rocks become sparsely porphyritic with feldspar, olivine disappears, and titaniferous iron ore becomes evenly dispersed throughout the rock in the form of thin skeletal rods. The non-appearance of olivine is due to rapid cooling at the margins, and shows, as also do the norms (Table II), that olivine is not a normal constituent of these rocks, and only appears as a result of slow cooling and the partial inhibition of reaction.

---

* N. L. Bowen, *The Evolution of Igneous Rocks*, 1928, p 58.

† J. D. Falconer, "The Igneous Geology of the Bathgate and Linlithgow Hills," *Trans. Roy. Soc Edin.*, vol. xlv, pt i, 1905–6, p. 137.

D. *The "White-trap" Modification*—The sills which penetrate the Triassic and Jurassic rocks show a decided preference for horizons of carbonaceous rocks, a feature which also characterises the Karroo sill-swarm of South Africa (p. 314). Dr A. L. du Toit has even suggested that the gases given off by the carbonaceous shales and coals have tended to facilitate the penetration of the molten rock along such horizons, acting as high-pressure wedges in front of the advancing magma. At any rate, contact with the carbonaceous materials has a very marked endo-metamorphic effect on the dolerite, reducing the iron in the magma to a soluble condition, causing the replacement of the ferro-magnesian minerals, and ultimately the feldspars, by carbonates of calcium, magnesium, and iron, and thus producing the well-known bleached rock known as "white-trap" Such bleached and softened rocks have been formed on the margins of many of the Spitsbergen sills, as at C. Dufferin (p. 293), C. Lee, and Wahlenberg Bay, North-east Land (p. 291). Backlund has thoroughly discussed a white-trap from the sill at Mt Edlund from the chemical point of view (15, pp. 28–9). The chemistry of ordinary weathering in Spitsbergen diabases has been described and discussed by E. Blanck, A. Rieser, and H. Mortensen (40, pp. 675–79).

3. *Chemistry.*

New analyses have been made for this work by Mr F. Herdsman of the normal fine-grained dolerite of Vuyl Island (Barents Island), and of the medium-grained dolerite of the Alk Range, Hinlopen Strait. In Table I, together with the above-named, there are collected all the "superior" analyses of Spitsbergen dolerites which we have been able to find. There are several old and "inferior" analyses in the literature which it is now expedient to consign to oblivion. For comparison with the intrusive rocks we have also assembled "superior" analyses of the corresponding lavas of undoubted Lower Cretaceous age from the Wyches Islands and those of less certain age from Franz Josef Land, as well as an analysis of the basaltic lava which rests on the Wood Bay peneplain. The norms of these analyses are assembled in Table II, together with the symbols of the analyses in the C.I.P.W.* method of indexing.

* Cross, Iddings, Pirsson, Washington.

Of the eight utilisable analyses of Spitsbergen dolerites (Table I, 1–8) the first three are the most recent and the most reliable, and Dr H. F. Harwood's analysis of the Sassen Valley rock is by far the most complete. Backlund's analyses (Table I, 5–8), which were made twenty-five years ago, are inferior to the above in that the alumina is almost certainly too high, whilst ferrous iron and titania are almost certainly too low. In the calculation of the norms (Table II, 5–8) this results in too high anorthite giving an exaggerated basicity to the plagioclase feldspars, and in two examples even leaving a residue of free alumina. Also, since all the lime is used up in forming anorthite from excess alumina, no diopside can be calculated from three of the analyses. Notwithstanding these deficiencies the Backlund analyses broadly confirm the results arrived at mainly from consideration of the three recent analyses. The analysis from Blanck, Rieser, and Mortensen's work (Table I, 4) is low in magnesia, soda, and titania compared with the first three analyses, and is reported to contain the extraordinary amount of 4·10 per cent. of Mn_3O_4. The authors state (translation): "The rock is a normally developed, rather fresh diabase of black colour, on the joint-planes of which occurs a thin coating of iron oxides." From the context it seems probable that the specimen was taken immediately below a sandy decomposed surface material which, from the analyses given, is progressively impoverished in manganese oxide towards the surface. Hence ferrous and manganiferous solutions may have filtered through the joint-planes of the underlying sound rock, and have deposited the oxides on the sides of the fissures Apart from this, Blanck, Rieser, and Mortensen's analysis agrees closely enough with the first three to be used in conjunction with them in discussion of the chemistry of Spitsbergen dolerites. It is noteworthy that two of the dolerites from Barents Island (Table I, 2, 7) are comparatively rich in MnO.

From consideration of the analyses and norms in Tables I and II it is clear that Nos. 1 and 2 go together as only slightly over-saturated rocks, and Nos. 3 and 4 as definitely over-saturated rocks with from 7 to 10 per cent. of free silica. Nos. 3 and 4 are also richer in the salic molecules (nearly 60 per cent.) as against the little more than 50 per cent. of Nos 1 and 2. It is reasonable, therefore, to suppose that analyses 3 and 4 were executed upon rocks (both from Sassen Bay region) somewhat more advanced in differentiation than Nos. 1 and 2, which are from eastern localities. From the considerations advanced on p. 306 it might therefore be expected that the hypersthene-enstatite molecules would be relatively more abundant with respect to the diopside molecule in Nos 3 and 4 than in Nos. 1 and 2. Actually only the norm of No. 4 shows this feature, and Dr Harwood's analysis, probably the best of the series, fails to show it. The fact illustrates, perhaps, the limitations of the norm method of analysing the physico-chemical conditions of differentiation rather than that the principle of the enrichment of the latter differentiates in the ferro-magnesian pyroxene molecules is wrong.

Disregarding the MgO and TiO_2 of analysis 4, Table I, which are almost certainly too low, also disregarding the Mn_3O_4 value, we may

TABLE I.—ANALYSES OF SPITSBERGEN DOLERITES AND BASALTS.

	1	2	3	4	5.	6	7	8	9.	10	11	12.
SiO_2	49·55	47·30	50·17	49·93	47·64	47·59	49·16	51·25	49·12	47·28	48·89	49·97
Al_2O_3	14·02	14·79	13·66	15·09	19·98	18·73	20·52	19·63	13·82	13·24	14·30	14·25
Fe_2O_3	1·65	2·48	5·40	4·09	4·83	5·12	6·89	5·07	6·76	4·44	4·71	2·74
FeO	11·96	11·81	6·59	10·11	7·26	7·65	7·31	7·24	12·53	10·50	7·60	9·20
MgO	5·46	5·42	5·31	2·52	7·14	5·92	5·64	5·23	3·19	5·94	6·11	8·96
CaO	10·41	9·50	9·26	8·30	9·62	8·86	6·06	8·61	8·70	11·04	11·04	7·81
Na_2O	2·21	2·06	2·06	1·59	1·23	2·11	1·30	1·12	2·49	2·62	2·31	3·18
K_2O	1·03	0·75	1·00	1·34	1·03	1·00	1·01	0·36	1·26	0·31	0·47	0·92
H_2O +	0·70	1·20	1·28	0·69	0·12	1·21	0·34	0·81	0·78	2·00	0·72	0·28
H_2O −	0·20	0·60	1·02	0·70							1·57	0·16
TiO_2	2·54	3·10	2·93	1·36	1·20	1·19	0·52	0·37	0·80	1·48	2·29	1·42
P_2O_5	0·10	0·11	0·28			..	..				0·24	0·62
MnO	0·25	0·73	0·15	4·10*		tr	0·78	tr.	0·08	0·40	0·18	0·17
CO_2	nil	nil	0·44	0·21	0·28	0·57	0·36				tr	tr.
(Ni, Co)O	nil	nil	nil		.	.	..	.	.		tr	0·06
S	nil	nil	0·24	.	..	..		.			tr.	..
Rest		.	0·13			..					0·08	0·02
	100·08	99·85	99·92	100·03	100·33	99·95	99·89	99·69	99·53	99·25	100·51	99·76
Sp Gr	.	..	2·86	..	2·98	3·02	2·96	.		..	2·91	..

1. Quartz-olivine-dolerite, Alk Range, Hinlopen Strait, Spitsbergen Analyst, F Herdsman, A R.S M New analysis
2. Quartz-olivine-dolerite, Vuyl Island, Barents Island, Spitsbergen. Analyst, F. Herdsman. New analysis
3. Dolerite, Sassendal, Spitsbergen Analyst, H F Harwood. Quoted from A Holmes (25, p 208) Summation incorrectly stated as 100·02 in original, and modified as follows: "less O for S, 0·09, 99·83" "Rest" includes V_2O_3, 0·05, BaO, 0·03; SrO, 0·05, Cr_2O_3, none, Cl, tr
4. Diabase, C Diabas, Sassen Bay, Spitsbergen Quoted from Blanck, Rieser, and Mortensen (40, p 676) *Mn_3O_4, 4·10; organic substances, tr.
5. Diabase, Förväxlings (Changing) Point, Barents Island, Spitsbergen Analyst, H G. Backlund Quoted from Backlund (15, p. 21). Summation incorrectly stated as 100·43 in original.
6. Diabase, foot of Mt Hellwald, Stor Fjord, Spitsbergen Analyst and reference as in 5.
7. Diabase, dike near Duckwitz Glacier, Barents Island, Spitsbergen. Analyst and reference as in 5
8. Diabase, Anderson Islands, Barents Island, Spitsbergen. Analyst and reference as in 5
9. Basalt lava, C Weissenfels, King Charles Land (Wyches Islands). Quoted from A Hamberg (11, p 523)
10. Basalt, C. Flora, Franz Josef Land. Analyst, J. J. H. Teall Quoted from E T. Newton and J. J. H Teall (10, p 647).
11. Basalt, Hooker Island, Frans Josef Land. Analyst, H. F. Harwood. Quoted from A. Holmes (25, p. 210). "Rest" includes: Cr_2O_3, 0·02; V_2O_3, 0·06; BaO, tr.; SrO, none; Cl, none.
12. Basalt, C. Augustus Victoria, Wood Bay, Spitsbergen. Analyst, Dittrich. Quoted from V. M. Goldschmidt (21) and from A. Holmes (25, p 206). The figures recorded are the mean of two closely accordant analyses "Rest" includes: Cr_2O_3, 0·02.

TABLE II.—NORMS OF SPITSBERGEN ANALYSES.

	1	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
Quartz	0·7	1·0	10·5	7·3	2·6	1·9	13·3	13·1	3·4	.	4·1	..
Orthoclase	6·1	4·4	6·1	7·8	6·1	6·1	6·1	2·2	7·8	1·7	2·8	5·9
Albite	18·3	17·3	17·3	13·6	10·5	17·8	11·0	9·4	21·0	22·0	19·4	27·0
Anorthite	25·3	28·9	25·0	30·0	45·9	38·4	27·8	42·8	22·5	23·4	27·2	21·5
Corundum		..					6·5	1·6		..	..	.
Diopside	21·3	14·6	13·2	9·5	.	1·6	.	..	17·4	25·8	21·1	11·1
Hypersthene	19·8	22·0	10·2	21·3	25·2	21·9	22·4	21·6	15·3	12·2	11·9	15·9
Olivine			..		.	.		.	..	2·6		10·0
Magnetite	2·3	3·7	7·9	6·0	6·9	7·4	10·0	7·4	10·0	6·5	6·7	4·0
Ilmenite	4·9	5·9	5·5	2·6	2·3	2·3	0·9	0·8	1·5	2·9	4·4	2·8
Apatite	0·3	0·3	0·7	..	.		.	.	.	.	0·3	1·3
Calcite	..	.	1·0	..	0·7	1·3	0·8	.	.		..	..
Silica number	12	17	175	122	44	32	223	219	57	−16	69	−62

The above norms are arranged in the same order as the corresponding analyses in Table I.

SYMBOLS IN C I P W SYSTEM OF INDEXING

1	III	5	″4	4
2	III	5	4	4
3	(II) III	4″	″4	4
4	(II) III	(4) 5	4	(3) 4
5	II (III)	5	4″	(3) 4
6	II″	5	4	4
7	II″	4	4	(3) 4
8	II	4	4 (5)	4″
9	III	5	3 (4)	4
10	III	5	(3) 4	″5
11	III	″5	4	(4) 5
12	III	5	3	4″

compute the average composition of Spitsbergen dolerites from the first four analyses of Table I. Backlund's analyses are left out of account for the reasons stated above (p. 309). The average obtained is shown in Table III, 1.

Turning now to the four analyses of the contemporaneous lavas, their very close similarity to those of the dolerites is to be noted. The chief differences are in MgO, which is higher in the lavas, and in the proportion of soda to potash, which is again higher in the lavas, total alkalis, however, remaining much the same. In a word, the lavas are more sodic and more femic than the dolerites, a feature which is clearly reflected in the norms and in the index symbols The above chemical differences are characteristic of the relations of basaltic lavas and the corresponding dolerites and gabbros in many parts of the world. They probably indicate that the basalts represent the parent magmas more closely than the dolerites, the latter having advanced a stage in differentiation.* The analysis given in Table III, 5, represents the average of the four basalt analyses of Table I, 9–12.

Comparison of the average Spitsbergen dolerite and basalt with dolerites and basalts of corresponding Mesozoic or near-Mesozoic ages from many localities, which is carried out in Table III, reveals an amazing degree of similarity in all respects. The Spitsbergen dolerites are there compared with the average Karroo dolerite of South Africa, the Gondwana dolerites of South America (both of Mid-Mesozoic age), and the North British quartz-dolerite suite (Late Palæozoic or possibly Early Mesozoic). The great similarities in mineral and chemical composition

---

* See G. W. Tyrrell, *Principles of Petrology*, 2nd ed., 1930, p. 107

TABLE III.—AVERAGE ANALYSES OF DOLERITES AND BASALTS

	1.	2.	3.	4.	5.	6.	7.	8.
SiO_2	49·2	52·2	51·2	50·2	48·8	50·2	50·4	50·5
Al_2O_3	14·4	14·6	13·2	14·7	13·9	13·6	13·5	13·7
Fe_2O_3	3·4	0·8	5·8	3·2	4·6	3·2	3·7	3·8
FeO	10·1	9·9	7·6	9·3	9·9	9·9	8·5	9·2
MgO	5·4	6·9	5·4	4·7	6·0	5·4	5·6	5·2
CaO	9·4	9·7	9·3	8·5	9·7	9·6	10·3	9·7
Na_2O	2·0	2·2	2·5	2·3	2·7	2·7	2·3	2·8
K_2O	1·0	1·0	1·1	1·1	0·7	0·7	0·8	1·1
H_2O	1·6	1·0	2·1	2·4	1·4	2·1	1·4	2·2
TiO_2	2·9	1·1	2·0	2·6	1·5	1·9	2·9	1·8
P_2O_5	0·2	0·2	0·2	0·3	0·4	0·3	0·4	0·3
MnO	0·4	0·4	0·2	0·2	0·2	0·3	0·2	0·3
	100·0	100·0	100·6	99·5	99·8	99·9	100·0	100·6

1. Average Spitsbergen dolerite, computed from four analyses (Table I, 1–4).
2. Average Karroo dolerite, computed from five analyses. Quoted from R. A. Daly and T. F. W. Barth, *Geol. Mag.*, vol. lxvii, 1930, p. 101.
3. Average Gondwana dolerite of South America, computed from five analyses. Analyses obtained from K. Walther, *Inst. Geol. Perforaç. Montevideo*, Bol. No. 9, 1927. (A study of the nine analyses of intrusive dolerites recorded in this paper shows that they fall into two groups, one which corresponds to the dolerites of South Africa, and another rather more acid group with SiO_2 about 56 per cent.)
4. Average of thirty North British quartz-dolerites—twenty-six from the Whin Sill, four from Midland Valley of Scotland. Compiled from works by J. Smythe, A. Holmes, S. I. Tomkeieff, J. J. H. Teall, J. D. Falconer, G. I. Finlayson, F. Walker, and G. W. Tyrrell. See references given on p. 314.
5. Average Spitsbergen basalt lava, computed from four analyses (Table I, 9–12).
6. Average Deccan basalt lava, computed from sixteen analyses, of which eleven are given in H. S. Washington's "Deccan Traps and Other Plateau Basalts," *Bull. Geol. Soc. Amer.*, vol. xxxiii, 1922, p. 774. The remaining five are unpublished analyses in the possession of one of the authors (G. W. T.).
7. Average Karroo basalt lava, computed from six unpublished analyses from Southern Rhodesia by Mr. E. Golding* and one from Natal published by A. L. Rogers and S. Haughton, *Trans. Roy. Soc. S. Africa*, vol. xi, 1924, p. 245.
8. Average of eight analyses of "non-porphyritic central" or "tholeiitic" magma type of the British Tertiary province. Computed from the first eight analyses of Table II, p. 17, in "The Tertiary and Post-Tertiary Geology of Mull, L. Aline, and Oban," *Mem. Geol. Surv. Scotland*, 1924.

---

* We are greatly indebted to Mr. H. B. Maufe, Director of the Geological Survey of Southern Rhodesia, for transmitting these analyses to one of us, and permitting their use.

between the quartz-dolerites of the world, and especially those of Mesozoic ages, were noted by one of us as early as 1909.[*] The similarities extend to the great suites of Late Pre-Cambrian quartz-dolerites in Scandinavia and Canada. This is obviously a fact of world-wide importance which has to be taken into account in any theory of petrogenesis.

The basaltic lavas associated with the Mesozoic or near-Mesozoic quartz-dolerite intrusions, whilst showing the constant difference of composition noted above, nevertheless amongst themselves exhibit the same astonishing degree of similarity. This point is illustrated in Table III, average analyses Nos. 5–8, in which the average Spitsbergen basalt is compared with the average Deccan basalt of Late Mesozoic or Early Kainozoic age, the average Karroo basalt of Mid-Mesozoic age (probably Early Jurassic), and "non-porphyritic central" or "tholeiitic" magma type of the British Kainozoic province. The chemical type of basalt exhibited here is quite different from the plateau type, as has been pointed out in the Mull Memoir (*op. cit*, *supra*), and in recent papers by Dr W. Q. Kennedy.[†] It represents an over-saturated type of basalt which differentiates towards a granitic or granophyric end-product. The plateau basalts, on the other hand, represent an under-saturated magma type which, by differentiation, leads to distinctly alkaline trachytic end-products. If the Spitsbergen basaltic magma type of Lower Cretaceous age may be regarded as a forerunner of the great basaltic eruptions within the North Atlantic region during the Early Kainozoic, its chemical and mineral similarity to the "tholeiitic" or over-saturated type of the British Kainozoic province may perhaps be regarded as evidence confirmatory of Dr Kennedy's view that the latter, and not the plateau-magma type, was the parent magma of the province.

4. *Comparisons with Other Regions of Mesozoic Activity.*

The petrographic similarities between the doleritic intrusions and the accompanying basaltic lava flows of different Mesozoic or near-Mesozoic occurrences throughout the world have been noted and discussed above. Further comparisons with regard to geological occurrence and modes of intrusion, etc., may now be made The Mesozoic is often thought of as a quiet era of sedimentation and abundance of life, and as relatively free from igneous activity compared with the Palæozoic or the Kainozoic. Yet, in the southern hemisphere especially, igneous action took place in the Mesozoic on an enormous scale. In time this activity ranged from the latest Palæozoic, through the Mesozoic, into the early part of the Kainozoic era, and occurred mainly in the kratogenic or stable blocks of the earth's crust, although orogenic igneous activity also occurred especially in the mountainous backbone of the two Americas (Alaska, Sierra Nevada, Andes) Great floods of basaltic lava broke out in South Africa and in the southern parts of South America, the underground manifestations of which were the vast sill- and dike-swarms of those regions Igneous activity of the same type and the same age also occurred in Peninsular India, Tasmania, and Antarctica. In the Triassic a basaltic flood took place in New Jersey, and was accompanied by a great dike- and sill-swarm which is found in most of the eastern states of the United States. In the very earliest part of the above-mentioned period there was intruded in the North of England and the Midland Valley of Scotland a great swarm of quartz-dolerite dikes and accompanying sills, but without a surface volcanism so far as is known *

---

[*] G. W. Tyrrell, *Geol. Mag.*, vol xlvi, 1909, p 363.

[†] "The Parent Magma of the British Tertiary Province," *Summ Prog. Geol Surv. for* 1930, pt. ii, 1931, pp. 61–73; "Trends of Differentiation in Basaltic Magmas," *Amer. Journ. Sci.*, vol. xxv, 1933, pp. 239–256.

Only two of the above-mentioned dike- and sill-swarms have been studied in close detail—the Karroo Swarm of South Africa,† and the North British Swarm.‡ In each of these regions the familiar features of sill-swarms are as well exemplified as in Spitsbergen—the occasional jump to higher or lower horizons along faults or other types of fracture or flexure, the connection with dikes both as feeders and as offshoots, and the preference for shale or coal horizons as the channels of intrusion. The petrographic resemblances are very striking, as has been shown above.

IV. AGE AND GEOLOGICAL RELATIONS.

1. *Age of the Dolerite Intrusions.*

The dolerite sills and dikes of Spitsbergen cut rocks ranging in age from Hecla Hoek (Hinlopen Strait) to Lower Volgian (= Upper Jurassic) at Cape Dufferin, Stor Fjord. They have never been found cutting

---

* For further details, see G W Tyrrell, *Volcanoes* (Home University Library), 1931, pp 191–194, 207; and *Geol Mag.*, vol. xlvi, 1909, pp 363–364.

† A. L. du Toit, "The Karroo Dolerites of South Africa· A Study in Hypabyssal Injection," *Trans. Geol. Soc S Africa*, vol. xxiii, 1920, pp. 1–42

‡ The North British quartz-dolerites are covered by an extensive literature, amongst which the following may be mentioned· G. W. Tyrrell, "Geology and Petrology of the Intrusions of the Kilsyth-Croy District, Dumbartonshire," *Geol. Mag.*, vol. xlvi, 1909, pp. 299–309, 359–366; "The Geology of the Glasgow District," *Mem. Geol. Surv. Scotland*, 2nd ed , 1925, pp. 153–159, 186–188; A. Holmes, "The Age and Composition of the Whin Sill and the Related Dikes of the North of England," *Min. Mag.*, vol. xxi, 1928, pp. 493–542; J A. Smythe, "A Chemical Study of the Whin Sill," *Trans. Nat. Hist. Soc. Northumberland, Durham, and Newcastle-on-Tyne*, vol. vii, pt. i, 1930, pp. 16–150.

Upper Cretaceous or Tertiary strata, but it is uncertain whether intrusive members of the suite cut Lower Cretaceous rocks in Thumb Point Island. Since the series of intrusions is unitary and homogeneous in respect both of geological relations and petrographical characters it is reasonable to assign it to a Late Jurassic or Early Cretaceous date. Confirmation of this view is to be found in the existence of basaltic lavas which are clearly the effusive equivalents of the dolerite intrusions (see Section III), interbedded with unbaked Lower Cretaceous sediments on Thumb Point Island in Hinlopen Strait (G. de Geer (33)). Identical basalts are also found resting upon a well-marked plane of erosion at Mt. Bifrost, near the head of Lomme Bay, and on both sides of Wood Bay (G. de Geer (27), map; Sandford (35), p. 651). This is the peneplain of marine erosion to which De Geer has assigned an Early Cretaceous age (27), but which may partly coincide with Carboniferous base-levelling. In his discussion of the evidences of age A. Hamberg (11, p. 527) assigns the basalts of Wyches Islands to the period between Jurassic and Early Cretaceous, and believes them to be contemporaneous with the basaltic lavas of Franz Josef Land. A. G. Nathorst comes to the same conclusions (13) The authors believe, therefore, that the dolerites and the accompanying basalt lavas are satisfactorily dated as between Jurassic and Lower Cretaceous.

2. *Morphology and Modes of Intrusion.*

The majority of the dolerite intrusions of Spitsbergen occur in the form of wide-spreading sills, some of which have been followed for more than forty miles (Wahlenberg Bay) Thicknesses which have been measured vary from 35 feet (C. Lee; Backlund (15)) to 300 feet (Wallenberg massif, Stevens, p. 294), but the greater number range from 100 feet to 150 feet. Thin dikes are associated with the sills as offshoots in disturbed zones, and as connections between sills on different horizons

Even more massive intrusions occur in the region about Hinlopen Strait. There is some evidence for the view that these enormous masses are vertically sided and that they have risen as stocks or perhaps ring-dikes through the strata. In one case at least (Alk Range), sills appear to have been thrown off in "cedar-tree" fashion from a thick stocky laccolithic mass which has apparently been intersected in the neighbourhood of its vertical feeder. A dike-like mass, 800 feet thick, with nearly vertical contacts against Triassic strata, occurs at Changing Point (Barents Island), and may represent one of the feeders of the great complex of sills in the northern part of the Stor Fjord region.

In that part of eastern Spitsbergen occupied by Carboniferous and Permian strata (Hinlopen Strait) the sills penetrate for great distances, retain remarkably uniform thicknesses throughout their courses, and keep very closely to given horizons (*e.g.* the *Cyathophyllum* Limestone at Lomme Bay, and the *Productus* Limestone in Wahlenberg Bay). It so happens that the Carboniferous and Permian strata are horizontal over most of the region, but where they are arched into a gentle anticline, as in Palander Bay, the sills conform to the structure. This simple, uniform, non-transgressive type of sill may be attributed to the fact that the intrusions had to penetrate a regular succession of limestones and shales which were probably deeply buried at the time of intrusion. These rocks were hard, rigid, and resistant compared with Triassic and later strata, and were devoid of lenticles of gypsum or of knoll-like facies of corals or shells which might have produced irregular intrusion effects.

The behaviour of the sills when intrusive into Mesozoic sediments is in great contrast to the above. They ramify through the strata, jumping from one horizon to another with great frequency, and sending off dike-like protrusions into breaks and archings of the beds which they themselves have produced. They occasionally come to abrupt terminations, and resume their courses equally abruptly but on different horizons This comparative freedom of movement may be attributed in part to the softness and relative incoherence of the Mesozoic rocks and to the abundance of carbonaceous horizons as compared to the Palæozoic strata; but it is perhaps mainly due to the fact that the Mesozoic rocks lay close to the surface at the time of injection. The dolerite magma was therefore able to displace and ramify through the comparatively thin Mesozoic cover with much greater ease than through the more deeply buried, harder, and more rigid Palæozoic sub-structure.

3. *Tectonic Relations.*

It may be well to point out that, in the present state of knowledge, explanation of observations is largely a matter of speculation. Coincidences may be misinterpreted as significant facts. Certain of these associations from which theory cannot be entirely divorced may be noted.

There is no visible connection between the dolerites and the great post-Devonian, pre-Carboniferous faults. The dolerite is markedly scarce in the exposed Older Rocks, and we are unable to recall a single instance in which it is seen to penetrate the Devonian rocks, although associated lavas were poured out over their peneplaned surface from unidentified and perhaps distant sources.

The dikes and sills are most abundant in the more or less horizontal Newer Rocks. To reach them from below the molten rock must have passed through sunken Older Rocks, but no observations indicate how or where. Some significance may be attached to the concentration of the dikes in three main areas (see p. 289), but these do not mark the hidden prolongations of the pre-Carboniferous faults, so far as can be judged.

Backlund's observations (28) suggest the deep submergence of the Older beneath the Newer Rocks of southern and eastern Spitsbergen, where the greatest amount of dolerite intrusion also took place. The coincidence of the zero isogam with the lines of Hinlopen Strait and Stor Fjord recalls attention to these highly critical regions. They are crossed by faults or fjord-like bays, and at the points of crossing the greatest intrusions of dolerite are to be found. The bays, certainly delineated by faults in certain localities, were indubitably formed after the dolerite had cooled In Stor Fjord dolerite seems to have followed a fault that was already in existence or was then forming It is tempting to think that the intrusions and the later faults are attributable to associated causes—instability of the sunken Older Rocks imparted to the soft Newer Rocks upon them, upward escape of basaltic magma along faults or fissures developed in this mass, and further instability resulting therefrom leading to post-doleritic readjustment of sections of the Newer Rocks and the production of narrow rifts within them. This is admittedly purely speculative, and rests chiefly on the coincidence of outcrops and faults. It is of enhanced significance, perhaps, when the flexure stretching across Spitsbergen is traced as pointed out by G. de Geer (17, 18, 27) and found to embrace the third great area of doleritic outcrop (the inner part of Ice Fjord), and to pass eastward by Sassen Bay towards the other two critical regions.

It is conceivable, in fact, that the flexure traced by De Geer is coeval with the growth of fractures up which the dolerite subsequently made its way, and along which post-doleritic faulting has taken place.

Between the periods of intrusion and of faulting and fjord formation were interposed first epeirogenic movements on a grand scale (29, 34) and secondly orogenic disturbances in West Spitsbergen (27), in which the sills and dikes were involved in common with the rocks into which they had been intruded.

V. SUMMARY.

1. A brief history of geological events in Spitsbergen is given to serve as a frame of reference for the discussion of the dolerites, followed by a history of the investigation of the latter

2. The intrusions are found in three main regions of the Spitsbergen archipelago. Field observations by the authors are given in detail,

together with data from earlier workers Most of the intrusions are sills, but there is some evidence for the occurrence of massive vertical-sided intrusions in the region of Hinlopen Strait. A brief statement of the relations of dolerite intrusions and associated basaltic lavas in the Wyches Islands and Franz Josef Land is also given.

3. Petrological investigation shows that the dolerite is remarkably uniform over the whole of the region Four facies are distinguished: the normal medium- to fine-grained rocks which constitute the great mass of the intrusions; coarse gabbroidal and pegmatitic types; dense, basaltoid contact rocks; and the endomorphic modification known as "white trap" Full petrographical descriptions are given with two new chemical analyses. A mineralogical feature of much interest is the co-existence of fresh olivine with a mesostasis containing abundant quartz. Comparisons of the Spitsbergen suite with Mesozoic and near-Mesozoic dolerites of other regions are instituted

4 The consensus of evidence is that the dolerite intrusions (and the accompanying basaltic lavas) are satisfactorily dated as between late-Jurassic and Lower Cretaceous.

5. The behaviour of the sills intruded into the Carboniferous and Permian on the one hand, and the Mesozoic on the other, is shown to be strongly contrasted. In the former the sills are uniform, non-transgressive, and very widely extended, whereas in the Mesozoic they ramify as a network through the strata, and send off dike-like protrusions into fractured and arched beds. It is believed that this contrast is due to injection under deep and shallow covers respectively

6. While it is only possible at present to offer speculation on the subject, the authors believe that the tectonic relations of the dolerites are explicable as due to instability within a buried fractured basement of Older Rocks which was transmitted to the superincumbent and weaker Newer Rocks, leading to escape of basaltic magma along faults or gaping fissures developed in the whole complex. Further instability following on the extrusion of masses of molten rock led to later readjustments within the complex of sediments and dolerite and the development of a system of rifts which outline the present-day fjords.

Acknowledgments.—The authors acknowledge with gratitude financial assistance from the Carnegie Trustees for the Universities of Scotland toward the cost of illustrating this paper; and also a grant of £12 from the Oxford University Expeditions, through Mr C. S. Elton, toward the general cost of printing the paper in the *Proceedings.*

REFERENCES TO LITERATURE.

Only works bearing directly on the geology and petrology of Spitsbergen are herein cited; all other references are given in full in the text.

1. KEILHAU, B M., 1829. *Reise til nordlige Finmarken, Beeren Eiland og Spidsbergen*, Kristiania.
2. ROBERT, E., 1842. *Voyages en Scandinavie, en Laponie, au Spitsberg et au Feroë, sur la Corvette "la Recherche,"* "Géologie, Minéralogie, et Métallurgie," par E. Robert, liv. iv, p. 87; liv. xxvi, p. 129
3. NORDENSKIÖLD, A. E., 1867. *Sketch of the Geology of Spitsbergen*, Stockholm, 55 pp. (Translated from *Kongl. Svenska Vetenskap. Akad.*, 1862, 1864, 1866)
4. ZIRKEL, F , 1870. "Mikromineralogischen Mittheilungen," *N J. f. Min.*, Jahrg , p. 801.
5. VON DRASCHE, R., 1874 "Petrographisch-geologische Beobachtungen an der West-Kuste Spitzbergens," *Tschermaks Min. Mitth* , pp 181–198, 261–268.
6. TÖRNEBOHM, A. E., 1874–75. "Mikroskopiska bergartestudier, IV. Nagra Grönstenar från Spetsbergen," *Geol. Fören. Förh. Stockholm*, pp 543–549.
7. NORDENSKIÖLD, A. E , 1876. "Sketch of the Geology of Ice Sound and Bell Sound, Spitsbergen," *Geol. Mag.* (II), vol. iii, pp 16–23, 63–75, 118–127, 255–267.
8. NEWTON, E. T., and J. J. H. TEALL, 1897. "Notes on a Collection from Franz Josef Land, made by the Jackson-Harmsworth Expedition, 1894-96," *Quart Journ. Geol. Soc.*, vol. liii, pp 477–519.
9. KOETTLITZ, R., 1898. "Observations on the Geology of Franz Josef Land," *Quart. Journ. Geol. Soc.*, vol. liv, pp. 620–645.
10. NEWTON, E. T., and J J. H. TEALL, 1898. "Additional Notes on Rocks and Fossils from Franz Josef Land," *Quart. Journ. Geol. Soc* , vol. liv, pp 646–652.
11. HAMBERG, A , 1899. "Ueber die Basalte des König-Karl Landes," *Geol. Fören. Forh. Stockholm*, vol. xxi, pp. 509–532.
12. CARLHEIM-GYLLENSKJÖLD, V., 1899. "Travaux de l'expédition suédoise au Spitsberg en 1898 pour la mesure d'un arc de méridien," *Ofvers K. Vet. Akad. Förh. Stockholm*, vol. lvi, pp. 887–890.
13. NATHORST, A. G., 1901. "Bidrag til Kung Karls Lands geologi," *Geol. Fören. Forh. Stockholm*, vol xxiii, pp. 341–378.
14. BRYANT, D. L , 1905 "Beiträge zur Petrographie Spitzbergens," *Inaug. Diss.*, Erlangen, 39 pp.
15. BACKLUND, H. G., 1907. "Les diabases du Spitsberg oriental," *Miss. scient. pour la mesure d'un arc de méridien au Spitsberg. Mission russe*, tom. ii, sect. ix, B, 1, St Petersburg, pp. 1–29.
16. BACKLUND, H. G., 1907. "Ueber einige Diabase aus arktischen Gebiet," *Tscherm. Min. Petr. Mitth.*, vol. xxvi, pp. 357–390.

17. DE GEER, G., 1909. "Some Leading Lines of Dislocation in Spitsbergen," *Geol. Fören. Förh. Stockholm*, vol. xxxi, pp. 199–208.

18. DE GEER, G., 1910. "Guide de l'excursion au Spitsberg," *XIᵉ Congrés. Géol. Internat*, Stockholm, 23 pp.

19. NATHORST, A. G, 1910. "Beiträge zur Geologie des Bären-Insel, Spitzbergens, und des König-Karl-Landes," *Bull. Geol Inst. Upsala*, vol. x, pp. 261–416.

20. HOEL, A., and O. HOLTEDAHL, 1911 "Les nappes de lave, les volcans et les sources thermales dans les environs de la Baie Wood," *Videnskapsselsk. Skr. M.-N. Kl. Kristiania*, No. 8.

21. GOLDSCHMIDT, V. M., 1911. "Petrographische Untersuchung einiger Eruptivgesteine von Spitzbergen," *Videnskapsselsk. Skr. M.-N. Kl. Kristiania*, No 9.

22. HOEL, A, 1914. "Nouvelles observations sur le district volcanique du Spitsberg du nord," *Videnskapsselsk Skr. M-N. Kl. Kristiania*, No. 9, 33 pp.

23. PHILIPP, H., 1914. "Geologische Beobachtungen" in: Ergebnisse der W. Filchnerschen Vorexpedition nach Spitzbergen," *Petermanns Mitth. Erganzungsheft*, No. 179, pp. 13–46.

24. HOEL, A., and O. HOLTEDAHL, 1914. "New Features in the Geology of North-western Spitsbergen," *Amer. Journ Sci*, vol xxxvii, pp. 415–424.

25. HOLMES, A., 1918. "The Basaltic Rocks of the Arctic Region," *Min. Mag.*, vol. xviii, pp. 180–223

26. STENSIÖ, E. A, 1918. "Zur Kenntnis des Devons und des Kulms an der Klaas Billen Bay (Spitzbergen)," *Bull. Geol. Inst. Upsala*, vol. xvi, pp. 65–80

27. DE GEER, G., 1919. "On the Physiographic Evolution of Spitsbergen," *Geografiska Annaler*, vol. ii, pp. 161–192.

28. BACKLUND, H. G., 1921. "Tektonik och Isostasi på Spetsbergen," *Geol. Fören. Förh. Stockholm*, vol. xliii, pp. 397–402.

29. WORDIE, J. M, 1921 "Present-day Conditions in Spitsbergen," *Geogr. Journ*, vol. lxiii, pp. 25–49.

30. NORDENSKIOLD, O., 1921. "Die Nordatlantischen Polarinseln," *Handb. d. Reg. Geol.*, Abt xxiv, Bd. iv, 30 pp.

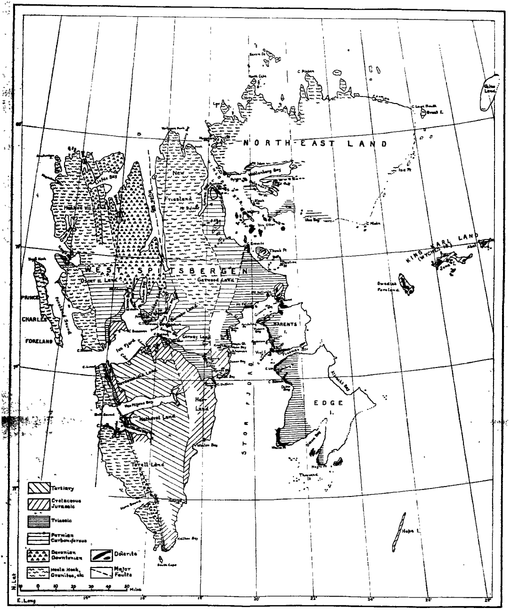
31. TYRRELL, G. W., 1921. "Geographical Observations in Spitsbergen, 1919 and 1920," *Scot. Geogr. Mag.*, vol. xxxvii, pp. 227–242.

32. TYRRELL, G. W., 1922. "The Pre-Devonian Basement Complex of Central Spitsbergen," *Trans. Roy. Soc. Edin.*, vol. liii, pt. i, pp. 209–229.

33. DE GEER, G., 1923. "Topographie et geologie," *Mission suédoise pour la mesure d'un arc de méridien au Spitsberg*, vol. ii, pt. ix, Stockholm.

34. TYRRELL, G. W., 1924 "The Geology of Prince Charles Foreland," *Trans. Roy. Soc. Edin.*, vol. liii, pt. ii, pp. 443–478.

35. HAGERMAN, T. H, 1925 "Stratigraphic and Structural Investigations within South-western Spitsbergen," *Geogr. Annaler*, Heft 3 and 4, Stockholm, pp. 195–221.



G. W. TYRRELL and K. S. SANDFORD. [PLATE

36. SANDFORD, K. S., 1926. "The Geology of North-east Land (Spitsbergen)," *Quart. Journ. Geol. Soc.*, vol. lxxxii, pp. 615–665.

37. QUENSTEDT, W., 1926. "Mollusken aus den Redbay- und Greyhook-Schichten Spitzbergens," *Resultater av de Norske Statsunderstöttede Spitsbergenekspeditioner*, No. 11, 107 pp.

38. ODELL, N. E., 1926. "The Geology of the Eastern Parts of Central Spitsbergen," *Quart. Journ. Geol. Soc.*, vol lxxxii, pp. 147–162.

39. STENSIO, A. E., 1927. "The Downtonian and Devonian Vertebrates of Spitsbergen," *Skrifter om Svalbard og Ishavet*, No 12

40. BLANCK, E., A. RIESER, and A. MORTENSEN, 1928 "Die wissenschaftlichen Ergebnisse einer bodenkundlichen Forschungsreise nach Spitzbergen in Sommer 1926," *Chem. d. Erde*, vol. iii, pp 588–698

41. HORN, G., and A. K. ORVIN, 1928 "Geology of Bear Island," *Skrifter om Svalbard og Ishavet*, No. 15, 152 pp

42. FALCON, N. L., in: H. G. WATKINS, "The Cambridge Expedition to Edge Island," *Geogr Journ.*, vol lxxii, pp. 117–143, Appendix III, "Geology," by N. L. Falcon, pp 134 139.

43. GRIPP, K., 1929. "Glaciologische und geologische Ergebnisse der Hamburgischen Spitzbergen-Expedition, 1927," *Abh Naturwiss Ver Hamburg*, vol. xxii, pp 146 249

44 SANDFORD, K. S, 1929 "The Glacial Conditions and Quaternary History of North-east Land," *Geogr Journ*, vol. lxxiv, pp 451–470, 543–552

45 FREBOLD, H, 1929 "Untersuchungen uber die Fauna, die Stratigraphie und Paläogeographie der Trias Spitzbergens," *Skrifter om Svalbard og Ishavet*, No. 26

46 FREBOLD, H, 1930. "Die Altersstellung des Fischhorizontes, des Grippianiveaus und des unterer Saurierhorizontes in Spitzbergen," *Skrifter om Svalbard og Ishavet*, No 28, 26 pp.

47. HORN, G, 1930. "Franz Josef Land. Natural History, Discovery, Exploration, and Hunting," *Skrifter om Svalbard og Ishavet*, No 29, 54 pp

48. BISSET, C B, 1930. "Geological Notes on North-east Land and Franz Josef Land: British Arctic Expedition of 1925," *Trans Edin. Geol Soc*, vol xii, pt. ii, pp. 196–212.

49 KNOTHE, H., 1931 "Spitzbergen: eine landeskundliche Studie," *Petermanns Mitth. Erg.-Heft*, No. 211, 109 pp

50. VON WOLFF, F., 1931. *Der Vulkanismus*, Bd. II, 2 Teil; "Die Alte Welt" 1, Lief. "Der Atlantische Ozean," Leipzig, pp. 829–1111; "Spitsbergen and Franz Josef Land," pp 845–849.

51. DALGETY, C. T, 1932. "Two Expeditions to Spitsbergen, North-east Land, and the Neighbouring Islands," *Geogr. Journ*, vol lxxxix, pp 131–134.

DESCRIPTION OF PLATE

Topographical and geological map of Spitsbergen, based mainly upon the map in 'H Knothe, "Spitzbergen, eine landeskundliche Studie" (*Petermanns Mitt. Erg*, Heft 211, 1931), with some alterations and additions

(*Issued separately October* 4, 1933)

XXII.—Studies on the Gametogenesis of *Stenophylax stellatus*, Curt. (Trichoptera).—Oogenesis. By R. A. R. Gresson, Ph.D.

(From the Department of Zoology, University of Edinburgh.)

Communicated by Professor J H ASHWORTH, F R.S. (With One Group Text-figures (1–14) and Two Plates)

(MS received June 1, 1933 Read July 3, 1933)

CONTENTS

PAGE
I Introduction and Previous Work 322
II Material and Methods 325
III Observations 325
1 The Growth of the Ovarioles 325
2 The Prophase of the First Maturation Division . 329
3 The Golgi Bodies and Mitochondria 330
(*a*) Undifferentiated Cells 330
PAGE
(*b*) Oocytes . . 332
(*c*) Nurse-cells 333
(*d*) Follicle-cells . . 333
4 The Vacuome 334
5 Yolk-formation . . 334
IV. Discussion . 336
V Summary 343
References to Literature 344
Description of Plates . 345

I. INTRODUCTION AND PREVIOUS WORK

PRESENT knowledge of the oogenesis of caddis-flies is scanty. Marshall's paper (17) on the ovary of *Platyphylax designatus* is the only detailed account of the growth and history of the ovarian elements during the larval and pupal stages, but he does not deal with the cytoplasmic inclusions

According to Marshall, the ovarioles of the youngest larva examined consisted of a terminal filament and an oval portion The cells of the terminal filament have an elongate nucleus and a single small nucleolus; chromatin granules are present which are connected by achromatic strands. The cells of the oval part vary according to their position in the tube—those at the sides being similar to the cells of the terminal filament, while those situated in the axial region of the anterior portion contain an oval or spherical nucleus, an "achromatic nucleolus," chromatin granules, and achromatic threads. In the nuclei of the cells situated at the posterior end of the ovarioles the threads have a beaded appearance, but chromatin granules are not present

In the middle portion of the ovarian tubes of an older larva many of the nuclei are in synapsis; the threads have a beaded structure, are closely packed together and difficult to make out. The nucleolus is still present, but does not always occur at the pole towards which the threads have contracted After this stage the threads become distributed through the nucleus; later, they are shorter and thicker, and each thread gives rise to two long chromosomes

In an ovary from a slightly older larva oocytes and nurse-cells could be identified; the former contain paired chromosomes which lie in an achromatic material which is identified as the remains of the threads; the "achromatic nucleolus" is still present. The chromosomes of the nurse-cells form tetrads and later fragment to form numerous granules which are at first arranged in distinct groups but ultimately spread out through the nucleoplasm.

Marshall states that the ovarian tubes of a young pupa contained five groups of oocytes and five groups of nurse-cells Chromosomes are not present in the older oocytes, but masses of achromatic material surround small chromatin granules or rods. The nucleolus of the most highly developed cells is vacuolated, while unstained bodies, some of which contain "a distinct chromatin spot or spots," occur scattered through the nucleus The two anterior oocytes have a blunt process arising from the anterior pole passing between the nurse-cells of the adjoining chamber

In the late pupæ two large oocytes filled with yolk are situated at the posterior end of the tubes The yolk-spheres are not figured, and are not described beyond stating that the yolk is composed of round bodies "which lie in a granular network."

Marshall does not give the chromosome number for either the oogonia or oocytes Lutman, however, in a paper on the spermatogenesis of *Platyphylax designatus* (16), states that the haploid number is thirty, and that an examination of the oogonia gave counts of fifty-five to sixty chromosomes.

Hirschler and Hirschlerowa (10), in a short note on certain large cells which they believe are somatic in origin, situated in the terminal filament of the ovary of *Phryganea grandis*, state that the Golgi elements are present as round or oval bodies, the elements of the vacuome as numerous grains which stain with neutral red, and the mitochondria as small granules which stain with Janus green

According to Hirschlerowa (11), in a brief account of the cytoplasmic inclusions of the ovary of *Phryganea grandis*, the majority of the elements of the vacuome are present, in the oocyte, as granules which adhere to the Golgi bodies. The latter are figured as rings, and after prolonged treatment are stained by neutral red, while the vacuome is blackened by osmic acid. Mitochondria are present as small grains which sometimes form dense masses. The Golgi elements and the vacuome of the follicle-cells and nurse-cells are similar to those of the oocyte.

Hosselet (15) states that the young nurse-cells of *Setodes tineiformis* have a spherical nucleus which contains small masses of chromatin and an irregularly shaped nucleolus. As the nucleus increases in size the nucleolus begins to break up into fragments which are extruded from the nucleus. The fragments are often surrounded by a basophil material, and it is suggested that the latter, which corresponds to a plasmosome, may be capable of forming communicating bridges by means of which the nucleolar fragments pass to the cytoplasm

In material fixed in Zenker-formol and subsequently treated with potassium bichromate, and in osmic acid preparations, small networks and irregular meshes occur at one pole of the nucleus of the young nurse-cells; these structures Hosselet identifies as the "chondriome." In the later cells the networks are reduced to two or three meshes and to some chondriosomes.

In the older nurse-cells, groups of two or three rings occur; the periphery of these bodies is at first regular, but later becomes thickened in places so that granules appear to be formed on the edges of the rings During the time that these changes are taking place the centre becomes clear. Later, the central vacuole is clearer and the "chondriome" finally disappears. A few elements do not take part in this process and are present as isolated granules.

Hosselet does not deal at any length with the oocyte of *Setodes tineiformis*. The "chondriome," in the form of a dense filo-reticular mass, occurs in contact with the nucleus, in the part of the cell next to the nutritive chamber, however, it is more dispersed and occurs in groups of individual elements situated between the nucleus and the neighbouring nurse-cells; some of these elements are ring-shaped A projection from the oocyte usually extends between two adjacent nurse-cells

From the above account it will be seen that comparatively little work has been carried out on the oogenesis of caddis-flies Marshall's paper, on *Platyphylax designatus*, contains the only description of the prophase of the first maturation division; Marshall, however, was chiefly concerned with the growth and history of the ovarian elements, and consequently does not give a detailed description of the prophase stages Although a certain amount of work has been carried out on the cytoplasmic inclusions of nurse-cells, follicle-cells, and oocytes, there appears to be no detailed account of the history of the Golgi elements and mitochondria during all stages of the development of the oocyte.

II. MATERIAL AND METHODS.

The material for this paper was obtained from larvæ and pupæ of *Stenophylax stellatus* collected from the Braid Burn, near Edinburgh, in the summer and autumn of 1932, and from larvæ collected in the spring of 1933.

For the study of the growth of the ovarioles, ovaries were fixed in Flemming's mixture and in Bouin's picro-formol For the preservation of late oocytes, filled with yolk-spheres, Smith's fixative was found to be the most satisfactory, and this fixative also gave good results for the younger cells in the ovaries of pupæ

Certain ovaries were treated according to Feulgen's technique for the identification of chromatin. The material was fixed in corrosive-acetic mixture; light green was used as a counter-stain

For the demonstration of the mitochondria, material fixed in Flemming (without acetic) and by the Mann-Kopsch method was found to be satisfactory The Golgi bodies were studied in material treated according to the method of Mann-Kopsch.

For the demonstration of the vacuome, ovaries were dissected out in saline solution and immediately placed in a weak solution of neutral red.

My thanks are due to Professor J H. Ashworth, F.R S., for help and advice, and to Mr Martin E. Moseley, of the Natural History Museum, South Kensington, for identifying specimens of *Stenophylax stellatus.* I wish to express my thanks for a grant in aid of this work from the Earl of Moray Endowment of the University of Edinburgh.

III. OBSERVATIONS

1. *The Growth of the Ovarioles*

In the ovarioles of the youngest larva examined three regions can be identified At the anterior end the cells are small and are, in most cases, approximately equal in size, each nucleus contains a small deeply stained nucleolus and minute deeply stained chromatin granules which are arranged on a faintly stained network (Pl. I, fig. 1). In the middle region of the ovarioles the majority of the cells have increased in size, and the nucleolus and chromatin granules are larger and more deeply stained than those present in the earlier cells. The nuclei of some of the cells in this region contain large masses of material, while others are so deeply stained that they resemble homogeneous bodies. Their appearance strongly suggests that they are undergoing degenerative changes (Pl. I, fig 1). At the posterior end of the ovarian tubes the cells are of about the same size as those in the middle region; the nucleolus is deeply stained, but in most cases the chromatin granules are more faintly stained than those of the slightly younger cells (Pl. I, fig. 1). In these young ovarioles it is not possible to distinguish between oocytes and nurse-cells. Small cells are situated towards the sides of the posterior region; it is probable that, at a later stage, they give origin to the cells of the follicular epithelium

In ovaries from more advanced larvæ, cells in corresponding stages of development to those of the younger ovarioles are situated in the anterior and middle part of the tubes. The succeeding cells are larger and their nuclei are in the early prophase of the first maturation division, while at the posterior end of the ovarioles paired chromosomes and a deeply stained nucleolus are present in each nucleus. Large deeply stained granules are present in the cytoplasm of the older cells (Pl. I, fig 2) It is not possible to distinguish with certainty between oocytes and nurse-cells, but some of the cells, situated at the posterior end of the ovarian tubes, are larger than their neighbours and are, apparently, oocytes which have not yet become clearly differentiated from the other cells

An examination of ovaries from young pupæ showed that the anterior and middle regions of the ovarioles are occupied by the same type of cells as are present in the late larvæ, while immediately posterior to the middle region it is possible to distinguish between oocytes and nurse-cells. Some of the cells have increased in size and contain a large nucleolus and paired chromosomes These cells are identified as the early oocytes, and are surrounded by smaller cells; the latter are, at a later stage, transformed into nurse-cells. Both types of cells contain large deeply stained cytoplasmic granules (Pl I, fig 3). The next stage in the differentiation of the nurse-cells is shown by slightly older cells situated below the very early oocytes and nurse-cells described above The chromosomes of these nurse-cells fragment into granules which at first remain in tetrads and small groups in or near the position previously occupied by the chromosomes from which the granules originated. As these changes are taking place the nucleolus becomes irregular in shape and some of its substance appears to be passed into the nucleoplasm in the form of nucleolar buds or extrusions (Pl I, fig. 4). In slightly older cells (situated at the posterior end of the ovarioles) the chromosomes have fragmented into numerous granules which are situated, for the most part, round the nucleolus and extend in clumps from the nucleolus to the periphery of the cell. In some cases the nucleolus is breaking up to form buds, but in others, although irregular in shape, has not yet entered upon a process of fragmentation (Pl. I, fig. 5) In the nucleus of the oocyte situated in contact with the older nurse-cells the chromosomes are paired and the nucleolus is breaking up into several smaller bodies. In both oocyte and nurse-cells deeply stained cytoplasmic granules occur close to or in contact with the outside of the nuclear membrane (Pl I, fig. 5).

In a slightly more advanced pupa the posterior ends of the ovarioles are occupied by a large oocyte and a group of nurse-cells (Pl I, fig 6) Paired chromosomes are present in the nucleus of the oocyte and the nucleolus has fragmented to form two or more deeply stained bodies The nuclei of the nurse-cells have increased in size and become less regular in outline; the nucleolus is deeply stained but in most cases has broken up into several pieces; numerous scattered chromatin granules are present The cytoplasm of both oocyte and nurse-cells contains granules, and in many of the oocytes these granules tend to form a chain at the posterior pole of the cell (Pl I, fig. 6). The high degree of nuclear activity (as evidenced by the fragmentation of the nucleolus of oocyte and nurse-cells) and the presence of granules on the outside of the nuclear membrane indicate that nucleolar material is extruded to the cytoplasm where it becomes visible in the form of deeply stained granules Definite follicle-cells occur at the posterior end of the ovarioles of the young pupa described above, but oocyte and nurse-cells have not yet been separated into their respective chambers

In slightly older pupæ two groups of oocytes and nurse-cells are present in the posterior part of the ovarian tubes The nurse-cells of the more highly developed group are now much larger than the adjacent oocyte, while the latter has increased in size and become irregular in shape so that its anterior end projects between the adjoining nurse-cells (Pl I, fig. 7) Paired chromosomes occur scattered through the nucleus and the nucleolus is differentiated into two parts—a large faintly stained spherical body and a darkly stained mass of material (Pl I, fig. 7) In some cases the deeply stained part is very large and almost completely covers the lightly stained portion. Small granules occur distributed through the nucleoplasm and in some cases are in contact with the nucleolus; in all probability they arise from the latter as nucleolar buds or extrusions Cytoplasmic granules are present and are most numerous in the vicinity of the nucleus and in the posterior part of the cell The nuclei of the nurse-cells contain numerous small granules and large irregularly shaped bodies; the former have arisen by the fragmentation of the chromosomes and the latter are derived from the nucleolus of the younger cells Owing to the large number of nucleoli present it is evident that they grow and divide after their liberation from the original nucleolus

In the ovarioles of late pupæ nine oocytes are present. The nuclei of the first and second oocytes, situated at the anterior end of the tubes, contain paired chromosomes and a deeply stained nucleolus. This region is followed by four groups of oocytes and nurse-cells which are in stages of development comparable with those of the younger ovarian tubes described above. The follicle-cells in association with the early nurse-cells are similar to those of the adjoining oocyte; in the seventh group, however, they are elongate, but have not yet formed a partition between the nutritive chamber and the oocyte. The seventh oocyte is larger, but in other respects is similar to the sixth.

The nucleus of the eighth oocyte contains a lightly stained vacuolated nucleolus and paired chromosomes. Large vacuoles are present in the ooplasm, and towards the periphery yolk-spheres are situated amongst the vacuoles (Pl. I, fig. 8).

The nuclei of the nurse-cells have increased in size and become very irregular in shape. Several nucleoli and numerous chromatin granules are present; the latter are greater in number and the nucleoli larger than in the younger cells (Pl. I, fig. 10). Granules are situated in the cytoplasm. The nutritive chamber is separated from the adjoining oocyte by a single layer of elongate follicle-cells.

The ninth oocyte has increased greatly in size, the yolk-globules are larger, have become slightly more numerous and now extend towards the central region of the cell (Pl. I, fig. 9). Chromosomes and a vacuolated nucleolus are present. The nurse-cells are similar to those of the eighth nutritive chamber.

In late pupæ, which have darkened preparatory to emergence of the imago, the last two oocytes are completely filled with yolk-globules. A cup-shaped depression is present at the anterior end of each oocyte, and into this concavity projects the group of neighbouring nurse-cells. The layer of follicle-cells separating the oocyte from the nutritive chamber, owing to the elongation of the cells, has become very thin. The chromosomes are faintly stained and clumped towards the middle of the egg nucleus; a nucleolus was not observed. The nurse-cells and their nuclei are identical in structure with those of the eighth and ninth nutritive chambers of the younger pupæ.

The ovarioles of certain pupæ were treated according to Feulgen's technique for the identification of chromatin. The chromatin granules and network of the undifferentiated cells and the prophase threads and chromosomes of the later cells gave a positive reaction; the chromosomes of the fully formed oocytes, however, were stained less brightly than those of the younger cells. The history of the chromatin of the nurse-cells was found to agree with the account based on the examination of other material

—thus the nuclei of the early nurse-cells contain paired chromosomes and a nucleolus which stains with light green; later, the chromatin is represented by groups of granules which increase in number and finally are distributed singly and in groups throughout the nucleus. In the nuclei of the older cells the groups of granules are connected with one another by threads which give the chromatin reaction; chromatin granules occur at intervals along the threads. In some cases, at least, it is probable that the appearance of threads is due to the close proximity of chromatin granules arranged in a line stretching from one group to another. In other cases there is little doubt as to the presence of chromatin threads. In the larger nuclei chromatin granules are situated on the inside of the nuclear membrane (text-fig. 1).

Spherical and irregularly shaped bodies were observed among the chromatin granules of the older cells; these bodies do not contain chromatin but stain faintly with light green and are identified as the nucleolar fragments described on p. 327. The chromatin granules often closely surround the nucleoli so that the latter are difficult to observe.

A nucleolus which stains with light green, chromatin granules, and a chromatin network were observed in the nuclei of the follicle-cells (text-fig. 2).

Although oocytes and their accompanying nurse-cells were carefully examined at all stages of development up to the appearance of yolk-globules, granules which give the chromatin reaction were not observed in the cytoplasm of the nurse-cells or in the ooplasm.

Certain nuclei situated among the undifferentiated cells appear as homogeneous masses of material which give the chromatin reaction, but are darker in colour than the chromatin of the surrounding cells (text-fig. 3). These nuclei are identical with those observed in sections stained in iron hæmatoxylin and identified as the nuclei of cells undergoing degenerative changes (p. 325). The whole nucleus appears to be filled with chromatin and no other structures are visible. The cytoplasm of the degenerating cells contains several bodies or granules which give the chromatin reaction, and are, apparently, small masses of chromatin which have been extruded from the nuclei during the degenerative changes (text-fig. 3).

2. *The Prophase of the First Maturation Division.*

At the stage of development of the undifferentiated cells shown in Pl. I, fig. 2, the chromatin granules become more faintly stained and give rise to thin leptotene threads; large granules of chromatin persist for some time (text-fig. 4). The threads now contract towards one pole of the nucleus to form a dense synizetic knot from which a few individual threads extend towards the opposite pole (text-fig. 5). Owing to the closeness of the knot it is impossible to make out any structure; a deeply stained area, however, observed in certain nuclei, is probably a nucleolus. In slightly older cells thick pachytene threads and a large nucleolus are present (text-fig. 6). The pachytene threads rapidly contract and give rise to the chromosomes of the diplotene stage, the members of a pair lying side by side. As already described (p. 326), the chromosomes of the nurse-cells fragment while those of the oocytes are present in the nuclei of the oldest cells examined. In the younger oocytes the chromosomes tend to lie towards the periphery of the nucleus, but in the fully developed cells they form a clump, usually in the vicinity of the nucleolus.

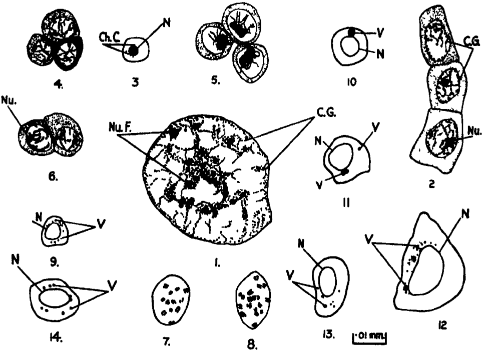
Owing to the presence of the nucleoli and nucleolar fragments which take up stain similar to the chromosomes, it was found difficult to obtain accurate counts of the chromosome number in material stained in iron hæmatoxylin. The oogonia contain about sixty chromosomes, but the smallness of the cells and of the chromosomes rendered counts unsatisfactory. Ovarioles treated according to Feulgen's technique were examined and a large number of oocytes gave counts of sixty chromosomes (text-figs. 7 and 8). In order to check these findings, sections of the testes of larvæ were examined and the haploid number was found to be thirty.

A nucleolus was not always observed in the Feulgen material during the leptotene stage; when visible, it was slightly stained with light green. The sections did not contain many nuclei in synizesis, but an examination of those present revealed the nucleolus stained green and closely surrounded by the chromatin threads. During the pachytene stage the nucleolus was identified with ease, and its subsequent behaviour agrees with the findings recorded on pp. 326–8. The nucleolus does not contain chromatin at any stage in the history of the oocyte.

3. *The Golgi Bodies and Mitochondria.*

(a) *Undifferentiated Cells.*—In material treated according to the Mann-Kopsch technique the Golgi elements of the undifferentiated cells, situated at the anterior end of the ovarioles of larvæ and early pupæ, are in the form of granules and small rings distributed round the nuclear membrane. In older cells the majority of the Golgi bodies form one or more clumps at one pole of the nucleus. Some of the larger spheres consist of a chromophilic rim and a chromophobic central substance (Pl. II, fig. 23).

Mitochondria could not be identified with certainty in this material,



TEXT-FIGS 1–14.

LETTERING.

Ch.C, chromatin granules in cytoplasm; C.G, chromatin granules; N, nucleus; Nu, nucleolus; Nu.F, nucleolar fragments; V, elements of the vacuome.

FIG. 1.—Nucleus of nurse-cell. Pupa. Feulgen.
FIG. 2.—Follicle-cells. Pupa. Feulgen
FIG. 3.—Degenerating cell. Pupa. Feulgen.
FIG. 4.—Leptotene stage. Early pupa. Bouin
FIG. 5.—Synizesis. Early pupa. Bouin.
FIG. 6.—Pachytene stage. Early pupa. Bouin.
FIG. 7.—Nucleus of young oocyte, showing twenty-four chromosomes. Pupa Feulgen.
FIG. 8.—The next section of the same nucleus, showing thirty-six chromosomes. Feulgen.
FIG. 9.—Undifferentiated cell. Larva. Neutral red.
FIG. 10.—Undifferentiated cell. Larva. Neutral red.
FIG. 11.—Later stage. Larva. Neutral red.
FIG. 12.—Young oocyte. Pupa. Neutral red.
FIG. 13.—Nurse-cell. Pupa. Neutral red.
FIG. 14.—Follicle-cell. Pupa. Neutral red.

partly, no doubt, owing to the occurrence of the Golgi material in that part of the cell in which the mitochondria are situated. In Flemming preparations, however, the mitochondria are revealed as a dark mass at one pole of the nucleus (Pl. II, fig. 21).

(b) *Oocytes.*—In young oocytes which have recently differentiated from the surrounding nurse-cells, the Golgi elements are for the most part situated at one pole of the nucleus. They are arranged in one or more clumps and are in the form of irregularly shaped granules and of rings; a few elements remain outside these clumps (Pl. II, fig. 11). Many of the spheres are large and somewhat irregular in shape, and consist of a deeply impregnated rim surrounding a chromophobic substance.

The mitochondria are well shown in the Flemming preparations. The mitochondrial mass is now breaking up and many of the individual granules, of which it was composed, are scattered through the ooplasm while a certain number are arranged in rows to form chains or filaments (Pl. II, fig. 22). Large granules which stain deeply with iron hæmatoxylin are present; these are identified as the cytoplasmic granules already described for other material.

In slightly older oocytes the clumps of Golgi material break up and the individual elements become distributed through the surrounding ooplasm. At this stage the mitochondria are scattered fairly evenly through the cell (Pl. II, fig. 12).

The Golgi bodies increase in number and in size and are situated throughout the ooplasm, in many cases they are less numerous in the posterior part of the cell. At this stage two or three Golgi elements are often closely applied to one another to form a larger body, and the substance of some of the ring-shaped Golgi bodies becomes broken up to form several small granules which surround an area which is not blackened by osmic acid. The mitochondria, as in the slightly younger oocytes, are scattered throughout the cell (Pl. II, fig. 14).

In older oocytes, in which a few yolk-globules occur, the majority of the Golgi bodies are situated towards the periphery and in the posterior part of the cell. Many of the Golgi bodies are large and consist of two or more closely associated rings or granules; several are deeply impregnated and are almost as large as the yolk-globules. Appearances indicate that the Golgi material is transformed into fatty yolk (Pl. II, fig. 15). Mitochondria are present but are less numerous in the neighbourhood of the nucleus.

In a later oocyte, in which fatty yolk-formation is well advanced, the Golgi bodies are situated chiefly towards the periphery, and in many cases two or more elements are associated to form a large body. The mitochondria in the form of single granules and chains are scattered through the ooplasm; they are less numerous in the vicinity of the nucleus (Pl. II, fig. 17).

In the fully formed oocytes the ooplasm is packed with yolk-globules so that observation of the Golgi elements and mitochondria is rendered very difficult. Small deeply osmophil granules and rings are present between the yolk-spheres, and are identified as Golgi bodies, but many of the smaller granules may possibly be mitochondria (Pl. II, fig. 20). Large bodies with chromophilic rims similar to those of the younger cells were not observed in the late oocytes.

(c) *Nurse-cells.*—The Golgi elements of the early nurse-cells are situated chiefly towards one pole of the nucleus; a few bodies may occur in other parts of the cytoplasm. They are in the form of granules and rings (Pl. II, fig. 11). As in the early oocytes, the mitochondrial mass breaks up and the individual granules become distributed throughout the cell.

In the more advanced nurse-cells the Golgi bodies are situated round the nucleus, while the mitochondria are fairly evenly scattered through the cytoplasm (Pl. II, fig. 13).

As the nurse-cells increase in size the Golgi bodies spread out to the periphery and are usually absent from the cytoplasm immediately surrounding the nucleus. The distribution of the mitochondria is similar to that of the slightly younger cells (Pl. II, fig. 16).

An examination of nurse-cells in association with fully formed oocytes showed that the Golgi material and mitochondria occupy the same positions as in the preceding stage. The Golgi bodies are, however, smaller and are granular in form; many of these granules arise from the Golgi rings in a similar manner to those of the late oocytes.

(d) *Follicle-cells.*—In the young follicle-cells, situated towards the anterior end of the ovarioles, the Golgi elements occur close to the nuclear membrane, and in many cases appear to be most numerous at the pole of the nucleus next to the adjoining oocyte. Mitochondria as a few small granules are present in the neighbourhood of the nucleus (Pl. II, fig. 18).

In follicle-cells surrounding fully formed oocytes and nurse-cells the Golgi bodies are more numerous, and occur singly and in clumps round the nucleus and also in parts of the cytoplasm towards the periphery (Pl. II, fig. 19). Some of the larger bodies consist of a chromophilic rim and a central clear substance. The mitochondria are more numerous than in the early cells; they are scattered through the cytoplasm but do not extend to the periphery of the cell.

4. *The Vacuome.*

The elements of the vacuome of the young undifferentiated cells were present as granules which stained after a few minutes' treatment with neutral red (text-fig. 9). In older cells the vacuome is represented by a single large spherical body situated close to the nuclear membrane (text-fig. 10). In certain preparations of cells at this stage the vacuome did not stain homogeneously but consisted of several deeply stained granules situated in a large spherical body which was stained but lightly by neutral red. An examination of slightly larger cells showed that the majority contain two or three bodies stained by neutral red, and in many cases appearances indicate that these bodies arise by division from the single element of the earlier cells (text-fig. 11).

In young oocytes and nurse-cells the vacuome is present as granules which stain brightly with neutral red, and, in size, closely resemble those of the early undifferentiated cells. They are more numerous in the oocytes than in the nurse-cells (text-figs. 12 and 13). The late oocytes and nurse-cells did not always give good results with neutral red; in some cases the elements of the vacuome could not be identified, while in others they were shown as small bodies similar to those of the young oocytes and nurse-cells, but more numerous and distributed throughout the cell. Late oocytes, in which yolk-globules were present, contained no structure stainable by neutral red. The elements of the vacuome of the follicle-cells are situated round the nucleus (text-fig. 14).

During the investigation of living cells stained with neutral red, refractile bodies were observed in the cytoplasm of oocytes and nurse-cells. These bodies are, in all probability, the Golgi elements; they were not stained by neutral red.

5. *Yolk-formation.*

The eighth oocyte in the ovarioles of advanced pupæ was the youngest cell in which yolk-globules were identified. At this stage the ooplasm stains but faintly (iron hæmatoxylin) and contains numerous vacuoles, while deeply stained granules are present towards the periphery; these granules are identified as fatty yolk-spheres (Pl. I, fig. 8). The spheres increase in size and spread towards the central region of the cell, so that in the ninth oocyte they occur throughout most of the cytoplasm (Pl. I, fig. 9). In late oocytes from mature pupæ the fatty yolk-globules have increased greatly in size and in number, and are closely packed together throughout the entire egg.

The details of fatty yolk-formation were revealed by an examination of ovarioles treated according to the Mann-Kopsch technique. The Golgi elements of the late oocytes become distributed through the ooplasm. Many of the Golgi bodies increase in size while others become clumped in groups consisting of two or more elements. Small yolk-spheres make their appearance and are, at first, about the same size or are slightly larger than the bigger Golgi bodies. These spheres are deeply impregnated with osmic acid and are the fatty yolk-globules (Pl. II, fig. 15). In a later oocyte the majority of the Golgi elements are situated at the periphery, while yolk-globules occur scattered among them and, to a less extent, in the cytoplasm towards the central region of the egg (Pl. II, fig. 17).

In the fully formed oocytes of mature pupæ most of the fatty yolk-spheres are large and in some cases contain a clear vacuole; they are not very numerous and are distributed fairly evenly throughout the ooplasm. Albuminous yolk-globules are now present; they are numerous and vary in size but are always smaller than the fatty yolk-spheres (Pl. II, fig. 20). Small Golgi elements and minute granules, which may be mitochondria, occur among the yolk-spheres and appear to be especially numerous towards the periphery.

There appears to be little doubt that the fatty yolk is formed from the Golgi elements of the late oocyte. The fatty yolk-globules arise in that part of the cell in which the Golgi elements are most numerous, and when formed are about the same size as the larger Golgi bodies. Additional evidence in favour of this view was produced by an examination of the larger Golgi rings. As already mentioned, the substance of some of the Golgi rings becomes broken up into granules which surround a clear area; appearances indicate that these granules coalesce and in some cases are joined by other Golgi elements, and are finally converted into fatty yolk-globules. In other cases the Golgi rings do not form granules, the central area becomes blackened with osmic acid and the whole structure gives rise to a fatty yolk-sphere. It is significant that Golgi elements are not numerous in mature oocytes, while those present are always small (Pl. II, fig. 20).

The albuminous yolk-globules were not observed to arise in connection with any of the cytoplasmic inclusions. The nucleolar extrusions disappear or become less numerous prior to the beginning of yolk-formation. In all probability they are dissolved in the ooplasm and their substance utilised in the formation of the albuminous yolk.

The mitochondria are less numerous in the late oocytes; their small size and the large number of yolk-globules present rendered their identification very difficult. They are numerous in oocytes in which fatty but not protein yolk is being formed (Pl. II, figs. 15 and 17). It is possible, therefore, that the mitochondria may play some part in the formation of the albuminous yolk.

IV. DISCUSSION.

The general account of the growth of the ovary of *Stenophylax stellatus* agrees fairly closely with Marshall's findings for *Platyphylax designatus* (17); the behaviour of the nucleoli of both oocytes and nurse-cells is given in more detail in the present paper, and the application of Feulgen's technique throws further light on the nature of the nucleoli, and of the granules and network of the early undifferentiated cells.

Marshall could not determine whether the numerous nucleoli of the late nurse-cells arose from the original nucleolus or were formed from other material. An examination of the nurse-cells of *Stenophylax stellatus* shows that the nucleolus of the young cells fragments into several pieces. The large number of these bodies present in the fully formed nurse-cells indicates that they cannot all have arisen from the nucleolus of the earlier cells and that the fragments grow and multiply at the expense of some substance derived from the nucleus, or that bodies, which stain in a similar manner to the nucleoli, are formed independently from some other nuclear material. While it was impossible to determine this matter with certainty, examination of the nucleolar fragments, many of which seem to be breaking up to form smaller bodies, supports the first view.

Hosselet (15) records fragmentation of the nucleolus and the occurrence of nucleolar extrusions from the nurse-cell nuclei of *Setodes tineiformis*, and believes that the "chondriome" is regenerated at the expense of these bodies. The present writer was unable to find any evidence that the nucleolar extrusions of *Stenophylax stellatus* take part in the regeneration of either the mitochondria or Golgi elements.

Marshall records the presence of deeply stained granules in the cytoplasm of the growing oocytes of *Platyphylax designatus*; he makes no suggestion as to their origin, but believes that they represent some product of the metabolism of the cell. During the present investigations similar bodies were observed in the ooplasm. In the young oocytes they are large and are in contact with the nuclear membrane; in the older cells they are smaller, and while a few may still be adjacent to the nucleus, the majority are scattered through the cytoplasm. These cytoplasmic bodies, although a few may be present in the very early cells, are most numerous at the time when the nucleolus is breaking up into two or more pieces. In all probability they are formed from nucleolar material which is passed in solution through the nuclear membrane. This appears to be the only recorded case of the occurrence of nucleolar extrusions in the eggs of caddis-flies. It is worthy of note that the extrusion of nucleolar material from the nucleoli of the spinning-gland cells of caddis-flies has been recorded by Nakahara (19), by Beams and Wu (1), by Hosselet (15), and by Wajda (31).

The darkly stained tracts of material mentioned by Marshall as extending from the neighbourhood of the nuclei of the nurse-cells to the adjoining oocytes of *Platyphylax designatus* were not observed during the present investigation.

Marshall records the occurrence of degenerating cells in the ovarioles of *Platyphylax designatus*. The present findings throw further light on degeneration in *Stenophylax stellatus* In the early stages the nucleus is occupied by large masses of dark material (in iron hæmatoxylin preparations), while later the entire nucleus is deeply stained. The Feulgen preparations did not contain nuclei in the early stages, but the nuclei of cells in an advanced stage of degeneration were filled with material which gave the chromatin reaction, and granules of chromatin were observed in the cytoplasm. It is evident that the beginning of nuclear degeneration is marked by the formation of large chromatin masses, which later increase in size and fill the whole nucleus, while some of the chromatin is extruded to the cytoplasm where it is visible as granules.

Nuclei in an advanced stage of degeneration are usually considerably smaller than those of the surrounding normal cells, this may account in part for the fact that they are completely filled by the chromatin. Later stages of degeneration were not observed, it is probable, therefore, that the cells may persist until very late in oogenesis.

The behaviour of the chromatin threads and chromosomes during the prophase of the maturation division of *Stenophylax stellatus* agrees fairly closely with Marshall's account for *Platyphylax designatus*. Marshall, however, found that chromosomes were not present in the older oocytes, while chromosomes, although stained faintly in Feulgen material, were identified in fully formed eggs during the present investigations A nucleolus was not visible in the oldest eggs of late pupæ; this agrees with Marshall's statement that nucleoli were absent from the fully developed oocytes.

The presence of chromosomes, as demonstrated by Feulgen's method, throughout the growth phase and stages of yolk-formation is an unusual condition for insect oogenesis. Thus the present writer working on Tenthredinids (7), and on *Periplaneta* (8), and Mukerji on *Apanteles* (18) record the absence of chromatin from the later oocytes, while Sanderson in a paper on a Tenthredinid (26) states that chromatin is present during the growth period as a kidney-shaped mass.

The present findings that the nucleolus, nucleolar buds, and extrusions of both oocytes and nurse-cells do not contain chromatin agree with the writer's former observations (7 and 8).

Steopoe (29) states that the nucleolus of the spermatocytes of *Nepa cinerea* and of *Naucoris cimicoides* gave a negative reaction with Feulgen's method.

Owing to the small size of the chromosomes of the oocytes, counts were difficult to obtain. The majority of counts gave the diploid number as sixty chromosomes, while in a few cases the nuclei appeared to contain only fifty-eight or fifty-nine chromosomes. An examination of the spermatocytes showed clearly that the haploid number is thirty. It is concluded, therefore, that the diploid number is sixty. Lutman (16), working on the oogonia and spermatogonia of *Platyphylax designatus*, obtained counts of from fifty-five to sixty chromosomes.

The structures called Golgi elements or Golgi bodies in the present contribution appear to be closely similar to the "chondriome" described by Hosselet (15) as occurring in the nurse-cells and oocytes of *Setodes tineiformis*; the present writer finds, however, that the juxta-nuclear masses consist of individual elements which are closely approximated in the early cells. Hosselet states his belief that the Golgi apparatus is an artifact; the structures often present in material treated by methods for the demonstration of the Golgi bodies he calls the "chondriome golgiesque." The present writer believes that the rings and large granules observed in the cells of the ovary of *Stenophylax stellatus* are identical with the material described as Golgi bodies by most workers, and, furthermore, that these bodies, as shown by osmic methods, are distinct from the mitochondria, which in the present investigations were revealed as minute granules. Both Golgi elements and mitochondria have previously been identified by Hirschler and Hirschlerowa (10), and by Hirschlerowa (11) in the ovarioles of the caddis-fly, *Phryganea grandis*.

As previously mentioned (p. 324), Hosselet describes the formation of a vacuole in the interior of the ring-shaped elements of the "chondriome" present in the older nurse-cells. The vacuole increases in size and finally the "chondriome" disappears with the exception of a few elements which do not take part in this process and remain as isolated granules. An examination of nurse-cells in association with fully formed oocytes of *Stenophylax stellatus* showed that the Golgi elements are still present, but are, for the most part, smaller than those of the earlier cells; vacuoles were not present. In slightly younger nurse-cells the substance of some of the ring-shaped Golgi bodies was observed to have become separated into a number of granules arranged round a clear area in a similar manner to that described by Hosselet for the osmophil granules of *Setodes*. In *Stenophylax* these granules do not disappear, but, apparently, are distributed through the cytoplasm to form the numerous granular Golgi elements of the late nurse-cells.

The ring-shaped Golgi elements of the oocytes, nurse-cells, and follicle-cells of *Stenophylax*, although consisting of an osmophil ring surrounding a central clear substance, are not identical in structure with the vesicular Golgi bodies recently recorded as occurring in the cells of the ovary of certain insects by Nath and his co-workers (21, 22, and 23), by Bhandari and Nath (3), by the present writer (6 and 8), and by Payne (25). The Golgi vesicles described by Nath and his collaborators and by the present writer increase in size, fat is deposited in the chromophobic central area and they are converted into fatty yolk-spheres. Payne finds that the Golgi bodies take no part in yolk-formation.

Many of the Golgi rings in the eggs of *Stenophylax* become broken up into a number of granules which, at first, surround the original clear central area, but later run together to form a body which increases in size, impregnates deeply with osmic acid, and is identified as a fatty yolk-sphere. In other cases the Golgi material does not appear to break up, the central area impregnates with osmic acid and the whole structure is converted into a fatty yolk-globule; two or more rings may become associated to form a globule. The actual substance of the Golgi bodies seems to be converted into fatty yolk, but owing to the rapid growth of the spheres it is highly probable that other material, derived from the ooplasm, plays an important part in the formation of fatty yolk.

Recent conclusions as to the participation of the cytoplasmic inclusions and nucleolar extrusions in yolk-formation in insects may be suitably summarised by means of the following table. In some instances the authors have not recorded the presence of fatty yolk, or have not determined its origin; in these cases the space under "fatty yolk" is left blank. The letter "G" under "Technique" indicates that methods suitable for the demonstration of the Golgi bodies have been employed.

Insect Studied and Author.	Protein Yolk.	Fatty Yolk.	Technique.
Periplaneta americana. Hogben (13).	Nucleolar extrusions.	..	..
Periplaneta americana. Nath and Mohan (23).	,, ,,	Golgi bodies.	G.
Periplaneta orientalis. Gresson (8).	,, ,,	,, ,,	G.
Gryllotalpa vulgaris. Voinov (30)	..	,, ,,	G.
Anisolobis annulipes. Payne (25)	Not from Golgi bodies or mitochondria.	Not from Golgi bodies or mitochondria.	G.
Oecanthus quadripunctatus. Payne (25).	Golgi bodies take no direct part in yolk-formation.	..	G.
Nepa cinerea. Spaul (27)	Nucleolar extr.	..	..
Nepa cinerea. Steopoe (28).	Golgi bodies	..	G.
Dysdercus cingulatus Bhandari and Nath (3)	Nucleolar extr	Golgi bodies.	G.
Galastocoris oculatus. Payne (25).	Not from Golgi bodies.	Not from Golgi bodies.	G.
Libellula depressa (*Anax* and *Plathemis*) Hogben (14)	Interaction of nucleolar extr, mitochondria and Golgi bodies.	..	G.
Anopheles maculipennis. Nicholson (24)	Rosette cells and oocyte nucleus.	..	..
Culex fatigans. Nath (20 and 21)	Material derived from nurse-cells and from surrounding fluid.	Some of the Golgi vesicles contain fat.	G.
Culex pipiens. Hosselet (15).	First stages not examined.	Elaborated by the "chondriome."	G.
Dicranomya sp Hosselet (15).	Close connection between "chondriome" and yolk-spheres.	..	G.
Chironomus plumbicornis Hosselet (15).	Arise in connection with "chondriome." Nurse-cells produce substance used in yolk-formation.	..	G.

Insect Studied and Author.	Protein Yolk.	Fatty Yolk.	Technique.
Luciola gorhami. Nath and Mehta (22).	Nucleolar extr.	Golgi bodies.	G.
Formica rufa (*Synergus*). Hogben (12).	Secondary nuclei	.	..
Apanteles glomeratus. Gatenby and Woodger (4).	Interaction of secondary nuclei and mitochondria.	None.	G.
Thrinix macula. Gresson (6).	Nucleolar extr.	Golgi bodies.	G.
Thrinix mixta. Gresson (6).	" "	" "	G.
Allantus pallipes Gresson (6).	" "	" "	G.

An examination of the table on yolk-formation shows that the Golgi bodies, in a large number of cases in which they have been identified in the insect egg, are connected with the formation of fatty yolk. There is considerable disagreement as to the origin of protein yolk, but in several cases and in widely separated groups it is believed that the yolk-spheres are formed, in part at least, from nucleolar extrusions.

Payne (25) examined the eggs of other insects besides those noted in the table but does not state the method of yolk-formation. In conclusion he mentions that the Golgi bodies seem to decrease in number during fat and yolk-formation, but there is no evidence that Golgi elements or mitochondria take part in this process. Payne states that the Golgi vesicles of *Oecanthus quadripunctatus* probably disintegrate and give rise to chemical substances which are used in other synthetic processes.

Hosselet (15) believes that, during yolk-formation, the "chondriome" acts as a catalytic agent. The "chondriome" disappears when the yolk-spheres are formed, and it is suggested that the elements fuse with the spheres.

In *Stenophylax* the Golgi bodies appear to be transformed into a fatty substance, and the latter together with other material derived from the ooplasm form the fatty yolk-spheres. With reference to the direct conversion of the Golgi elements into fat, it is of interest that Bell (2) has recently described the origin of neutral fats from the Golgi material of the spermatid of the dog, and that the present writer (9) believes that the Golgi bodies in atretic eggs of the mouse are transformed into fat-globules.

As already stated (p. 335), it is probable that the nucleolar extrusions take part in the formation of the protein yolk. Nucleolar extrusions have been described in the oocytes of several insects, and have been recorded by Nath and his co-workers (22 and 23), by Bhandari and Nath (3), and by the present writer (5, 6, and 8) as giving rise to albuminous yolk-globules.

Hirschlerowa (11), in a short note on the oocytes of *Phryganea grandis*, states that the elements of the vacuome are present as granules which adhere to the Golgi bodies, and that the vacuome of the nurse-cells and of the follicle-cells is similar to that of the oocytes. In the older oocytes the vacuome and Golgi bodies occupy similar positions. The present contribution shows that the elements of the vacuome of *Stenophylax stellatus* are at first numerous, and are situated round the nuclear membrane of the early undifferentiated cells. At this stage the Golgi bodies are distributed in a closely similar manner. At a later stage, corresponding to the localisation of the Golgi material, the vacuome is represented by a single spherical body situated at one pole of the nucleus. In the young oocytes and nurse-cells the elements of the vacuome and the Golgi bodies are again distributed through the cytoplasm. There is, therefore, a close similarity between the distribution of the vacuome and the Golgi material during the early stages of oogenesis. During the present investigations it was not determined with certainty if the elements of the vacuome adhere to the Golgi bodies as described by Hirschlerowa. It is of interest that Hosselet (15) finds that the vacuome of the young ovarian cells of *Culex pipiens* occurs in contact with the localised "chondriome"; later, the vacuome becomes distributed with the elements of the "chondriome."

Payne (25), working on the oocytes of certain insects, identified vacuoles which are stained by neutral red but are not elements of the vacuome. He believes that in insects there is no vacuome in the sense of a substance common to all cells. Nath (21), Nath and Mehta (22), and Nath and Mohan (23) do not record the presence of the vacuome in the eggs of the insects upon which they worked, and the present writer could not find structures which stained with neutral red in the oocytes and nurse-cells of Tenthredinids (6) or in the oocytes of *Periplaneta orientalis* (8). It would appear, therefore, that vacuoles or granules which stain with neutral red are absent from the ovarian cells of certain insects, or that in some cases they are very difficult to identify. During the present investigations the vacuome of the young cells was usually brightly stained after a few minutes' treatment with neutral red.

There is a certain amount of disagreement as to whether the neutral red vacuoles are definite pre-existing cell structures or are formed after treatment with the stain. This matter has been discussed fully in recent literature and need not be reviewed here. The present findings show that in *Stenophylax* the vacuoles are not identical with the Golgi elements, and that their distribution resembles that of the latter. Their definite distribution in cells at different stages of development would appear to support the supposition that they represent some pre-existing cell structure.

V. SUMMARY

1. The present account of the growth and history of the ovarian elements of *Stenophylax stellatus* agrees fairly closely with Marshall's findings (17) for *Platyphylax designatus*. The fragmentation of the nucleoli and the occurrence of nucleolar extrusions in the cytoplasm of the nurse-cells agrees with Hosselet's observations on *Setodes tineiformis*. The present writer found no evidence in support of Hosselet's statement that the "chondriome" is regenerated at the expense of the nucleolar extrusions. The presence of nucleolar extrusions in the ooplasm is recorded in the eggs of caddis-flies.

2. The application of Feulgen's technique has thrown further light on the nature of the granules and threads in the nuclei of the undifferentiated cells, nurse-cells, and follicle-cells, and on the nature of the nucleoli. The absence of chromatin from the nucleoli of the oocytes and nurse-cells agrees with former findings for other insects. The presence of chromosomes throughout the growth period of the oocyte appears to be unique in insect oogenesis. The presence of degenerating cells in the ovarioles of caddis-flies has previously been recorded by Marshall (17): in the present paper it is shown that the nuclei of these cells are completely filled with chromatin. Some of the chromatin is extruded to the cytoplasm where it is visible as granules which give the chromatin reaction.

3. The chromosome number of *Stenophylax stellatus*, as determined by counts in oocytes and spermatocytes, is sixty.

4. The structure and behaviour of the Golgi bodies and mitochondria of the undifferentiated cells, nurse-cells, follicle-cells, and oocytes are described. The material called Golgi bodies or Golgi elements in the present contribution corresponds with the "chondriome" described by Hosselet in the ovary of *Setodes tineiformis*. The present paper describes the behaviour of the mitochondria and Golgi elements during the growth of the oocyte and stages of yolk-formation.

5. It is suggested that material derived from nucleolar extrusions is utilised in the oocyte in the formation of the protein yolk. Mitochondria and substances derived from the ooplasm may also take part in this process. The origin of the fatty yolk-globules from the Golgi elements is described;

it is probable that material derived from the ooplasm is added to the growing globules. The findings of other workers on yolk-formation in insects are discussed.

6. Vacuoles which are stained by neutral red are present in oocytes, nurse-cells, and follicle-cells. Their distribution resembles that of the Golgi elements.

REFERENCES TO LITERATURE.

(1) Beams, H. W., and Wu, C. F., 1929. "Cytological Studies on the Spinning Glands of *Platyphylax designatus* Walker (Trichoptera)," *Journ. Morph. and Physiol.*, vol. xlvii.

(2) Bell, A. W., 1929. "The Origin of Neutral Fats from the Golgi Apparatus of the Spermatid of the Dog," *ibid.*, vol. xlviii.

(3) Bhandari, K. G., and Nath, V., 1930. "Studies in the Origin of Yolk. V. Oogenesis of the Red Cotton Bug, *Dysdercus cingulatus*," *Zeit. f. Zellforsch. u. mikr. Anat.*, Bd x.

(4) Gatenby, J., Brontë, and Woodger, J. H., 1920. "On the Relationship between the Formation of Yolk and the Mitochondria and Golgi Apparatus during Oogenesis," *Journ Roy. Micr Soc.*

(5) Gresson, R. A. R, 1929. "Nucleolar Phenomena during Oogenesis in certain Tenthredinidæ," *Quart. Journ. Micr. Sci.*, vol. lxxiii.

(6) Gresson, R. A. R., 1929. "Yolk-formation in certain Tenthredinidæ," *ibid.*

(7) Gresson, R. A. R., 1930. "Certain Phenomena of Tenthredinid Oogenesis as revealed mainly by Feulgen's Nuclear Reaction," *ibid.*

(8) Gresson, R. A. R., 1931. "Yolk-formation in *Periplaneta orientalis*," *ibid*, vol. lxxiv.

(9) Gresson, R A. R., 1933. "A Study of the Cytoplasmic Inclusions and Nucleolar Phenomena during the Oogenesis of the Mouse," *ibid.*, vol. lxxv.

(10) Hirschler, J., and Hirschlerowa, Z., 1928. "L'appareil de Golgi et le Vacuome dans une certaine catégorie de Cellules somatiques chez la Larve de *Phryganea grandis* L (Trichoptera)," *C.R. Soc. de Biol.*, t. xcviii.

(11) Hirschlerowa, Z., 1928. "Les composants plasmatiques des Cellules génitales femelles de *Phryganea grandis* L. (Trichoptera)," *ibid.*, t. xcix.

(12) Hogben, L. T., 1920. "Studies on Synapsis. I. Oogenesis in the Hymenoptera," *Proc. Roy. Soc.*, B, vol. xci.

(13) Hogben, L. T., 1920. "Studies on Synapsis. II. Parallel Conjugation and the Prophase Complex in Periplaneta, with Special Reference to the Premeiotic Telophase," *ibid.*

(14) Hogben, L. T., 1921. "Studies on Synapsis. III. The Nuclear Organisation of the Germ Cells in *Libellula depressa*," *ibid.*, vol. xcii.

(15) Hosselet, C., 1931. "Contribution a l'étude du Chondriome chez les Insects (Culicides et Phryganides)," *Arch. Zool. Exper. et Gén.*, t. lxxii.

(16) Lutman, B. F., 1910. "The Spermatogenesis of the Caddis-fly (*Platyphylax designatus* Walker)," *Biol. Bull.*, vol. xix.

(17) MARSHALL, W. S., 1907. "The Early History of the Cellular Elements of the Ovary of a Phryganid, ***Platyphylax designatus*** Walker," ***Zeit. Wiss. Zool.***, Bd. lxxxvi.

(18) MUKERJI, R. N., 1930. "The Nucleal Reaction in ***Apanteles*** sp., with Special Reference to the Secondary Nuclei and the Germ-cell Determinant of the Egg," ***Proc. Roy. Soc.***, B, vol. cvi.

(19) NAKAHARA, W., 1917. "On the Physiology of the Nucleoli as seen in the Silk-gland Cells of certain Insects," ***Journ. Morph.***, vol. xxix.

(20) NATH, V., 1924. "Egg-follicle of *Culex*," *Quart. Journ. Micr. Sci.*, vol. lxix.

(21) NATH, V., 1929. "Studies on the Shape of the Golgi Apparatus. I. Egg-follicle of *Culex*," *Zeit. f. Zellforsch. u. mikr. Anat.*, Bd. viii.

(22) NATH, V., and MEHTA, D. R., 1929. "Studies in the Origin of Yolk. II. Oogenesis of the Firefly, *Luciola gorhami*," *Quart. Journ. Micr. Sci.*, vol. lxxiii.

(23) NATH, V., and MOHAN, P., 1929 "Studies in the Origin of Yolk. IV. Oogenesis of *Periplaneta americana*," *Journ. Morph. and Physiol.*, vol. xlviii

(24) NICHOLSON, A. J., 1921. "The Development of the Ovary and Ovarian Egg of the Mosquito, *Anopheles maculipennis* Meig.," *Quart. Journ. Micr. Sci.*, vol. lxv.

(25) PAYNE, F., 1932. "A Study of the Cytoplasm in Insect Ova," *Journ. Morph*, vol. liii.

(26) SANDERSON, A. R., 1933. "The Cytology of Parthenogenesis in Tenthredinidæ," *St Andrews University Publication*, No. 33, *Genetica*, vol. xiv.

(27) SPAUL, E. A., 1922. "The Gametogenesis of *Nepa cinerea* (Water Scorpion)," *Journ. Roy. Micr. Soc*

(28) STEOPOE, J., 1926. "L'appareil de Golgi dans la vitellogenese chez la *Nepa cinerea*," *C.R. Soc. de Biol*, t. xcv.

(29) STEOPOE, J., 1932. "Étude sur le Plasmosome et les Chromosomes sexuels dans la Spermatogenese de *Nepa cinerea* et *Naucoris cimicoides*," *Arch. Zool. Exper Gén.*, t. lxxii.

(30) VOINOV, D., 1925. "Les elements sexuels de *Gryllotalpa vulgaris* Latr.," *ibid.*, t. lxiii.

(31) WAJDA, ST., 1931. "Cytologische Untersuchungen über die Spinnstoffsekretion der Trichopterenlarven," ***Bull. Internat. Acad. Polon. Sci. et Lettres***, Bd. ii.

DESCRIPTION OF PLATES.

A.Y., protein yolk; Ch., chromosomes; C.G., chromatin granules; De., degenerating cells; F.C., follicle-cell; F.Y., fatty yolk; G.E., Golgi element; M., mitochondria; M.C., mitochondrial mass; N.C., nurse-cell; Nu., nucleolus; Nu.B., nucleolar bud; Nu.E., nucleolar extrusion; Nu.F., nucleolar fragment; Oo., oocyte; Pr., nuclei in prophase.

PLATE I.

Fig. 1. Ovariole from young larva. Bouin.

Fig. 2. Ovariole from older larva; cells in middle region are in early prophase; chromosomes present in nuclei at posterior end Bouin.

Fig. 3. Early oocyte and neighbouring nurse-cells. Young pupa. Bouin.

Fig. 4. Nurse-cells showing granules formed by fragmentation of chromosomes; nucleolus beginning to break up. Young pupa. Bouin.

Fig. 5. Older oocyte and nurse-cells. Young pupa. Bouin.

Fig. 6. Oocyte and two nurse-cells from posterior end of ovariole of young pupa. Bouin.

Fig. 7. Oocyte and two nurse-cells from posterior end of ovariole of older pupa. Bouin.

Fig. 8. Part of oocyte to show early stages of fatty yolk-formation. Pupa. Smith.

Fig. 9. Part of oocyte to show slightly later stage of fatty yolk-formation. Pupa. Smith.

Fig. 10. Nurse-cell, showing chromatin granules and nucleolar fragments in nucleus, and granules in cytoplasm. Pupa. Smith.

PLATE II.

Fig 11. Early oocyte and nurse-cells from anterior region of ovarioles of pupa. Mann-Kopsch.

Fig. 12. Older oocyte. Pupa. Mann-Kopsch.

Fig. 13. Older nurse-cell. Pupa. Mann-Kopsch.

Fig. 14. Oocyte showing dispersal of Golgi elements and mitochondria. Pupa. Mann-Kopsch.

Fig. 15. Oocyte; a few fatty yolk-globules are present. Pupa. Mann-Kopsch.

Fig. 16. Nurse-cell. Pupa. Mann-Kopsch.

Fig. 17. Late oocyte, showing fatty yolk-globules, Golgi elements, and mitochondria. Pupa. Mann-Kopsch.

Fig. 18. Follicle-cells, showing Golgi elements and mitochondria. Pupa. Mann-Kopsch.

Fig. 19. Older follicle-cells. Pupa. Mann-Kopsch.

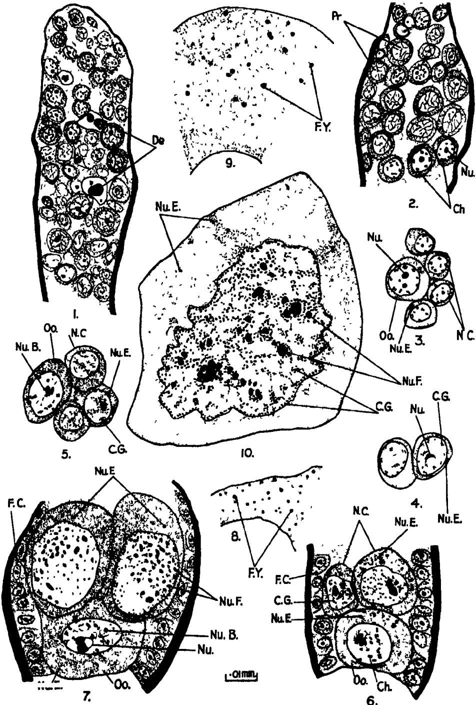
Fig. 20. Part of fully formed oocyte. Pupa. Mann-Kopsch.

Fig. 21. Undifferentiated cells. Larva. Flemming.

Fig. 22. Young oocyte. Larva. Flemming.

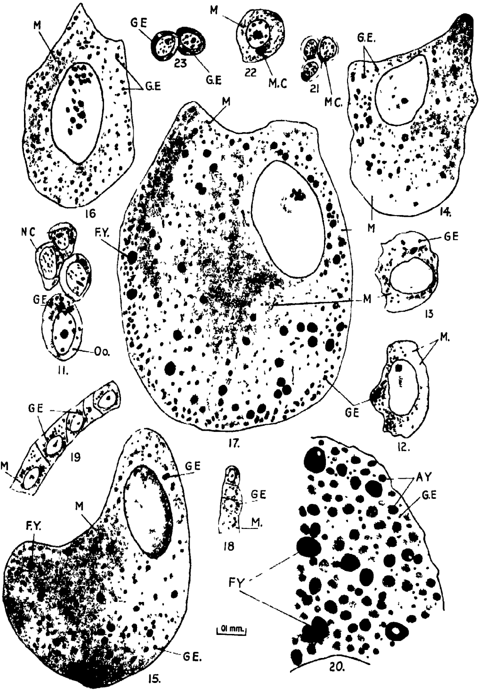
Fig. 23. Undifferentiated cells. Larva. Mann-Kopsch.

(*Issued separately October* 30, 1933.)



R. A. R. GRESSON

[PLATE I.



R. A. R. Gresson. [Plate II.

XXIII.—The Mathematical Representation of the Energy Levels of the Secondary Spectrum of Hydrogen. By Ian Sandeman, Ph.D., Late Carnegie Research Scholar, University of St Andrews.

(MS received May 10, 1933 Note received September 4, 1933 Read June 5, 1933)

1. *Term Representation of Diatomic Spectra.*

THE mathematical symbolism now employed by the practical spectroscopist for the representation of the energy term F(J) of the diatomic molecule has recently been fixed with precision as a result of the labours of Mulliken and others.* The formula employed to represent the rotational term is

$$F(J) = B_eJ(J+1) + D_eJ^2(J+1)^2 + F_eJ^3(J+1)^3 + H_eJ^4(J+1)^4 + \ . \qquad (1)$$

where J is the quantum number and B_e, D_e, . . . are constants. $B_eJ(J+1)$ is the kinetic energy, expressed in wave-numbers, which the molecule would have if it were a rigid rotator; and the remaining small correction terms in powers of $J(J+1)$ correspond to the expansion of the rotator under centrifugal force.†

If the effect of vibration of the nuclei is taken into account, we are not able as yet to obtain a representation from the new mechanics. The older theory of Kratzer,‡ however, gives us an idea of the changes which have to be made in formula (1) to allow for the effect of vibration on the rotational term. We have in fact to substitute B_v for B_e, D_v for D_e, . . . , v being the vibration quantum number distinguishing the state. Thus the formula becomes

$$F(J) = B_vJ(J+1) + D_vJ^2(J+1)^2 + F_vJ^3(J+1)^3 + H_vJ^4(J+1)^4 + \ . \ . \ . \qquad (2)$$

The following relations also hold between the constants of this formula and their equilibrium values used in formula (1):—

---

* See Professor O W. Richardson's paper on "Nomenclature and Symbolism for the Spectra of Diatomic Molecules," *Trans. Faraday Soc*, vol. xxv, 1929, p. 628.

† E. Schrödinger, *Collected Papers on Wave Mechanics*, 1929, p. 34 *et seq.* Here the results are, however, not given in the form familiar to spectroscopists quoted in equation (1).

‡ A. Kratzer, *Zeits. für Physik*, vol. iii, 1920, p. 289; *Phys. Zeits.*, vol. xxii, 1921, p. 552; and *Ann. der Physik*, vol. lxvii, 1922, p. 127.

$$\left.\begin{aligned} B_v &= B_e - \alpha(v+\tfrac{1}{2}) + \\ D_v &= D_e - \beta(v+\tfrac{1}{2}) + \\ F_v &= F_e - \gamma(v+\tfrac{1}{2}) + \\ H_v &= H_e - \delta(v+\tfrac{1}{2}) + \\ &\cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \end{aligned}\right\} \quad . \quad . \quad . \quad . \qquad (3)$$

where α, β, . . . are small quantities of rapidly diminishing magnitude, and v is the vibration quantum number. In the normal treatment of band spectra only the first small quantity α is taken into account.

The above formulæ (2) and (3) are hybrids, since they are based essentially on the old mechanics and borrow the form $J(J+1)$ and the half-quantum values of v from the new mechanics. They nevertheless represent diatomic data with great accuracy, and must be regarded as very near the mark.

2. *Difficulties in the Application of Band Theory to H_2.*

The difficulty of applying formula (2) to the known rotational terms of the hydrogen spectrum lies in the fact that the successive constants of this formula do not diminish with sufficient rapidity to compensate for the increase in the successive values of $J^n(J+1)^n$. It is always possible to obtain a very rough approximation to the term constants by neglecting the higher powers of $J(J+1)$. By doing so, however, we can never hope to achieve an accuracy comparable to the accuracy with which the wave-numbers of the lines have been measured; nor can the relations (3) be applied with any certainty. The writer has spent much time on the question and feels that the hydrogen terms can only be represented adequately by the infinite series (2). Any attempt to neglect the higher terms leads to unsatisfactory results, because the number of constants that can be calculated is limited by the data available for each band. If we stop, for example, at the coefficient of $J^3(J+1)^3$, we obtain values for the constants appreciably different from those obtained if we include the term in $J^4(J+1)^4$. It is thus not possible to compare the constants derived from a weak band with those derived from a strong band, nor to be sure of the accuracy of those derived from even the strongest band.

It is therefore necessary to approach the question afresh, and the writer believes that established band theory opens up a possibility of doing so.

3. *A Mathematical Transformation of the Term Form.*

A convenient summary of the present state of our knowledge of band theory is given in W. Jevons's *Report on Band-spectra of Diatomic*

Molecules (1932). The values of the successive constants of formula (1) taken from this Report are as follows:—

$$D_e = -4B_e^3/\omega_e^2,$$
$$F_e = 2D_e^2/B_e - \alpha\omega_e D_e^2/6B_e^3,$$
$$H_e = 3D_eF_e/B_e - 5D_e^3/B_e^2 + F_e^2/D_e - 8D_e^2x_e/3\omega_e.$$

The value of D_e given here is that of Kratzer; the values of F_e and H_e are due to Birge.* In the above expressions ω_e is the equilibrium value of the frequency of vibration, *i.e.* the frequency for infinitesimal amplitude; α is the constant appearing in formula (3), which is equal to $6B_e^2/\omega_e$ on Kratzer's theory; and x_e is another familiar constant of band spectra, which is equal on the same theory to $3B_e/\omega_e$.

We may use the known values of the constants, and eliminate every one except B_e and ω_e from the above expressions When this is done, we obtain

$$B_e = B_e.(2B_e/\omega_e)^0,$$
$$D_e = -B_e.(2B_e/\omega_e)^2,$$
$$F_e = B_e.(2B_e/\omega_e)^4,$$
$$H_e = -B_e.(2B_e/\omega_e)^6.$$

Accordingly, putting for simplicity $2B_e/\omega_e = u$, we may write formula (1), as far as the term in $J^4(J+1)^4$, in the form

$$F(J) = B_eJ(J+1)\{1 - u^2J(J+1) + u^4J^2(J+1)^2 - u^6J^3(J+1)^3\}.$$

Unfortunately we have no data regarding the values of the coefficients of the higher powers of $J(J+1)$. It seems not a very far-fetched assumption to allocate the following values to these small quantities:—

$$J_e = B_e\ (2B_e/\omega_e)^8,$$
$$L_e = -B_e.(2B_e/\omega_e)^{10},$$
$$N_e = B_e.(2B_e/\omega_e)^{12}.$$
$$\cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot$$

The successive constants would then be of the correct order of magnitude; and the assumption that they are alternately positive and negative seems a very likely one. The term form could then be written very simply as

$$F(J) = \frac{B_eJ(J+1)}{1 + u^2J(J+1)}. \qquad . \quad . \quad . \quad . \quad . \qquad (4)$$

If the assumption made above is correct, this formula should hold exactly.

---

* See "Molecular Spectra in Gases," *Bull. Nat. Res. Coun.*, 1926, p. 237.

We may attempt to apply the same form to formula (2), and write

$$F(J) = \frac{B_v J(J+1)}{1 + u_v^2 J(J+1)} \quad . \quad . \quad . \quad . \quad (5)$$

It is clear that we are not now dealing with an exact relation, although the expressions (3) show that, if (4) is right, (5) ought to hold to a close degree of approximation owing to the small size of the constants α, β, ...

The writer has found that expression (5) represents the terms of all the simpler states of the visible spectrum of hydrogen to a very good approximation. Expression (4) must therefore be, if not correct, a very close approximation to the truth.

4. *Application to the Hydrogen States.*

One illustration of the closeness of the fit will be sufficient. The null band of the Fulcher System provides six term differences of the form

$$\Delta_2 F(J) = F(J+1) - F(J-1)$$

for the final state ($2s^3\Sigma$)—as large a number as is provided by any of the visible bands. Using formula (5), and taking $B_0'' = 33{\cdot}37808$ and $u_0''^2 = 0{\cdot}000652059$, we obtain the following values of $\Delta_2 F(J)$:—

J.	$\Delta_2 F''(J)$ (calc.).	$\Delta_2 F''(J)$ (obsd.).	Obsd. – Calc.
1	199·49	199·51	+0·02
2	330·76	330·76	0·00
3	459·48	459·43	–0·05
4	584·70	584·64	–0·06
5	705·54	705·57	+0·03
6	821·19	821·24	+0·05

The upper states of the same band, for which there are five data, yield the following results, when we take $B_0' = 29{\cdot}83569$ and $u_0'^2 = 0{\cdot}000723569$:—

J.	$\Delta_2 F''(J)$ (calc.).	$\Delta_2 F''(J)$ (obsd.).	Obsd – Calc.
2	295·36	295·36	0·00
3	409·96	409·91	–0·05
4	521·10	521·05	–0·05
5	627·94	627·95	+0·01
6	729·68	729·67	–0·01

In calculating the last set a least-square method has not been employed, and the fit could be improved.

Expression (5) has been applied to some of the $1s^1\Sigma$, $2s^1\Sigma$, $2s^3\Sigma$, $2p^3\Pi$, and $3p^3\Pi$ states of H_2, and has been found to fit these states with similar accuracy.

The expression breaks down for the upper states of the $3d^3\Delta \rightarrow 2p^3\Pi$ bands which were discovered by O. W. Richardson and P. M. Davidson * and have recently been rearranged by the writer.* In this case the analogous formula representing the $3d^3\Delta$ states is very curious: it appears to be

---

* *Proc. Roy. Soc.*, A, vol. cxxxi, 1931, p. 658.

$$F(J) = \frac{B_v J(J+1)}{1 + u_v^2 J(J+1)} + \frac{B_v^2 J(J+1)}{1 + u_v^4 J(J+1)} \quad . \quad . \quad . \quad (6)$$

If we apply this formula to the state having the greatest number of term differences, viz. to the upper B state of the (1, 1) Band, and take

$$B_1' = 5{\cdot}48207, \qquad u_1'^2 = 0{\cdot}0302166,$$
$$B_1'^2 = 30{\cdot}05309, \qquad u_1'^4 = 0{\cdot}000913043,$$

we obtain

J.	F(J).	Differences	$\Delta_2F'(J)$ (calc)	$\Delta_2F'(J)$ (obsd.).	Obsd. – Calc.
2	207·18				
		197·83 (197·81)			
3	405·01		451·44	451·45	+0·01
		253·61 (253·64)			
4	658·62		558·81	558·80	–0·01
		305·20 (305·20)			
5	963·82		658·46	658·46	0·00
		353·26 (353·26)			
6	1317·08		751·32	751·33	+0·01
		398·06 (398·00)			
7	1715·14				

The figures placed in brackets after the differences are those obtained by applying an approximate representation to the lower states of the band. They are given on p. 407 of the paper quoted in the previous footnote. The fact that the formula gives a good approach to these, while fitting the $\Delta_2F(J)$ almost perfectly, gives ground for supposing it to be very near the mark.

The co-existence of two independent terms in the same level would be inadmissible on the old mechanics On the new mechanics, properly speaking, we can postulate no such things as energy levels. The possibility cannot therefore be entirely ruled out As, however, the terms

$$\frac{BJ(J+1)}{1 + u^2J(J+1)}$$

and

$$\frac{B^2J(J+1)}{1 + u^4J(J+1)}$$

are not independent, a more likely explanation is that the appearance of two terms is purely mathematical in origin, and points to the fact that in Kratzer's theory sufficient correction terms have not been taken into account. Instead of having $D_e = -u^2B_e +$, as on Kratzer's theory, it may be that we can write $D_e = -u^2 B_e(1 + u^2 B_e + \ )$. If this be so, it is a cause for satisfaction that the ultimate band law is capable of a second approximation in so simple a form.

---

* I. Sandeman, *Proc. Roy. Soc.*, A, vol. cxxxviii, 1932, p. 395.

Summary.

It is suggested that the term form for the rotational spectra of diatomic molecules may be written in the simple form

$$F(J) = \frac{B_e J(J+1)}{1 + u^2 J(J+1)},$$

where u is a contraction for $2B_e/\omega_e$.

This form fits the simpler states of H_2 with great accuracy. An extension of it is suggested for the $3d^3\Delta$ states to which the simple form is not applicable.

NOTE ADDED ON 24TH JULY 1933.

The above paper is based entirely on band theory. Such assumptions as are made concerning the small correction terms in the rotational expansion (1) are justifiable because these terms are well below the level of experimental determination. There are, however, certain steps in established band theory which must be looked upon with some suspicion, as they involve the omission of small terms which may not in fact be negligible. The theoretical approximate values of the constants α and x_e come under this category. Birge* has found that as a general practical rule, applying to all electronic states and all molecules,

$$x_e B_e/\alpha = \text{a constant} = 0{\cdot}7 \pm 0{\cdot}1.$$

The theoretical values of α and x_e which have been quoted previously give

$$x_e B_e/\alpha = 0{\cdot}5.$$

In the preceding analysis α affects the F_e constant, and α and x_e both affect the H_e constant, and it should be stated frankly that at least the values of the H_e constant given by the formula in this paper appear to be too small.

A comparison of the results following from this paper with those of O. W. Richardson and P. M. Davidson † on the energy functions of the hydrogen molecule is of interest. The writer's simplified band formula is equivalent to Kratzer's [*] potential energy expression

---

* See Jevons's *Report on Band-spectra of Diatomic Molecules*, 1932, p. 29.

† *Proc. Roy. Soc.*, A, vol. cxxv, 1929, p. 23.

$$E_{pot} = -a(A + 1/\rho - 1/2\rho^2 + b\xi^3 + c\xi^4 + \ldots),$$

when the small coefficients b, c, . . . are left out of account, *i.e.* precisely the same small constants which have been neglected in the values of a and x_e.

The variable part of this potential-energy expression can be thrown into the general form [†]

$$U = \text{const. } \xi^2(1 - a_1\xi + a_2\xi^2 - \ldots).$$

The writer's formula is equivalent to [‡]

$$U = \text{const. } \xi^2(1 - 2\xi + 3\xi^2 - \ldots).$$

Richardson and Davidson have estimated the values of the constants of the general formula for several of the hydrogen states. The expression found by them for the $1s3p'\Pi$ state (equation 26 of their paper) is, for example,

$$U(\text{volts}) = 5{\cdot}16\xi^2(1 - 2{\cdot}01\xi + 3{\cdot}19\xi^2 - 7\xi^3).$$

Successive coefficients of the first three terms are closely in the ratio 1 : 2 : 3. The expression found for the $1s2p^1\Sigma$ state is similar to this, but those found for the $1s3p^3\Pi$ and $1s2p^3\Sigma$ states differ somewhat widely.

A further word of caution is necessary with regard to the formula (6) quoted for the $3d^3\Delta$ states. It may be that the vibration quantum number should play a part in this formula, but is not in evidence because of being included in a factor equal to unity for this particular vibrational state.

---

[*] A. Kratzer, *loc. cit.*

[†] Since the relation between the variables ρ and ξ in Kratzer's expression is $\rho = \xi + 1$, substitution and expansion give

$$E_{pot} = -a(A + \tfrac{1}{2} - \tfrac{1}{2}\xi^2 + (1 + b)\xi^3 - (3/2 - c)\xi^4 + (2 + d)\xi^5 - \ldots),$$

which is the same as the general expression above, and the same as the writer's expression, if b, c, . . . are neglected.

[‡] This may be seen by substituting the writer's values of D_e, F_e, and H_e in the equations on p. 236 of Birge's article in the *Report on Molecular Spectra in Gases*, 1926, integrating Birge's force equation to obtain the equation of potential energy, and substituting the appropriate variable $\xi = (r - r_e)/r_e$.

(*Issued separately December* 22, 1933.)

OBITUARY NOTICES.

William Inglis Clark, D.Sc.

DR CLARK was born in Bombay on June 4, 1855, being the son of the Reverend Thomas G. Clark. He was educated at the Royal High School and University of Edinburgh. He passed as Pharmaceutical Chemist and took his degree in Science at an exceptionally early age. Clark entered the service of Messrs Duncan, Flockhart & Co. as an apprentice in 1871, and by his ability and application became a partner of the firm in 1880. He had a good knowledge of physics and his strong point was chemical engineering, the construction of large-scale plant for the production and purification of chloroform being an important part of his work. Dr Clark was for nine years a member of the Board of Examiners for Scotland of the Pharmaceutical Society, his special subject being Botany. His interest in plant life was shown by the manner in which he converted the comparatively uninteresting grounds of his house at Roslin into a charming resort. He had travelled extensively in Europe, Asia, and America. Another of his outside activities was mountaineering He had engaged in many climbs both in Scotland and abroad, was for some years President of the Scottish Mountaineering Club, and presented that institution with a hut on Ben Nevis in memory of his son who died of wounds in Mesopotamia in 1918. Colour photography was a subject to which Dr Clark devoted much attention, leading to the production of interesting lantern slides. These were used to illustrate his lectures, which were in much request, and by means of which considerable sums were raised for charitable purposes.

Dr Clark died at his house in Edinburgh on 21st December 1932. He is survived by his widow and a married daughter.

He was elected a Fellow of the Society in 1911.

D. B. D.

Sir Dugald Clerk, K.B.E., M.Inst.C.E., D.Sc., LL.D., F.R.S.

SIR DUGALD CLERK, who died on November 12, 1932, was one of the pioneers in the early development of the internal combustion engine. He was born in Glasgow on 31st March 1854. After completing his workshop training he spent four years studying, especially Physics and Chemistry, at the Andersonian College, Glasgow, and the Yorkshire College of Science, Leeds. Returning to Glasgow, he joined Messrs Thomson, Sterne & Co., and in 1877 began to devote himself to the development of the gas engine. It is interesting to recall that the four-stroke cycle Otto engine had been exhibited for the first time at the Paris Exposition in 1878. Clerk, realising the disadvantages of having only one working stroke in four, set to work to design a gas engine operating on a two-stroke cycle; his first engine working on this cycle made its appearance in 1881, and eventually many hundreds of this type were placed on the market. From this period right up to the time of his death Clerk devoted most of his work to the practical development of the internal combustion engine and to researches on the specific heat of gases and on explosion pressures. His researches had great influence on modern advance in the science of thermodynamics.

For these researches, which had placed him in the front rank of scientific investigators in the field of applied science, he was elected a Fellow of the Royal Society of London in 1908; in 1924 he was awarded the Royal Medal of the Society. He received honorary degrees from the Universities of Glasgow, St Andrews, Leeds, Liverpool, and Manchester. In 1932 he was elected President of the Institution of Civil Engineers, the highest professional distinction to which an engineer can attain; unfortunately ill-health prevented him from taking office. He had already in 1917 been created K.B.E.

On the outbreak of war in 1914, though by that time in his sixtieth year, he placed his exceptional talents in research at the disposal of the Government; he became Director of Engineering Research at the Admiralty, President of the Internal Combustion Engine Committee at the Air Ministry, and was a member of Committees in other fields of research.

He took great interest in the development of the motor vehicle, and did much important work in this branch of Mechanical Engineering.

For many years he was in partnership with Lord Marks as a consulting engineer and patent agent, and no other engineer was so frequently called upon for service as expert witness in patent cases.

As a member of the University Grants Committee, Clerk took a very real and keen interest in the teaching of Pure and Applied Science in the Universities, and University Engineering departments always benefited by the wise and sound advice he gave to the staff on the occasion of official visits.

He was elected a Fellow of the Society in 1922.

T. H. B

William Grant Craib, M.A.

PROFESSOR WILLIAM GRANT CRAIB, who died on September 1, 1933, was born at Banff on March 10, 1882. His death occurred after a few hours' illness, at Kew, where he had been spending part of his long vacation working in the Herbarium on the Siamese flora.

Craib received his early education at Banff and Fordyce Academies, and later entered Aberdeen University, taking the degree of M.A. in 1907. In 1908 he received a temporary appointment as Acting Curator of the Herbarium at the Royal Botanic Gardens, Calcutta. While in India he made large collections of plants from the North Cachar Hills, which he subsequently named. In 1909 he accepted the post of Assistant for India at Kew. At this time large consignments of Siamese plants were being received, and Craib's interest was roused in the flora of that area.

From 1911 onwards he published, under the title *Contributions to the Flora of Siam*, descriptions of numerous species. In 1925 he commenced the publication of what must be considered his principal work, his *Flora Siamensis Enumeratio*, and at the time of his death the first volume had been issued as well as the first part of Volume II. The manuscript of the second part of Volume II had also been sent to press.

During his undergraduate days Craib had shown his interest in the plants around Banff, and in 1912 he published the *Flora of Banffshire*, a list of flowering plants and ferns with their local distribution.

In 1915, on the invitation of Bayley Balfour, he became Lecturer in Forest Botany and Indian Trees in the University of Edinburgh, and while holding this post he published a series of papers on the *Regional Spread of Moisture in the Wood of Trees.*

While in Edinburgh in 1917 he married Mary Beatrice, daughter of the late Mr James Turner of Acton. Mrs Craib entered whole-heartedly into his work and was able to assist him in many ways.

He was elected a Fellow of the Royal Society of Edinburgh in 1920, and during the same year became a Fellow of the Linnean Society.

In 1920 he was appointed to the Regius Professorship of Botany at Aberdeen. Here he entered enthusiastically into the work of teaching and research. He devoted himself largely to Systematic Botany, and encouraged special research in the subject among his students, several of whom have made contributions to the flora of Siam. Unfortunately a serious accident in 1921, which resulted in the loss of a leg, prevented an active life.

Professor Craib was a stimulating teacher, and was able to inspire his students with some of his own enthusiasm. His botanical colleagues have lost a distinguished systematist who was always willing to give help and valuable advice.

M. W.

John Cuthbertson, M.B.E.

MR JOHN CUTHBERTSON was born at Kilmaurs, Ayrshire, in 1859. He was trained as a schoolmaster at the Free Church Training College, Glasgow, and at the University of Glasgow.

After holding various teaching appointments, he was appointed teacher of mining for Ayrshire and organised classes in this subject throughout the county. Later Mr Cuthbertson was made organiser of technical education under the Ayrshire County Council.

In 1889, on the foundation of the West of Scotland Agricultural College, Mr Cuthbertson was appointed Secretary and Treasurer, and for the remainder of his life was closely identified with the work of the College. He played an important part in its development from small beginnings to the important position it now holds in the scheme of Scottish agricultural education.

In addition to his professional work, Mr Cuthbertson took a wide interest in the public affairs of Ayrshire and was an active member of many committees and societies. He was awarded the M.B.E. for his work on the Food Production Committee of Ayrshire and on various organisations concerned with the welfare of the troops.

Mr Cuthbertson was a man of wide general culture and a gifted artist in water-colour. He retired in 1931, and his death on April 8, 1933, at the age of seventy-three, so soon after giving up his official duties, is regretted by a wide circle of friends.

He was elected a Fellow of the Society in 1904.

A. L.

William Hope Fowler, C.V.O., M.D., M.R.C.P., F.R.C.S.Edin.

DR WILLIAM HOPE FOWLER died in Edinburgh on October 4, 1933, after a long and painful illness due to the effects of exposure to X-rays.

Born in 1876, he received his early education at Daniel Stewart's College, and after completing his medical course at Edinburgh University he graduated M.B., Ch.B. in 1897. Immediately after his graduation he was appointed House Surgeon with Mr C. W. Cathcart.

Roentgen had discovered X-rays during the time when Fowler was a senior student, and after graduation he became one of the early pioneers of the use of X-rays in medical diagnosis.

In 1901, while still in general practice, he became an assistant radiologist to the Royal Infirmary, Edinburgh, under the late Dr Dawson Turner.

In 1911 he was appointed, in association with Dr Archibald M'Kendrick, as chief of the Radiological Department of the Infirmary, and in 1926 he was made a Consulting Radiologist to the Institution.

In 1924 he became a Fellow of the Royal College of Surgeons, Edinburgh, and in 1931 a Member of the Royal College of Physicians, Edinburgh.

From 1911 till 1926 he was Honorary Consulting Radiologist to the Admiralty, and during the Great War he was a member of the War Office X-ray Commission.

He was Lecturer and Examiner in Radiology to the Royal Colleges, and held numerous consulting radiological appointments. He was a Fellow of many learned societies.

His death is a loss which it is difficult to appraise. Long and intimate contact with him makes it even more difficult to say whether the loss sustained by the medical profession and the scientific world in general or by the general public is the greater.

He had many friends, and no enemies. As one of those fortunate friends I pay tribute to his memory. We started together in the Royal Infirmary, Edinburgh, in the days when our equipment was little more than a simple spark coil and a totally unprotected X-ray tube, and the X-ray department an old store-room in the basement. Up to the time of building the present X-ray department we worked together as colleagues, and strong though his personality was, it was impossible to quarrel with him. Even in those early days, working in dismal surroundings, when most of us exhibited frayed nerves, he emanated a cheery optimism which was infectious.

Ambitious he certainly was, but it was not a personal ambition. He did not strive to appear better than his fellow-workers. It was his ambition that his medical and surgical colleagues should be afforded the greatest help obtainable by improved X-ray technique. His private laboratory (and his lumber-room) bear witness to this side of his nature. He was always keen to procure the latest improvement, and he would purchase it, test it, and either add it to his already large equipment or relegate it to his lumber-room. It is questionable if a more complete history of the advancement of X-ray technique could be found anywhere than in his private museum.

Perhaps one of his greatest assets was his phenomenal memory. It was like a well-kept diary. Old Edinburgh students returning to the Royal Infirmary after many years of absence used to seek him out in the old department. In an almost unbelievable way he could recall the names of the students of any single year, when they qualified, where they had gone, how they had progressed in their profession, and a host of details which would always appeal to the returned student.

He maintained an intimate connection with his old school, and recently presented four Bursaries for Stewartonians at Edinburgh University; for several years also he was President of Stewart's Athletic Club, and President of the former pupils club of this school.

He took all the happiness out of life, and made his life a source of happiness to others. Even in the more acute stages of his painful affliction he was never known to complain. He has bequeathed to all who knew him nothing but the happiest of memories.

He was elected a Fellow of the Society in March 1933.

A. M'K.

Frederick Gardiner, M.D., B.Sc., F.R.C.S.E.

DR FREDERICK GARDINER died in Edinburgh on September 8, 1933, after a long illness. He was born in Edinburgh in 1874, and studied medicine at Edinburgh University, graduating M.B., C.M. in 1895. After holding a surgical resident post in Stirling Royal Infirmary, he became Resident Medical Officer at the Dundee Royal Asylum. Later he returned to Edinburgh and took up general practice in the city. He graduated B.Sc. in Public Health in 1901 and M.D. in 1902. In 1904 he was admitted a Fellow of the Royal College of Surgeons. As soon as Dr Gardiner returned to Edinburgh he began to take an interest in X-rays, and did a great deal of pioneer work on that subject. His thesis for the M.D. degree was written on the X-ray appearances in the lungs in pulmonary tuberculosis, and was awarded special commendation. He was appointed radiologist to Leith Hospital in 1901, but gave up that post some months later on his Infirmary appointment. In addition to his other activities he worked as clinical assistant in the Skin Department of the Royal Infirmary, and in 1904 he was appointed an assistant physician to that department. In 1912 he became a full physician to the department and started teaching Dermatology as a lecturer in the University and Extra-mural School. After serving his period of fifteen years as physician and lecturer he was reappointed annually for a further period of six years.

Up till the time of his last illness Dr Gardiner spent a very active life. In addition to attending to a large general and consulting practice, he found time to work in the Laboratory of the Royal College of Physicians. He investigated several dermatological subjects, chief of which was the penetrating power through the skin of various greases. The results of his work were published in various medical journals. For twenty-one years he taught dermatology to successive classes of University and College students, and also took part regularly in the post-graduate teaching. His *Handbook of Skin Diseases* for students reached a third edition. During his whole life he kept up his interest in X-rays, and his early radiological experience enabled him to do much pioneer work in the treatment of skin diseases with X-rays. During the War he acted as one of the Consultants in Dermatology for the Edinburgh area at the Edinburgh War Hospital at Bangour. Everything Gardiner undertook he did with great keenness and energy, and, owing to his cheerful disposition, he was very popular with his patients and students.

He was elected a Fellow of the Society in 1923.

R. C. L

John Glaister, M.D., F.R.F.P.S.G., LL.D.

DR JOHN GLAISTER died at his residence on December 17, 1932.

Born in Lanark in 1856, John Glaister began his career in a law office, but shortly afterwards gave this up to enter upon the study of medicine at Glasgow University, from which he graduated as Bachelor of Medicine and Surgery in 1879. Almost from the time of his graduation he made a special study of the subjects of Forensic Medicine and Public Health, and in 1881 he commenced to teach Forsenic Medicine at the Royal Infirmary School in Glasgow.

Seven years after this he was elected Professor of Forensic Medicine in St Mungo's College, which position he held until 1898, when he succeeded Professor Simpson in the Chair of Forensic Medicine and Public Health in the University of Glasgow. When the University decided to separate the departments of Forensic Medicine and Public Health, Glaister elected to retain the former, and this he held until his retirement about a year before his death.

Glaister was one of the best-known practitioners in Medical Jurisprudence in this country, and acted for the Crown in almost all the medico-legal cases in Glasgow for a great many years. His services have been of the greatest value in the elucidation of many criminal cases, and his unique experience of crime and criminals will be greatly missed by the legal authorities.

Although Glaister did not interest himself particularly with scientific research, his enormous practical experience led to his recognition as an authority on his particular subject, and his textbook on Medical Jurisprudence has become one of the standard works in the English language.

Professor Glaister was a Fellow and former President of the Royal Faculty of Physicians and Surgeons of Glasgow, a Licentiate of the Royal College of Physicians and Surgeons of Edinburgh, and a Diplomate in Public Health of the University of Cambridge. He was also a Fellow of many societies both at home and abroad, among others at home being the Royal Society of Edinburgh (1898), the Chemical Society, the Royal Sanitary Institute, and the Royal Institute of Public Health. He was a Vice-President of the Medico-Legal Societies of New York and of Great Britain, and an Honorary Academician of the Physico-Chemical Academy of Palermo.

Dr Glaister's many activities in connection with medicine did not prevent him taking part in public life. His services as President of the Conservative Association of the St Rollox Parliamentary Division were for many years of great value to the cause of Unionism. He was in close touch with many civic institutions. He represented the Faculty of Physicians and Surgeons on the Board of Governors of the Glasgow School of Art, and was one of the first lay members of the Glasgow Art Club. He was interested in the subject of cremation, and was connected with the Scottish Burial and Cremation Society.

Dr Glaister had a distinguished career, and the valuable work which he did will long be remembered.

S. S.

William Jacob Holland, LL.D.

By the death on December 13, 1932, of Dr W. J. Holland, Director-Emeritus of the Carnegie Museum in Pittsburgh, Pennsylvania, America lost a distinguished man of science, and a large circle a genial and charming associate and friend. Born in Jamaica on August 16, 1848, of American parents, his thoughts first turned towards the Church, and after graduating M.A. with highest honours at Amherst College, he entered Princeton Seminary to study for the ministry. From boyhood he had been interested in entomology, and during a pastorate of seventeen years in Pittsburgh he continued his scientific studies to such purpose, with numerous publications, that he was selected as naturalist to the United States Government Eclipse Expedition to Japan in 1887. Four years later he was appointed Chancellor of the Western University of Pennsylvania, now the University of Pittsburgh.

In 1898 he assumed the duties of Director of the Carnegie Museum, Pittsburgh, which offered more scope for his scientific interests, and he held the post till 1922, when he became Director-Emeritus. Meanwhile he founded the American Association of Museums in 1907, of which he was President until 1909. His wide culture and organising abilities led to his being a coveted member for various associations and corporations. Thus he was Vice-President of the Carnegie Hero Fund from 1904 to 1922, and President thereafter. As such he was, *ex officio*, a member of the Carnegie Corporation of New York, and a trustee for many academic institutions.

During several years Dr Holland directed palæontological explorations in Utah, Wyoming, Montana, and North Dakota which resulted in highly important scientific finds, including the *Diplodocus carnegiei*, the original specimen of which is set up in the Carnegie Museum. He continued to publish great numbers of papers in his particular branch, now being regarded as the dean of American entomologists, while at the same time he edited the *Annals* and *Memoirs* of the Museum. Of his many books, *The Butterfly Book* and *The Moth Book*, popular guides to the North American forms, are well known in the States. Membership, ordinary and honorary, of a host of scientific societies (mainly entomological) at home and abroad, recognition by decorations from five European States of his contribution to science, and at least half a dozen honorary degrees, including one greatly valued from St Andrews University, left the rugged simplicity of his character absolutely unspoiled. He worked hard up to the end in the rooms reserved for him in the Carnegie Museum, and it is characteristic of the man that the same number of the *Carnegie Magazine* that contained an article on "The Passing of Dr Holland" should also contain possibly his last paper, a popular account of how he secured the skin and skull of the mounted specimen of the White Rhinoceros (*R. simus* Burchell) that is a feature of the Museum.

He was elected a Fellow of the Society in 1911.

J. Y. S.

Robert T. A. Innes.

DR R. T. A. INNES was born in Edinburgh in 1861. He showed early his interest in astronomy, and was elected a Fellow of the Royal Astronomical Society in 1879, when he was no more than eighteen years of age. Going to Australia while still a young man, his zeal for the science continued, and he showed by his own discoveries of double stars the immense field of unexplored work that the southern heavens contained. He attracted the notice of Gill, then in charge at the Cape, at whose invitation to a post in the Royal Observatory he came to South Africa. He remained at Cape Town until 1903, when on Gill's recommendation he was put in charge of the Observatory at Johannesburg, where he remained until his retirement in 1927. During that period he quite transformed the Observatory, which up till then had been little more than a meteorological station. Besides making good use of the instruments he found available, he acquired in succession, by gifts or government grants, important additions. The late Mr Franklin Adams, choosing Johannesburg for the site of his survey of the Southern skies, made a present of the instruments with which he executed it to the Observatory. The same donor made other important gifts also. The chief instrument of the Observatory is a 26½-inch refractor by Grubb, long in construction and delivered only in 1925, but yet put to good and effective use. An instrument of which Innes early impressed the use was a "blink" comparator, for the discovery of exceptional objects upon a crowded photographic plate.

Innes's interests were by no means solely observational. He pursued theoretical researches too, including the rotation of the Earth, perturbational work, and Jupiter's satellites. He was in all respects an unconventional man, caring little whether his words or acts met with the approval of the powers that are supposed to be, provided that he was satisfied that what he said and did was accurate. Subsequent to his retirement, he came to the London district, where he died on March 13, 1933.

He was elected Fellow of the Society in 1904, and has contributed to our *Proceedings* (vols. xxvii and xlii, 1907 and 1922).

R. A. S.

John Edward Marr, Sc.D., F.R.S.

THE death of Professor Marr is a great loss to geological science. During the past generation he maintained the high reputation of the "Cambridge School" and was pre-eminent as a teacher. His former students are scattered all over the world, and many of them fill important posts in the teaching profession and on the staff of Geological Surveys. Born in 1857, he was a Lancashire man and most of his geological work was carried out in the Lake District. He was educated at Lancaster Grammar School and at St John's College, Cambridge, where he became a Scholar. He obtained a First Class in the Natural Science Tripos and was elected a Fellow of his College. Much of his training was obtained from Professor Bonney, who was his College Tutor. For some time he acted as a Lecturer in Geology under the University Extension Scheme, and for a period he acted as deputy for Professor A. H. Green at Leeds. In 1886 he was appointed University Lecturer in Geology at Cambridge, a post which he filled with such distinction that on the death of Professor Hughes in 1917 he was elected as successor to the Chair of Geology, which he held until 1930 when he retired on account of ill-health.

His special spheres of work were in physical and stratigraphical geology. He published numerous papers, the most important of which dealt with the older rocks of the Lake District and North-West Yorkshire. During his later years he devoted much attention to the Pleistocene and Recent Deposits of Cambridge and the adjoining areas.

Professor Marr was the recipient of many distinctions. He was elected a Fellow of the Royal Society of London in 1891, served on the Council for two years, and received a Royal Medal in 1930. For a period he served as Secretary to the Geological Society of London, from which he received the Lyell Medal and the Wollaston Medal He was President of the Society in 1904–1906. In 1930 he was elected an Honorary Fellow of the Royal Society of Edinburgh.

He died at Cambridge on October 2, 1933, at the age of seventy-six.

Professor Marr will be greatly missed by a host of geologists to whom his teaching proved an inspiration.

T. J. J.

Ralph Richardson, W.S.

MR RALPH RICHARDSON, who died on June 26, 1933, at the advanced age of eighty-seven, was one of the oldest Fellows of the Society. A son of the late Mr James Richardson, F.R S.E., he was elected a Fellow in 1875, and during his long lifetime continued to interest himself in various subjects of scientific importance. He was one of the founders of the Royal Scottish Geographical Society in 1884, and for the first ten years an Hon. Secretary, and a Vice-President from 1924. He was also for several years Hon. Secretary of the Edinburgh Geological Society and a Fellow of the Scottish Society of Antiquaries.

He was a man of wealth and leisure in his later years, and since 1883 had held the office of H.M. Commissary Clerk, which did not prevent him from devoting much time to travel and observation of things of interest abroad and at home and to the welfare of the community in Edinburgh

Between 1889 and 1914 he communicated numerous papers to the *Scot. Geogr. Mag.*, and previously also to the *Trans. Edin. Geol. Soc.* By profession a Writer to the Signet, his legal help and advice was freely given and much valued, both while he acted as Hon. Secretary to these societies and in after years. One of his first papers, written in 1874, was on "Peat as a Substitute for Coal," at that time in every sense a burning question while the coal famine was at its height after the Franco-German War. On January 18, 1889, Edinburgh was slightly shaken by an earth tremor that was felt over the whole district. Mr Richardson took much pains to collect all the evidence from persons affected, and published an account of all their experiences in the *Scot. Geogr. Mag.* for 1889, which is of real scientific value.

He married a daughter of the late Dr Andrew Fleming of the Indian Medical Service by whom he is survived.

H. M. C.

Johannes Schmidt.

JOHANNES SCHMIDT died in Copenhagen on February 22, 1933, at the all too early age of fifty-six years. He was a sturdy man to all appearance, and young-looking to the last; but heart trouble, which had long threatened him, laid him low. Schmidt was a very great naturalist, in a time when great naturalists have been few. His diligence kept pace all his life long with his opportunities, and both were ample; his learning and his originality were one as conspicuous as the other. He had the good fortune to solve, and to solve completely so far as we can see, a problem as old as Natural History itself, the ancient mystery of the place and manner of reproduction of the eel. This was his chief and crowning triumph, and brought him world-wide fame; but it was far from being the only work he did.

Nothing in biological science came amiss to him, and he was eager to borrow from the other sciences what they could lend to biology. He was botanist, zoologist and physicist by turns; he was one of the first to see that the distribution of a fish in the sea was a problem in oceanography, and called for all one could discover of the varying temperature, salinity and oxygen-content, and the wide-flung currents of the sea. He was no less quick to use statistical methods and the theory of probabilities in apparently simple but actually subtle and recondite questions as to local races or allied species of fish.

Two thousand years ago and more, Aristotle knew a good deal about eels, how to keep and feed them, how, where and when to capture them; but he thought that they proceeded neither from pairing nor from the egg, and was content to suppose that they came forth out of the mud, as worms which the sun had begotten of putrefaction. A great Italian, Redi, had a glimpse of the truth nearly three hundred years ago; he saw, I suppose, the eels going downstream in the dark moonless nights of autumn and the elvers coming up the rivers in spring; and he drew the bold, if simple, inference that they bred somewhere out at sea. The story took a long time to tell. A little thin transparent fish, sent in the eighteenth century from Anglesey to Pennant, was recognised a hundred years later to be the peculiar larva of an eel; and this, the so-called *Leptocephalus*, was presently found in numbers, and of several kinds, in the Mediterranean, especially in the Straits of Messina, where Charybdis turns the waters upside down and brings strange creatures out of the depths into the light of day. Here, for many years, Grassi had studied the problem of the eel, and had discovered much before, some thirty years ago, Johannes Schmidt's work began. On a fishery-cruise on board the ***Thor***, Schmidt happened to find a *Leptocephalus*-larva to the westward of Faeroe; and searching the North Atlantic for two whole summers (1904–5) he found the same larvæ all the way from Spain to Iceland and far to the westward, but never off the Norwegian coast nor in the North Sea. In the spring he found only larvæ, but later on he began to find them metamorphosed into elvers or little eels. It was clear that the eels were breeding far off in the west: it was remarkable that no eels at all were found in the South Atlantic, nor even as far south as the Equator. Two or three years later Schmidt began to explore the Mediterranean, and found that there were no eel-larvæ farther east than the west coast of Italy, or the Straits of Messina; and even there, there were no young larvæ but only large ones, not far off the time of their metamorphosis In short the Mediterranean, which had seemed in Grassi's time to be a breeding-ground for eels, was found to breed none at all; they all came in from the Atlantic, helped along by the steady current running in through the Straits of Gibraltar. The larvæ take three months or so on their way from Gibraltar to Messina, and there are few eels or none in the Black Sea; they have all turned aside, into one river or another, before they get there

Johannes Schmidt's voyages of discovery were interrupted by the War, but by 1921 he was out in the Atlantic again; and in this year and the next he tracked down the eel to its actual breeding-place, south-west of Bermuda, in the salt warm waters of the Sargasso Sea. Once in their lives the old eels resort to this far-off breeding-place; they spawn there, but they come back no more. Only the little larvæ, wafted by the currents, come eastward home on a voyage three years long; and turning into elvers as they go, they grope their way at last into this river or that, from Iceland to the Baltic, from the Baltic to the Nile, as chance may lead them Not the least curious part of the story is that the American eels, differing slightly but unmistakably (so Schmidt declares) from the European stock, breed in the self-same waters or close by; but their progeny are somehow guided, westward instead of eastward, to American rivers instead of ours. In the South Atlantic there seem to be no waters so warm or so salt as those of the Sargasso Sea, nor is there any system of currents setting to the shore; these seem to be the reasons why no larval eels are to be met with there. But in many other parts of the world, in the East Indies, in the archipelagos of the Pacific, and very notably in New Zealand (where eels were very important to the Maori)—in all of these regions eels are plentiful; and in his later years Schmidt made a round-the-world cruise with the special object of tracking these many eels to their breeding-places in the Southern Oceans.

Johannes Schmidt was a born traveller. After his student days in the University of Copenhagen he went at once to Siam, as botanist to a Danish Expedition; he brought home very large collections, and for many years after, while busily engaged in other work, he continued to edit the *Flora of Koh Chang*, and contributed several sections to that work, on the Mangroves, the Combretaceæ, and other families. He had already written, with a colleague, a useful textbook on the Bacteria, soon translated into German; and an elaborate account of the Danish Cyanophyceæ or Blue-green Algæ.

Johannes Schmidt was a member for many years of the International Council for the Investigation of the Sea. In this capacity he did a deal of work, chiefly in connection with the cod and other Gadoid fishes, whose spawning-grounds, from Iceland to Spain, he traced out as diligently as the eel's. In this investigation he made full use of his knowledge of hydrography; and showed how the distribution and migrations of the cod were simply determined by the physical phenomena of the sea

As Director of the Carlsberg Laboratory he became interested in hops, and published a series of papers on their growth, breeding, and variation. Lastly, he made a number of laborious and important studies of variation in particular fishes, eels among the rest. He was especially interested in those many cases where two races, varieties or so-called species, were so nearly alike that a single individual of either could not safely be identified; but statistical evidence, the average characters of a large random sample of individuals, gave characters about which there could be no mistake. The European and American races of the common eel are a case in point; both vary in such a way that their characters overlap, and a single specimen cannot be safely referred to either. But a count of the vertebræ in a hundred individuals of each shows an average of nearly ten vertebræ more in the one group than in the other. In a certain inshore fish, common in the Baltic, the viviparous blenny, he found a similar difference between the fish in the fjords and those in the open sea, even between those in one fjord and another; he found that these small differences were in the main hereditary, as he followed them through six generations or more; but yet, after all, he discovered that a change of conditions, or environment, did produce a small but perceptible change of characters. These statistical studies, and the whole concept of racial groups on which they are based, have an importance that as yet we are slow to realise.

Johannes Schmidt was a fortunate man. His marriage was singularly happy, and his wife became a friend of his friends and a worker by his side; it associated him closely with the Carlsberg Brewery, and helped him to a share (without which his work could never have been done) in the munificent trust under which the profits of that great business go to the advancement of knowledge. He was a member of many Academies besides ours, into which he was elected in 1927; he had decorations from ten countries (I believe) besides his own He was at home in several languages. His life was as simple as it was happy; he was modest as he was learned; he was big-hearted and most generous. I found him a rarely lovable man during a friendship of more than thirty years.

D. W. T.

John Stephenson, C.I.E., F.R.S.

JOHN STEPHENSON was born at Padiham, near Burnley, Lancashire, in 1871 and was educated at the Burnley Grammar School and the Owens College, Manchester, which he entered as a student of science in 1887. In 1890 he passed the examinations for the degree of B.Sc., London, with second-class honours in Zoology. In the same year he was awarded the English Essay Prize in the Owens College for an essay on the poetry of Wordsworth; he had throughout his life a strong interest in English literature.

Stephenson then entered the Medical School of the Owens College, in which he had a distinguished career; a fellow-student states that he was easily the best man of his year. In 1891 he passed the London Intermediate M.B. in the first class with Exhibition and Medal in Anatomy and Medal in Physiology; in 1893 he graduated M.B., Victoria University, and in 1894 M.B., London, both with first-class honours. During his last year he held appointments as house physician for six months, first in the Royal Infirmary, Manchester, and later in the Royal Hospital for Diseases of the Chest, London.

In 1895 Stephenson entered the Indian Medical Service, being first of his batch. In the same year he married Gertrude, elder daughter of Mr Thomas Bayne, of Burnley. For the next five years he was on military duty with the 22nd Punjab Cavalry, including service in the North-West Frontier Expedition of 1897. During the following years he was on plague duty, and held appointments as Civil Surgeon at four stations in the Punjab. His spare time during these years was devoted to the study of Persian, and while Civil Surgeon of Murree in 1901 he passed the High Proficiency Persian examination. In 1905 he was home on leave and passed the examinations for the Fellowship of the College of Surgeons of England.

In October of the following year Captain Stephenson, then Civil Surgeon of Ambala, was "appointed temporarily" at about a week's notice to the newly created Chair of Biology in Government College,* Lahore, an appointment which was to fill his life for thirteen years.

---

* The teaching of biology was instituted in the College in 1902, when an Assistant Professor was appointed; he was transferred to the Agricultural College, Poona, in the vacation of 1906.

Stephenson had studied zoology in the Owens College under the stimulating influence of Professor Milnes Marshall, and building on this foundation his instruction in the subject soon reached a high standard, and within a few years resulted in a productive school of zoology. The expansion of the work in biology rendered necessary new laboratories, which Stephenson planned with regard to the special conditions of climate, and they were ready for occupation in 1914. That his presentation of the subject was successful in appealing to his students is shown by the fact that several of them became inspired with his ideals of critical work, and responded to his suggestions for the investigation of the fauna of their country. They are now holding influential zoological posts in India and have published important papers on their researches. Stephenson thus made an important contribution to the development of zoological teaching and research in India.

For his own investigations Stephenson chose the anatomy and biology of the freshwater oligochætes occurring near Lahore, and began work on them soon after his appointment to the Chair. His first paper, on an oligochæte allied to *Chætogaster* which he had found in February 1907, was published in August of that year, and was quickly followed by another paper on two freshwater oligochætes—an *Æolosoma* and a new species of *Chætogaster*. In 1909 he submitted to the University of London a thesis, which was approved for the degree of Doctor of Science, on the aquatic Oligochæta of the Punjab, the most interesting part of which is an account of the phenomena of antiperistalsis and ascending ciliary action in the intestine of these worms, with some general considerations arising therefrom. Stephenson showed that the inhalent function of the intestine is common in these worms, and suggested that it is possibly the primitive function—the intake of oxygen in solution—and he added some critical considerations on the evolution of the vascular system in oligochætes. In 1909 when on leave he spent some time at the Marine Biological Station, Millport, working on the littoral Oligochæta of the Clyde, on which he contributed a paper to the Society's *Transactions* in 1911. On his next leave he worked for a month in the summer of 1912 at the Marine Laboratory, Plymouth, chiefly on intestinal respiration in Polychæta, and he then expanded his thesis of 1909 so that it constituted a study of intestinal respiration in annelids, with considerations on the origin and evolution of the vascular system in this group. This paper was communicated to the Society and published in 1913.

About this time Stephenson received a collection of earthworms, made, during the Abor Expedition, in Assam and the Abor country—

the first of the many collections from the Oriental Region which came into his hands—and his interests henceforth widened to the oligochætes in general. Throughout his period in Lahore he maintained a steady output of published work on collections of these worms from India, Japan, and China, and on structural details, *e g.* the cells of the pharyngeal glands and septal glands, the prostate glands (with Haru Ram) and the calciferous glands (with B. Prashad).

In 1912 Stephenson was appointed Principal and in 1918 Vice-Chancellor of Government College. The strain of his duties during the war years—an anxious period for those in charge of youth in India—began to tell upon his health, and in the spring of 1919 he decided to resign his appointments in Lahore, which he vacated in September of that year to the great regret of staff and students. He was a highly successful Professor and Principal, kind, firm, and understanding. His dealings with the students were characterised by unfailing courtesy and by scrupulous fairness, which gained their admiration and respect, and his quiet effective personality left a deep impression upon them.

In June 1920 I was able to offer Lieutenant-Colonel Stephenson a Lectureship in Zoology which had been recently established in the University of Edinburgh, the duties of which consisted principally in directing students in their work on invertebrates. This appealed to him as a congenial post, and he commenced duty in October of that year. For nine years he shared with me the lectures on invertebrates and on a number of special subjects given to the advanced students. His teaching was clear and concise, and characterised by a wide scientific outlook; in the laboratory his critical supervision was greatly appreciated, especially by the students reading for honours, to whom he gave unsparingly help and encouragement. I may be permitted to add that he was a most efficient and trusted colleague and a loyal friend.

Stephenson was assiduous and methodical in research. He continued his investigations on earthworms, and published while in Edinburgh a score of papers. The majority of these were on systematics and distribution, others described the septal and pharyngeal glands (a continuation of previous work), blood glands, the diffuse production of sexual cells in a species of *Chætogaster*, and the sexual organs of *Naidium*. In a paper published in 1921 Stephenson discussed evolution in the family Megascolecidæ; he showed that, knowing which characters are primary and which secondary, it is possible to work out the sequence of evolution of the principal genera and that a number of the genera have had a multiple origin—that is, are polyphyletic. In the same paper he set forth his views on problems of distribution in the Oligochæta, stating his objections to several of the land-bridges, *e.g.* the Indo-Australian, which had been postulated in order to explain the present range of distribution of certain genera and offering alternative views. In 1923 he published a volume of 518 pages on the Oligochæta in *The Fauna of British India.* After the list of references to the literature, which formed a complete bibliography for the species found in India, he gave a useful account of the methods of examination of oligochætes, followed by a summary of the geographical distribution of Indian Oligochæta, with remarks on their bionomics, and then passed to the systematic description of the fifty-five genera and three hundred and forty-five species recorded from India. This work is notable for its clarity and method, and for the practical treatment of the subject which was the outcome of the author's intimate knowledge of his material derived from innumerable dissections and other preparations.

His last work in Edinburgh was the preparation of a monograph on the Oligochæta. In 1927 he was appointed a Carnegie Teaching Fellow and relieved of all duties except those in connection with the senior students, and henceforward he devoted the major part of his time to the preparation of the monograph (published by the Clarendon Press in January 1930), which he completed and saw through the press before he left Edinburgh. He brought to the preparation of this volume of almost a thousand pages a thorough mastery of the structure and classification of oligochætes, an extensive knowledge of the literature, and a flair for concise and clear presentation of facts and conclusions. He produced a monograph of exceptional merit which, in addition to the anatomical and systematic descriptions, contains a summary of his conclusions on the evolution of oligochætes, on convergence and polyphyly, and on the bearing of the known facts of geographical distribution of oligochætes on palæogeographical problems such as the former existence of a more extensive antarctic continent and of Indo-Australian and other land-bridges. His nine years in Edinburgh were his most productive period in published work; he became one of the two recognised authorities on the Oligochæta.

In 1928 Stephenson was appointed Editor of *The Fauna of British India* in succession to the late Sir Arthur Shipley, and one of the Editors of *The Annals and Magazine of Natural History.*

On leaving Edinburgh in November 1929 he went to reside in London, and continued until a few weeks before his death his investigations on oligochætes as an unofficial worker in the British Museum (Natural History). These resulted in several systematic papers, including one on a new species of *Nais* living parasitically under the eye and in the

Harderian glands of East African frogs. In 1931 he became Zoological Secretary of the Linnean Society and took an active interest in its affairs, and especially in the publication of the zoological papers.

Stephenson was made, in 1919, a Companion of the Order of the Indian Empire in recognition of his services in Lahore. He was elected a Fellow of the Royal Society of Edinburgh in 1912 and was awarded the Keith Medal in 1920. He was elected a Fellow of the Royal Society of London in 1930.

He regarded as a recreation his studies—which were carried out with his usual thoroughness and care—on Persian and Arabic literature. In 1910 the Asiatic Society of Bengal published his translation of the first book of the *Hadīqatu' l-Haqīqat* ("Enclosed Garden of the Truth") of Sanā'ī. The original poem, written probably about 1140 A.D., is Sanā'ī's most important work and one of the early Persian text-books of the Sufi philosophy. On finding there was no European edition or translation Stephenson collated half a dozen manuscript and lithographic editions in the preparation of the text and made a translation, appending numerous notes. He also translated the zoological section of the *Nushatu-l-Qulūb* ("Heart's Delight"), which was published by the Royal Asiatic Society in 1928. This work, which was compiled by a Persian writer, Hamdullāh Mustaufī of Qazwīn, about 1340, gave a conspectus of zoological knowledge of the time, and "constitutes one of the very few extant zoological treatises of the Islamic East."

Stephenson was held in high esteem for his attractive personality, his sound judgment, and his readiness to help in matters relating to the advancement of zoology in Britain and in India.

He died in London on the 2nd February 1933.

J. H. A.

Sir J. Arthur Thomson, M A., LL.D.

ON February 12, 1933, within two years of his retiral from the Chair of Natural History at the University of Aberdeen, where he had taught with great effect for thirty-one years, J. Arthur Thomson succumbed to a heart illness at his home at Limpsfield, Surrey. He will be most widely remembered for the success of his endeavours to bring scientific knowledge home to the plain man, but his zoological investigations and philosophical essays bear witness besides to much activity in the cause of knowledge for itself.

Born in 1861 at Saltoun in East Lothian, Thomson graduated M A at the University of Edinburgh in 1880 A son and grandson of the manse, he aimed at the ministry and completed the divinity course at New College; but the pull of heredity (his grandfather was Dr David Landsborough of the little books on seaweed and zoophytes), and the influence of his teachers, Alleyne Nicholson, Dickson, Cossar Ewart, Geikie, Rutherford, and others, determined his course towards the study of Nature. His post-graduate experience was wide: in 1883–4 he was working in Ernst Haeckel's zoological laboratory at Jena, in 1884 he moved on to Professor Delage's marine laboratory at Caen, and in 1885 he was concentrating upon the histology of sponges in F. Eilhard Schultze's laboratory in Berlin.

On his return to Edinburgh in 1886 he began the teaching career which was to place many generations of students of biology deeply in his debt. Following upon a variety of lecture courses in Edinburgh and occasional experience as substitute for Professor Haddon in Dublin, Professor Prince in Glasgow, and Professor Henry Drummond in Glasgow Free Church College, in 1899 he was appointed by the then Secretary for Scotland, Lord Balfour of Burleigh, to the Regius Chair of Natural History at the University of Aberdeen, left vacant through the death of Henry Alleyne Nicholson He taught with clearness, sympathy, and imagination, and one of the indications of his success is the presence of his pupils in many important zoological posts.

Even during the stress of his extraordinary teaching activity, Thomson kept in close touch with scientific developments. For many years he contributed an annual section to the *Zoological Record*, and was responsible for the zoological extracts in the *Journal of the Royal Microscopical*

Society. With the greater opportunities at Aberdeen, he turned to the investigation of the coral-like Alcyonaria, and contributed a large number of systematic papers on collections of the group from all parts of the world, notable among them being memoirs on the collections of the Australian *Thetis* Expedition, on the Prince of Monaco's collections, and on those of the Dutch *Siboga* Expedition to the East Indies.

There can be no doubt, however, that Sir Arthur Thomson gained recognition most widely as a writer of books. The quantity and the variety of his written work was enormous. Some of his books, like *The Evolution of Sex* (1889), written with the late Sir Patrick Geddes, made a lasting impression upon scientific thought; others, like *The Science of Life* (1899), *Heredity* (1908), and *The System of Animate Nature* (St Andrews Gifford Lectures of 1920), classified and systematised aspects of scientific progress or modern concepts of life; while his last great work, also written with Geddes, *Life: Outlines of General Biology* (1931), is a rich storehouse of facts bearing upon a very wide variety of biological themes. Besides this more austere aspect of his writing, Thomson had the art, rare and not easily acquired, of expressing scientific truths in language simple, clear, and beautiful, so that the plain man could understand and appreciate. He became more and more an evangelist proclaiming to the people the gospel of the interest and beauty and wonder of living things.

In 1887, when he was appointed Lecturer in the School of Medicine in Edinburgh, Thomson became a Fellow of the Royal Society of Edinburgh, and during two terms (1906–8 and 1920–23) he served on the Council. He was an LL.D. of his own University of Edinburgh, as well as of the Universities of McGill and California, and in 1930, upon his retiral from the Chair of Natural History in Aberdeen University, a Knighthood was conferred upon him. Sir J. Arthur Thomson is survived by Lady Thomson, who shared in his labours and achievements, and by a family of three sons and one daughter, all of whom have written books of value on different aspects of biological science.

J. R.

William Arthur Williams, F.I.C.

After a short illness, Mr W. A. Williams, Works Manager of the North British Rubber Co., Ltd., Castle Mills, Edinburgh, passed away on December 29, 1932. He had not enjoyed robust health for a few years, but a three months' tour in South Africa in 1928 did much to restore his former vitality, and he continued to interest himself in the scientific as well as the managerial side of his life's work until within a short time of his sudden death. By his death the rubber industry has been deprived of one of its pioneers in the application of scientific methods and principles to the introduction and control of manufacturing operations

Born in 1879, he received his school education in Finsbury, whence he passed to the City and Guilds Technical College to study chemistry under the late Professor Raphael Meldola, F.R S., and Mr Streatfeild. The sound technical training he there acquired remained one of Mr Williams's main assets throughout his business career, and his outlook on industrial problems was inspired by his early technical training. He was awarded the Senior Scholarship as the best final student of his year on the completion of his course in 1898, and he became research assistant to Professor Meldola, with whom he collaborated in a paper on polyazo compounds read before the Chemical Society in 1899

Mr Williams was the first chemist appointed to the staff of Castle Mills, taking up duties there in May 1900. He was soon able to show the value of the application of his scientific training in the elucidation of works problems, and in experimenting with and standardising new tests for control of factory operations. Improvements were made in the laboratory accommodation and equipment, and a staff of qualified assistants was appointed.

His organising ability found further scope when he was appointed Assistant General Works Superintendent, and shortly afterwards Works Manager, a position which he held till his death.

During the period of his association with the rubber industry (which previously had been largely guided by "rule of thumb" methods) important progress was made towards the standardising and scientific control of factory processes, and, in collaboration with the rubber producers, the manufacturers succeeded in overcoming the many problems attendant on the introduction and development of plantation rubber.

Mr Williams contributed several papers on the subject of variability of plantation rubbers during this period of transition from wild rubbers to plantation rubbers.

He was highly esteemed by all those who came under his charge, and was keenly interested in improving the working conditions throughout the factory.

He was elected a Fellow of the Institute of Chemistry in 1918 and a Fellow of the Royal Society of Edinburgh in 1924. In 1925 he was elected Chairman of the Edinburgh section of the Society of Chemical Industry. He was a Member of Council of the Institution of the Rubber Industry and a member of its Board of Examiners, and took an active part in the work of its Scottish section. He also held the post of President of the Royal Scottish Society of Arts and received the Gold Medal of that Society on relinquishing this post in 1929.

C. L. A.

FRANCIS ERNEST BRADLEY, M.A., LL.D.(Lond.), Judge of County Courts, Circuit No. 4 (N.W. Lancashire), since 1921, was a member of the Court of the University of Manchester as representative of Convocation from 1921 to 1930, and was Chairman of Convocation for 1928–1929. He was the author of *Handbook to the Companies Act* (1900); *Debtors and the War* (1914); *Private Companies, their Constitution and Management* (1917); *Company Principles and Precedents* (1920); and of essays in several Law journals.

He was elected a Fellow of this Society in 1916, and died on September 3, 1933.

REV. JOSEPH ROBERT FRASER, F.S.A Scot., was Minister of the New Parish Church of Scotland, Kinneff, Inverbervie, Montrose. He was elected a Fellow of the Society in 1915, and died on January 22, 1933.

JOHN HENDERSON, D.Sc., A.M.Inst E E., was elected a Fellow of the Society in 1900, and died on June 9, 1933.

THEOPHILUS BULKELEY HYSLOP, M.D, C.M., M R.C.P.E., late Physician Superintendent to the Royal Hospitals of Bridewell and Bethlem; P.M.O. King Edward's Schools, London and Witley, Lecturer on Psychological Medicine, St Mary's Hospital and London School of Medicine for Women, was educated at Edinburgh, London, and Paris Dr Hyslop was a remarkably versatile man He was a player in several orchestras, and wrote a number of orchestral works He was also an artist, his pictures being accepted by the Royal Academy on several occasions. He was the author of *A Text-Book on Mental Physiology; The Borderland* (1924); and of numerous articles on neurology, psychology, and medicine.

He was elected a Fellow of the Society in 1908, and died on February 12, 1933.

ROBERT JARDINE, M.D.(Edin.), M.R.C.S., F.R F.P.S.(Glas.), was born in Jardineville, New Brunswick, on January 30, 1862, and was educated at Mount Allison College, New Brunswick, Edinburgh University, and the London Hospitals. After holding several posts in Glasgow, he was appointed Professor of Midwifery and Dean of St Mungo's College, Glasgow. He was the author of *A Text-Book on Midwifery* (1899); *Clinical Lectures on the Hæmorrhages of Pregnancy* (1899); *Clinical Obstetrics* (1903); *Delayed and Complicated Labour* (1911); and of numerous papers on obstetrics and gynæcology in medical journals.

He was elected a Fellow of the Society in 1901, and died on December 17, 1932

SPENCER MORT, M.S., M.D., F.R.C.S.E., studied at the Universities of Glasgow, Paris, Dresden, Stockholm, Munich, and Amsterdam, and for twenty-two years held the appointment of Medical Superintendent and Surgical Director of the North Middlesex County Hospital, Edmonton. During the War he was a Lieut -Colonel in the Royal Army Medical Corps, and Officer Commanding and Chief Consulting Surgeon to the Special Military Surgical Hospital at Edmonton, being twice mentioned in dispatches Among his published works are a thesis on *Resistance to Peritoneal Bacterial Infection* (1913); and on *Surgical Experiences at North Middlesex Hospital* (1914). He was also the author of numerous articles on operative surgery and radium therapy.

He was elected a Fellow of the Society in 1914, and died on November 21, 1932.

JOHN W. TAIT, B.Sc., was Rector of Leith Academy for over twenty years, and retired about ten years ago. He was elected a Fellow of the Society in 1904, and died on December 17, 1932, in his seventy-first year.

JAMES EDWARD TALMAGE, D.Sc., LL.D., F.R.M.S., F.G.S., formerly President and Professor of Geology, University of Utah, was born at Hungerford, Berks, on September 21, 1862, and died on July 27, 1933. He was elected a Fellow in 1895.

JOHN MILLAR THOMSON, LL.D.(Glas.), F.R.S., F.C.S, F.I.C., Hon. Fellow King's College and Queen's College, London, was born in the precincts of the Old College of Glasgow on March 7, 1849. He was Professor of Chemistry, King's College, London, from 1887 to 1914, and Vice-Principal from 1905 to 1914.

He was elected a Fellow of the Society in 1880, and died on March 22, 1933. (For details of his life and work see *Obituary Notices of Fellows of the Royal Society*, No. 2, December 1933.)

APPENDIX.

CONTENTS.

PAGE

PROCEEDINGS OF THE STATUTORY GENERAL MEETING, OCTOBER 1932 . 387

PROCEEDINGS OF THE ORDINARY MEETINGS, SESSION 1932–1933 390

PROCEEDINGS OF THE STATUTORY GENERAL MEETING, OCTOBER 1933 . 396

THE KEITH, MAKDOUGALL-BRISBANE, NEILL, GUNNING VICTORIA JUBILEE, JAMES SCOTT, BRUCE, AND DAVID ANDERSON-BERRY PRIZES, AND THE BRUCE-PRELLER LECTURE FUND 399

AWARDS 403

ACCOUNTS OF THE SOCIETY, SESSION 1932–1933 . . . 411

VOLUNTARY CONTRIBUTORS UNDER LAW VI (END OF PARA. 3) . . 419

THE COUNCIL OF THE SOCIETY AT OCTOBER 23, 1933 . . . 420

FELLOWS OF THE SOCIETY AT OCTOBER 23, 1933 . . . 421

HONORARY FELLOWS OF THE SOCIETY AT OCTOBER 23, 1933 . . 447

CHANGES IN FELLOWSHIP DURING SESSION 1932–1933 . . . 449

FELLOWS OF THE SOCIETY ELECTED DURING SESSION 1932–1933 . 449

LAWS OF THE SOCIETY 450

ADDITIONS TO THE LIBRARY—PRESENTATIONS, ETC.—1932–1933 . 458

INDEX 461

INDEX, UNDER AUTHORS' NAMES, OF PAPERS PUBLISHED IN "TRANSACTIONS" 464

PROCEEDINGS OF THE STATUTORY GENERAL MEETING

Beginning the 150th Session, 1932–1933.

At the Statutory General Meeting of the Royal Society of Edinburgh, held in the Society's Rooms, 24 George Street, on Monday, October 24, 1932, at 4.30 P.M.,

Professor Sir E. A. Sharpey-Schafer, F.R.S., President, in the Chair,

the Minutes of the Statutory Meeting held on October 26, 1931, were read, approved, and signed.

The GENERAL SECRETARY submitted the following Report.—

SECRETARY'S REPORT, OCTOBER 24, 1932.

By request of the Council, addresses were delivered on "David Hume and Modern Science," by Professor R. A. SAMPSON, LL.D., F.R.S., on December 7, 1931, "Research on Whales in the Antarctic," by Dr N. A. MACKINTOSH, A.R.C.S., on March 7, 1932, and on "Conduction and Automatism in the Mammalian Heart," by Professor PIERRE RIJLANT, University of Brussels, on June 20, 1932. On July 4, 1932, in commemoration of the Centenary of the death of Sir WALTER SCOTT, who was President of the Society from 1820 to 1832, Professor H. J. C. GRIERSON, M.A., LL.D., delivered an address on "Scott and the Historical Novel."

40 papers were read, as compared with 34 in the previous session. The papers were divided among subjects as follows: Mathematics, 7, physics, 1; chemistry, 4; engineering, 1; geology and mineralogy, 7, palæontology, 5, botany, 3, plant physiology, 1; zoology, 3, human genetics, 3; animal genetics, 3, physiology, 2. 9 papers have been published or are being published in the *Transactions*, and 23 in the *Proceedings*. 7 papers were read but have not yet been submitted in final form, and 1 was declined for publication. Several papers were received during the session which were found unsuitable for presentation to the Society.

The Society has lost by death 24 Fellows and 4 Honorary Fellows, and 6 Fellows have resigned. 31 Fellows were elected.

Invitations were received, and the Society was represented as follows on the occasions mentioned:—

1. University of Amsterdam, Tercentenary Celebration, June 27, 1932. Professor D'ARCY W. THOMPSON, C.B., F.R.S., presented address.
2. Royal Society of Canada. 50th Anniversary, May 20–28, 1932. Professor J. MEAKINS, M.D., LL.D., presented address.
3. National Academy of Sciences, Washington. 100th Anniversary of the Electrical Discoveries of JOSEPH HENRY. April 25, 1932. Letter sent.
4. Tartu University (Dorpat). Tercentenary of its Foundation, June 30 and July 1, 1932. Professor J. Y. SIMPSON, D.Sc., presented address.
5. Congress of Entomology, Paris, July 18–23, 1932. Letter sent.
6. International Congress of Mathematicians, Zurich, September 4–12, 1932. Professor E. T. WHITTAKER, F.R.S., Professor H. W. TURNBULL, F.R.S., and Professor J. E. A. STEGGALL, M.A.
7. Spinoza Tercentenary. The Hague, September 1932. Letter sent.
8. National Trust for Scotland. Society's representative, the Hon. LORD SANDS.

A medal to be awarded in 1933, together with a sum of money, as the **DAVID ANDERSON-BERRY PRIZE**, has been designed by Mr PERCY METCALFE, designer of the coinage of the Irish Free State. The dies have been made at the Royal Mint, and a bronze replica is now in the possession of the Society.

The **KEITH PRIZE** for the period 1929–1931 was awarded to Dr A. W. GREENWOOD of the University of Edinburgh.

The **NEILL PRIZE** for the period 1929–1931 was awarded to Dr C. H. O'DONOGHUE of the University of Edinburgh.

The BRUCE PRIZE for the period 1930–1932 was awarded to Mr **HENRY GINO WATKINS**, who, unfortunately, met his death by drowning in Lake Fjord, Greenland, on August 20, 1932, while leading the British Arctic Air Route Expedition.

After careful consideration by a Committee, a more durable paper has been decided upon for the *Transactions* and *Proceedings*, and a new type has been selected for the *Proceedings*. This will result in a saving of approximately 10 per cent. in the cost of printing per sheet. The *Transactions*, already in monotype, will be charged as formerly.

Reference was made in my last report to the collaboration of the Society with the Royal Society of London, to arrange for the Second Polar Year and to administer the grant of £10,000 for this purpose made by the Air Ministry. The Joint Committee, reinforced by some outside members, has continued its work and has dispatched a party to Fort Rae in Canada and a second one to Tromsö in Norway. The Society, with the approval of the Committee, has itself arranged for auroral photographs and observations to be made at Urafirth in Shetland, and these are now in progress

The Council of the Society drew the attention of the Town Council of Edinburgh to the neglected condition of the David Hume monument in the Old Calton Burial Ground. As a consequence steps have been taken to improve its condition.

The Calendar of the Hume MSS. in the possession of the Royal Society of Edinburgh, by Dr J. Y. T. GREIG and HAROLD BENYON, has now been published as Vol. 52, Part I, of the *Proceedings*, and has been circulated.

A number of new exchanges of publications were initiated during the session.

In connection with the Scott Centenary Exhibitions in Edinburgh and Glasgow, it may be mentioned that the Society lent to these exhibitions its portrait of Sir Walter Scott by Graham Gilbert.

The acknowledgment of the Society is due to the Carnegie Trust for the Universities of Scotland for grants to authors towards the costs of the illustrations of papers published by the Society, amounting to £87, 16s 7d ; to the University of Liverpool Dunning Lawrence Research Fund for a grant of £25 towards the cost of publication of Dr TUDOR JONES's recent paper in the *Transactions*; and to the Royal Society of London for a sum of £250 from the Government Publication Grant in aid of the cost of printing the Society's *Transactions* and *Proceedings* during the session 1931–1932.

TREASURER'S REPORT.—

The TREASURER stated that there were no specially outstanding features with regard to the finances of the past year.

There had been an increase in the amount received for contributions, so that while last year there was an adverse balance on the year of over £62, the Accounts of this year show a surplus of over £20.

The PRESIDENT in the course of his remarks drew attention to the fact that the Society had never confined itself to science, but had always taken a prominent part in the encouragement of literature as well. While referring to the value of addresses given from time to time by eminent scientists, instancing especially that of BOHR in the previous session, and in the present session that of RIJLANT, who gave the Society a lucid exposition of the researches of DEMOOR and himself, the PRESIDENT recalled in the literary field the address above referred to on David Hume, and especially that on Walter Scott, with which Professor H. J. C. GRIERSON had favoured the Society. He also referred to the losses of the Society by the death of Ordinary and Honorary Fellows, details of which may be found in the Obituary Notices already published. To these must be added, though not a Fellow, the name of the Bruce Medallist, HENRY GINO WATKINS, whose tragic death by drowning in Greenland occurred on August 20, 1932.

The PRESIDENT nominated as Scrutineers of the Ballot, Dr ROBERT CAMPBELL and Mr J. W. BUTTERS.

The Ballot for the Election of Council and Office-Bearers was then taken.

Mr JOHN BARTHOLOMEW moved the adoption of the Reports and the reappointment of Messrs LINDSAY, JAMIESON & HALDANE, C.A., as auditors for the ensuing session. These motions were approved.

The Scrutineers reported that the Ballot Papers were in order, and the PRESIDENT declared that the following Council and Office-Bearers had been duly elected:—

Professor Sir E. A. Sharpey-Schafer, M.D., D Sc., LL.D , F.R.S., President
Professor J. H. Ashworth, D.Sc., F.R.S.
Arthur Logan Turner, M.D., LL.D , F.R C.S.E.
J. B. Clark, M.A , LL.D., J.P.
Professor James Ritchie, M.A , D.Sc.
Principal Sir Thomas H. Holland, K.C.S I , K C.I.E., D L , D.Sc., LL.D , F R.S.
The Hon. Lord Sands, Kt , LL D., D D
} Vice-Presidents.
Professor R. A. Sampson, M A , D Sc , LL D , F R.S , General Secretary
Professor C. G. Darwin, M A , F.R.S.
Professor F. A. E Crew, M D , D Sc , Ph.D.
} Secretaries to Ordinary Meetings.
James Watt, W.S , LL D , Treasurer
Professor D'Arcy W. Thompson, C.B , Hon. D Sc , D Litt , F R S , Curator of Library and Museum

ORDINARY MEMBERS OF COUNCIL.

Murray Macgregor, M A., D Sc.
A. Crichton Mitchell, D.Sc.
Professor P. T. Herring, M.D , F.R.C.P.E.
Professor James P. Kendall, M A., D Sc., F.R.S.
Professor Thomas M. MacRobert, M.A., D.Sc.
Professor Godfrey H. Thomson, D Sc., Ph D
Malcolm Wilson, D Sc., A R.C S., F L.S
Professor E B. Bailey, M.C., M A , F R.S.
Professor J. C Brash, M.C., M.A., M.D.
Professor A. J. Clark, M C., B.A , M.D., F R S.
Professor A G. Ogilvie, M.A., B.Sc.
Professor E M Wedderburn, M A., D Sc , LL B , W S

The President, before closing the meeting, thanked the Scrutineers for their services.

PROCEEDINGS OF THE ORDINARY MEETINGS,
Session 1932–1933.

FIRST ORDINARY MEETING.

Monday, November 7, 1932

Professor Sir E. A Sharpey-Schafer, F R.S., President, in the Chair.

The Minutes of the last Ordinary Meeting were taken as read.

The following Communications were submitted·—

1 Supernumerary Pectoral Fins in *Raia circularis* Loudon. By W. F. HARPER, Ph D., M.B. Communicated by Professor A. D PEACOCK, D.Sc *Proc* , vol. 53, pp. 26–30

2. The Innervation of the Uterus By Miss AMY M. FLEMING, M.D., M.C O.G. Communicated by Professor E. P CATHCART, C B E , F.R S *Trans* , vol 57, pp. 473–490.

3 On the Development of the Olfactory Organ in Protopterus By Miss MARGARET H. FULLARTON, B Sc Communicated by Professor J GRAHAM KERR, F.R.S. *Proc* , vol. 53, pp. 1–6.

4. Development and Probable Evolution of the Suctorial-disc in the Tadpoles of *Rana afghana* Günther. By SUNDER LAL HORA, D Sc *Trans* , vol 57, pp. 469–472.

5. Evolution of the Catopterid Fishes. By JAMES BROUGH, B.Sc Communicated by Professor H. GRAHAM CANNON, M A , Sc D (Read by title)

6 Contributions to the Study of the Old Red Sandstone Flora of Scotland. Part VIII· On Arthrostigma, Psilophyton, and some Associated Plant-remains from the Strathmore Beds of the Caledonian Lower Old Red Sandstone. By Professor W H LANG, F R S *Trans* , vol 57, pp 491–521 (Read by title)

SECOND ORDINARY MEETING

Monday, December 5, 1932.

The Hon. Lord Sands, LL D , D D., Vice-President, in the Chair.

The Minutes of the last Ordinary Meeting were taken as read.

The following Communications were submitted:—

1. The Metamorphism of the Deeside Limestone By ARTHUR G HUTCHISON, D.Sc Communicated by Professor H H. READ, D.Sc. *Trans* , vol. 57, pp 557–592

2 Notes on the Kidston Collection of Fossil Plant Slides. No. 1: The Anatomy of the Axis of *Lepidodendron Brownii* Unger sp , with special reference to the Relationship between this Stem and *Lepidostrobus Brownii* Unger sp By MARY G. CALDER, B.Sc Communicated by Dr S WILLIAMS. *Trans* , vol 57, pp. 547–555.

3. The Restoration and Regeneration of the Epithelium and Endometrium of the Uterus of *Cavia* Post Partum in Non-pregnant Animals. By WILLIAM J. HAMILTON, M.Sc , M B., B.Ch. Communicated by Professor T H BRYCE, F R.S. *Trans.*, vol. 57, pp. 593–600

4. Expansion and Contraction of Coal caused respectively by the Sorption and Discharge of Gas (Preliminary Notice) By Professor H BRIGGS, D.Sc., and R P. SINHA, B.Sc *Proc.*, vol. 53, pp 48–53

. The undernoted Papers were read by title:—

5 On Properties of Null Geodesics, and their Application to the Theory of Radiation. By W. O. KERMACK, M A., D.Sc , W. H. M'CREA, M A , Ph.D , and Professor E. T. WHITTAKER, F.R.S. *Proc* , vol. 53, pp 31–47.

6. A Matrix Notation for Mendelian Populations. By Professor LANCELOT HOGBEN, M.A., D.Sc. *Proc.*, vol. 53, pp. 7–25.

7. On the Graduation of Data by the Orthogonal Polynomials of Least Squares. By A. C. AITKEN, M.A., D.Sc. *Proc.*, vol. 53, pp. 54–78.

THIRD ORDINARY MEETING

Monday, January 9, 1933

Professor Sir E A. Sharpey-Schafer, F.R S , President, in the Chair.

The Minutes of the last Ordinary Meeting were taken as read.

At the request of the Council Dr John E. Mackenzie and Dr Harry W. Melville gave an experimental demonstration of the Measurement of the Diffusion Coefficients of Bromine-Hydrogen and Bromine-Carbon Dioxide, and Professor James G. Gray, D Sc., gave an address, with demonstration, on "Self-erecting Gyrostats"

The undernoted Paper was read by title —

On the Measurement of Spatial Distance in a Curved Space-time By H S Rusk, M.A , D.Sc *Proc.*, vol 53, pp. 79–88

FOURTH ORDINARY MEETING

Monday, February 6, 1933.

Principal Sir Thomas H Holland, K.C S.I., K C I E , LL D , F.R.S ,
Vice-President, in the Chair.

The Minutes of the last Ordinary Meeting were taken as read

The Vice-President referred to the illness of the President, and intimated that the news of him was good

The following Communications were submitted·—

1. A Case of Non-disjunction in the Fowl. By Professor F. A E. Crew, M D , D.Sc , Ph.D *Proc* , vol 53, pp 89–104

2 Studies on the Scottish Marine Fauna the Natural Faunistic Divisions of the North Sea, illustrated by the Quantitative Distribution of the Mollusca By A C. Stephen, B Sc *Trans.*, vol. 57, pp. 601–616

3. The Diurnal Incidence of Disturbance in the Terrestrial Magnetic Field By A Crichton Mitchell, D Sc *Trans* , vol 57, pp 617–632

The undernoted Papers were read by title —

4 The Intellectual Resemblance of Twins. By Louis Herrman, M A , and Professor Lancelot T. Hogben, M.A , D Sc *Proc* , vol 53, pp 105–129

5. Spermatogenesis in *Drosophila obscura* Fallen. I —The Cytological Basis of Suppression of Crossing-over By P. Ch Koller and Thelma Townson Communicated by Professor F A E Crew, M D , D.Sc , Ph D *Proc* , vol 53, pp. 130–146

FIFTH ORDINARY MEETING

Monday, March 6, 1933

Dr J B. Clark, M A , Vice-President, in the Chair.

The Minutes of the last Ordinary Meeting were taken as read

The election of Fellows then took place The Vice-President, from the Chair, nominated Dr Alexander Lauder and Lt -Col. L. M. Davies as Scrutineers After the ballot the following candidates were stated by the Vice-President as having been duly elected Fellows of the Society:—

Frederick Latham Arnot, Prabodh Chandra Banerjee, James Livingstone Begg, Thomas Robert Bolam, Finlay James Cameron, John Irvine Carswell, Arthur Melville Clark, Arthur Lancelot Craig-Bennett, Edmund Gilbert Dymond, John Maxwell Erskine, Thomas Ferguson, Charles Forrester, William Hope Fowler, Augustus Wm. de Rohan Galbraith, Alfred Charles Gardner, Robert Pollock Gillespie, Thomas Haining Gillespie, William Gilmour Guthrie, Edgar Erskine Hume, David Lees, James Macleod, John Malcolm, William John Milne Menzies, Louis Melville Milne-Thomson, John Murray, Thomas Nicol, Charles Wynford

Parsons, Alfred William Noel Pillers, Frank Anderson Baillie Preston, John Livingston Somerville, John Barclay Tait, George Taylor, Geoffrey Timms, John Mitchell Watt, William Frederic James Whitley, James Wright.

The Vice-President, from the Chair, thanked the Scrutineers for their services.

The Vice-President intimated that Council had decided to make the following awards:—

The MAKDOUGALL-BRISBANE PRIZE (1930–1932) to Dr A. C. AITKEN, for various contributions to Mathematics published in the Society's *Proceedings*

The GUNNING VICTORIA JUBILEE PRIZE (1928–1932) to Emeritus Professor Sir JAMES WALKER, F R.S , for numerous contributions to Physical and General Chemistry.

The awards to be made at the Ordinary Meeting of the Society on July 3.

The Vice-President announced that the BRUCE-PRELLER LECTURE (1933) would be given by Professor C. H. LANDER, C.B E., on October 23, 1933, on the subject of the Utilisation of Coal.

The following Communications were submitted:—

1. The Late-glacial Readvance Moraines of the Highland Border west of the River Tay. By J. B. Simpson, B Sc (Agr), D.Sc *Trans* , vol 57, pp. 633–646

2. Help from America in reading Scottish Tectonics By Professor E. B Bailey, M.C., F.R.S.

3. On the Pituitary in *Lepidosiren* and its Development By T. Kerr, M A. Communicated by Professor J. Graham Kerr, F R S. *Proc* , vol. 53, pp. 147–150 (Read by title.)

SIXTH ORDINARY MEETING

Monday, May 1, 1933

The Hon. Lord Sands, LL D , D.D., Vice-President, in the Chair

The Minutes of the last Ordinary Meeting were taken as read

The following were admitted to the Society.—Dr F. L. Arnot, Mr F. J Cameron, Mr J I Carswell, Dr A M. Clark, Dr A L Craig-Bennett, Mr E G Dymond, Dr Thomas Ferguson, Dr W. H Fowler, Mr A. C Gardner, Mr T. H. Gillespie, Dr W G. Guthrie, Mr James Macleod, Mr W J M Menzies, Dr John Murray, Mr J L. Somerville, Dr G Timms, and Dr W F J. Whitley.

The Chairman announced the following nominations to Honorary Fellowship:—

Foreign Honorary Fellows· John Jacob Abel, Professor of Pharmacology, Johns Hopkins University, Baltimore; Filippo Bottazzi, Professor of Experimental Physiology, Royal Institute of Physiology, Naples; Edwin Grant Conklin, Professor of Biology, Princeton University; Nikolaj Konstantinovic Koltzoff, formerly Professor of Zoology, State University, Moscow; Albrecht Penck, Geheimrat Professor (Emeritus) of Geography, Friedrich-Wilhelms-Universität, Berlin; Pieter Zeeman, Professor of Physics, University, Amsterdam.

British Honorary Fellows: Sir George Macdonald, K.C.B , LL.D., Edinburgh; Sir William Napier Shaw, Kt , LL D , F.R S , London

The Chairman presented the JAMES SCOTT PRIZE (1933) to Professor Dr ARNOLD SOMMERFELD of the University of Munich, who then delivered a lecture entitled: "Ways to the Knowledge of Nature."

SEVENTH ORDINARY MEETING.

Monday, May 15, 1933.

Professor J. H. Ashworth, F.R.S., Vice-President, in the Chair.

The Minutes of the last Ordinary Meeting were taken as read.

The following were admitted to the Society:—Mr Thomas Nicol, Mr C. W. Parsons, and Dr T. R. Bolam.

The following Communications were submitted:—

1. The Early Stages in Development of *Cavia*. By Norman H. W. Maclaren, Ph.D., F.L.S., and Professor T. H. Bryce, M.D , F R.S. *Trans.*, vol. 57, pp. 647–664.

2. Formation of Rock Joints and the Cleat of Coal. By Emeritus Professor P. F. KENDALL, D.Sc., F.R.S., and Professor HENRY BRIGGS, D Sc , Ph.D *Proc.*, vol 53, pp. 164–187.

3. Notes on the Kidston Collection of Fossil Plant Slides —No II. The Anatomy of the Axis of *Bothrodendron mundum* Williamson sp By MARY G. CALDER, B Sc. Communicated by Dr S. WILLIAMS, M.Sc. *Trans* , vol 57, pp. 665–673.

The following Papers were read by title.—

4. Matrices and Continued Fractions By Professor H. W TURNBULL, M A , F.R.S *Proc.*, vol. 53, pp. 151–163 and pp 208–219.

5. Studies in Genetic Psychology.—I. Correlations between Collateral Relatives with respect to Intellectual Resemblance. By J. L. GRAY, M A , and PEARL MOSHINSKY, B Sc. Communicated by Professor LANCELOT HOGBEN, M A , D Sc *Proc* , vol 53, pp 188–207.

EIGHTH ORDINARY MEETING

Monday, June 5, 1933

Dr A. Logan Turner, LL D , Vice-President, in the Chair

The Minutes of the last Ordinary Meeting were taken as read

The following Communications were submitted —

1. On the Feeding Mechanism of certain Marine Ostracods By Professor H. GRAHAM CANNON, M A., Sc.D *Trans.*, vol. 57, pp 739–764

2 Studies in the Scottish Marine Fauna —*A* The Quantitative Distribution of the Echinoderms in the North Sea. *B*. The Validity of Petersen's Community Hypothesis in the North Sea. By A. C STEPHEN, B.Sc *Trans.*, vol 57, pp 777–787.

3. The Sequence of Floras in the Upper Carboniferous, with special reference to South Wales. By EMILY DIX, D Sc , F G S Communicated by JOHN PRINGLE, D.Sc , F G S *Trans* , vol 57, pp. 789–838

The undernoted Papers were read by title –

4. Studies on the Reproductive System in the Guinea-Pig.—I Variations in the Œstrous Cycle in the Virgin Animal, after Parturition, and during Pregnancy By THOMAS NICOL, M B., Ch.B., F.R.C.S E. *Proc* , vol. 53, pp. 220–238

5. The Mathematical Representation of the Energy Levels of the Secondary Spectrum of Hydrogen. By IAN SANDEMAN, Ph D *Proc* , vol 53, pp. 347–352.

6. On the Theory of Statistical Regression. By M S BARTLETT, B A , B Sc. Communicated by J. WISHART, M.A., D Sc. *Proc* , vol 53, pp. 260–283.

NINTH AND LAST ORDINARY MEETING.

Monday, July 3, 1933

Principal Sir Thomas Holland, K.C S I , K C I.E., LL D., F R.S ,
Vice-President, in the Chair.

The Minutes of the last Ordinary Meeting were taken as read.

Mr WM. MAXWELL was admitted a Fellow of the Society.

The following were elected Honorary Fellows, Professor Sir T. H. BEARE and Professor T. J. JEHU acting as Scrutineers:—

Foreign Honorary Fellows: JOHN JACOB ABEL, Professor of Pharmacology, Johns Hopkins University, Baltimore; FILIPPO BOTTAZZI, Professor of Experimental Physiology, Royal Institute of Physiology, Naples, EDWIN GRANT CONKLIN, Professor of Biology, Princeton University; NIKOLAJ KONSTANTINOVIC KOLTZOFF, formerly Professor of Zoology, State University, Moscow; ALBRECHT PENCK, Geheimrat Professor (Emeritus) of Geography, Friedrich-Wilhelms-Universität, Berlin; PIETER ZEEMAN, Professor of Physics, University, Amsterdam.

British Honorary Fellows: Sir GEORGE MACDONALD, K.C.B., LL.D., Edinburgh; Sir WILLIAM NAPIER SHAW, Kt , LL.D., F.R.S., London.

The GUNNING VICTORIA JUBILEE PRIZE for the period of 1928–1932 was presented to Emeritus Professor Sir JAMES WALKER, F.R.S. The Vice-President read the following statement·—

> Sir JAMES WALKER has been one of our Fellows for forty-two years, and during that time he has contributed largely to our publications and served the Society for many years on the Council and as a Vice-President Although we can rely with safety on the number of honours which have been conferred on him by workers in his own sphere of chemistry, it is satisfying to know that during his period of Fellowship Sir JAMES WALKER has established himself as a leading apostle and worker, especially in the region of physical chemistry—a subject which was little recognised, almost unknown, forty years ago To every student of the history of physical chemistry two names most frequently recur—the late WILHELM OSTWALD of Leipzig and JAMES WALKER of Edinburgh Their contributions to research may well be ranked together, but Sir JAMES WALKER stands out especially as an exponent of the new ideas.
>
> Certainly the first name that springs to the mind of any English-speaking scientist when physical chemistry is mentioned is that of Sir JAMES WALKER, and throughout the forty-odd years that have elapsed since the subject began its struggle for recognition he has played a leading part in its triumphant development Opposition to the new ideas, almost universal at the start, rapidly changed to eager acceptance, so that in 1892 OSTWALD, returning to Leipzig after attending a meeting of the British Association, could write thus —"I left Edinburgh with the certainty that it was not necessary to take any more special measures for the further advancement of our cause in Great Britain JAMES WALKER, a born teacher, was training a host of convinced Ionists."
>
> The ionic theory has matured considerably since 1892, but among the crowd of convinced ionists that continue to spread the WALKER gospel the Edinburgh School has always remained prominent. Although he retired officially from the chair of Chemistry in 1928, Sir JAMES WALKER maintained until quite recently the active direction of research within his old department, and freedom from teaching duties has rendered the last few years not the least fruitful of his long career He has indeed erected for himself a monument that will be admired for many decades
>
> I should like to add a reference to Sir JAMES WALKER's work during the war, which showed him in a new light as an administrator under novel and difficult conditions May I mention a phase of his versatile character too little known among scientific men his appreciative love of music and the fine arts
>
> Our list of Gunning Victoria Jubilee prizemen includes- KELVIN and TAIT, JAMES DEWAR, CHRYSTAL, NORMAN COLLIE, and E T WHITTAKER This latest addition maintains a standard which the Society can regard with real satisfaction

The MAKDOUGALL-BRISBANE PRIZE for the period 1930–1932 was presented to Dr A. C. AITKEN The Vice-President read the following statement —

> In the last ten years Dr AITKEN has published over thirty original papers, which have effected considerable advances in three branches of mathematical science, namely, numerical mathematics, the theory of matrices and determinants, and the mathematical theory of statistics
>
> The earlier papers relate chiefly to numerical mathematics, including interpolation, numerical integration and differentiation, and the solution of algebraic and simultaneous equations. The original impetus towards this field of work doubtless came from his own extraordinary powers in mental arithmetic for the benefit of those of us who are not gifted in this way, he has devised methods by which most of the problems with which the practical mathematician is confronted can be reduced to repetitions of a process peculiarly suited to an arithmometer, namely, a cross-multiplication followed by a division.
>
> As a by-product of his interest in interpolation, finite differences and operational methods, he discovered in 1929 a remarkable and powerful general theorem which comprehends in one formula Taylor's Theorem and the other expansion theorems of the Differential Calculus, all "single-line" difference interpolation formulæ, and a multitude of other possibilities.
>
> A paper which he published in our *Proceedings* in 1926, on BERNOULLI's method of solving algebraic equations and extensions of it, led him by way of symmetric functions to the wider field of algebra In 1927 he discovered a dual theorem on determinantal symmetric functions which contained theorems of JACOBI and MACMAHON as special cases, and joined the determinantal theory of symmetric functions to the combinatory theory Regarding algebraic equations from another point of view, namely, as the characteristic equations of matrices, he made many original contributions to the general theory of matrices, particularly in regard to matrices with multiple latent roots Much of this work has appeared in the book on "Canonical Matrices" which he published in collaboration with Professor TURNBULL last year, and which is recognised by mathematicians as opening a new era in the history of the subject
>
> Lastly, mention must be made of his researches on mathematical statistics and frequency functions. In 1930, working with Dr OPPENHEIM, he showed that though the frequency function of Type A in Hermite polynomials has terms of irregular order of magnitude, the logarithm of the function, so expanded, has terms regularly descending. In 1931 he proved the analogous result for Type B and its logarithm, with the appropriate polynomials In his most recently published paper he has examined the fitting of polynomials to data by least squares with Tchebychef polynomials, finding new forms for the polynomials and new and rapid methods based thereon
>
> The admiration with which mathematicians regard the formal beauty and architectural elegance of Dr AITKEN's work in algebra is reinforced by the gratitude they feel for the many new practical methods which he has placed at the disposal of the computer.

The following Communications were submitted·—

1. The Abnormal Atomic Weight of Calcium contained in Two very old Potassium-rich Deposits at Rhiconich, Sutherlandshire, and Portsoy, Banffshire. By WM. W. SMITH, Ph.D., and THOMAS TAIT, Ph D. Communicated by Professor J. P. KENDALL, F.R.S.

2. Geology of the Outer Hebrides. Part V —North Harris and Lewis. By Professor T. J JEHU, M.D., F.G S., and R. M. CRAIG, M.A., D.Sc., F.G.S. *Trans,*, vol. 57, pp. 839–874.

The undernoted Papers were read by title:—

3. The Effect of Consanguineous Parentage upon Metrical Characters of the Offspring. By Professor LANCELOT HOGBEN, M.A., D.Sc. *Proc.*, vol. 53, pp. 239–251.

4. The Fæcal Pellets of *Hippa asiatica*. By HILARY B. MOORE, Ph.D Communicated by Professor J. GRAHAM KERR, F.R.S. *Proc*, vol 53, pp. 252–254.

5. The Structure of the Foot in certain Mosses and in *Anthoceros lævis*. By NELLIE M. BLAIKLEY, M Sc Communicated by Professor J. R. MATTHEWS, M A., F.L.S. *Trans*, vol. 57, pp. 699–709.

6. A Contribution to the Experimental Morphology of *Lycopodium Selago*, with special reference to the Development of Adventitious Shoots. By S WILLIAMS, M Sc, Ph D. *Trans.*, vol. 57, pp. 711–737.

7. Studies on the Gametogenesis of *Stenophylax stellatus*, Curt. (Trichoptera)· Oogenesis. By R. A. R. GRESSON, Ph.D. Communicated by Professor J H ASHWORTH, F R.S. *Proc.*, vol 53, pp. 322–336

8. The Diffusion Coefficients of Bromine-Argon, Bromine-Methane, Bromine-Hydrogen Chloride, Bromine-Nitrous Oxide By JOHN E MACKENZIE, D Sc., and HARRY W. MELVILLE, Ph D *Proc.*, vol. 53, pp. 255–259

PROCEEDINGS OF THE STATUTORY GENERAL MEETING

Ending the 150th Session, 1932–1933.

At the Statutory General Meeting of the Royal Society of Edinburgh, held in the Society's Rooms, 24 George Street, on Monday, October 23, 1933, at 4.30 P M.

Emeritus Professor Sir E A. Sharpey-Schafer, F.R S., President, in the Chair,

the Minutes of the Statutory Meeting held on October 24, 1932, were read, approved, and signed.

The General Secretary submitted the following Report·—

GENERAL SECRETARY'S REPORT, OCTOBER 23, 1933.

On January 9, 1933, by request of the Council, an "Experimental Demonstration of the Measurement of the Diffusion Coefficients of Bromine-Hydrogen and Bromine-Carbon Dioxide" was given by Drs J E Mackenzie and H W. Melville, and an address (with demonstration) was delivered by Professor J. G. Gray, D Sc, on "Self-erecting Gyrostats" On May 1, 1933, in terms of the award of the JAMES SCOTT PRIZE, and by request of the Council, Professor Dr Arnold Sommerfeld, For Mem. R.S, gave an address on "Ways to the Knowledge of Nature."

41 papers were read, as compared with 40 in the previous session The papers were divided among subjects as follows·—Mathematics, 5; physics, 3; chemistry, 2; geology and mineralogy, 5; palæontology and palæobotany, 5, botany, 2; zoology, 9; human genetics, 4; animal genetics, 2; physiology, 3, and anatomy, 1. 14 papers have been published, or are being published in the *Transactions*, and 20 in the *Proceedings*. 6 papers were read but have not yet been submitted in final form, and 1 has been withdrawn Several papers have been declined as unsuitable for publication by the Society

The Society has lost by death 24 Fellows and 2 Honorary Fellows; 3 Fellows have resigned, and 1 has been removed from the Roll of Fellows 36 Fellows and 6 Foreign and 2 British Honorary Fellows have been elected. Dr William Hope Fowler, it is regretted, has died since his election

Invitations were received, and the Society was represented as follows on the occasions mentioned —

1 Deutsche Gesellschaft für Natur- u. Völkerkunde Ostasiens in Tokyo 60th Anniversary, March 22, 1933 Professor A. Tanakadate presented address.
2. Warsaw, Society of Sciences and Literature Celebration, November 23, 1932. Letter sent.
3. Entomological Society of London 100th Anniversary in London, May 3 and 4, 1933. Sir T. Hudson Beare and Dr R. Stewart MacDougall presented an address.
4. International Geological Congress at Washington, July 22 to 29, 1933. Professor W. T. Gordon, D Sc, and Sir John Flett, F R.S.

It has been decided that the first award of the DAVID ANDERSON-BERRY PRIZE for the best recent work on the nature of X-Rays in their therapeutical effect on human diseases should be made in the summer of 1935.

The GUNNING VICTORIA JUBILEE PRIZE for the period 1928–1932 was awarded to Sir James Walker, F.R.S.

The MAKDOUGALL-BRISBANE PRIZE for the period 1930–1932 was awarded to Dr A. C. Aitken and the JAMES SCOTT PRIZE to Professor Dr Arnold Sommerfeld, For. Mem. R.S.

The BRUCE-PRELLER LECTURE will be delivered by Professor C. H. Lander, C.B.E., on the "Liquefaction of Coal."

The British Expedition in connection with the Polar Year 1933, in the organisation of which the Society occupies an exceptional position, has been conducted with conspicuous success at Fort Rae in Canada.

Professor T. J. Mackie will represent the Royal Society of Edinburgh and the University of

Edinburgh, jointly, for a further term of three years, as from February 1933, on the India Office Committee for the recruitment of medical research workers

In view of the present adverse exchanges and the high cost of German Periodicals, the Council decided to discontinue, in the meantime, the following journals.—Neues Jahrbuch fur Mineralogie u. Paläontologie (all sections); Centralblatt fur Mineralogie, Geologie u. Palaontologie (A and B); Zeitschrift f. Kristallographie, Mineralogie u. Petrographie (A only); and Zentralblatt f. Bakterologie, Parasitenkunde u Infektionskrankheiten (all sections) All these journals are accessible elsewhere in Edinburgh.

During the session additional steel shelving has been added in the Front Reading Room (Street Floor), the Gallery has been slightly extended, and a lighter steel stair erected. Painting and some other alterations have also been carried out on the North Wall of the Lecture Room, to the improvement of its appearance

A number of new publications were added to the Library by exchange and purchase during the session.

The Assistant Librarian, Mr R J B Munro, has been placed under the Federated Superannuation System for Universities

The acknowledgment of the Society is due to the Carnegie Trust for the Universities of Scotland for grants to authors towards the cost of the illustrations of papers published in the *Transactions* and *Proceedings*, amounting to £154, 18s 11d, to the University College, Dundee, for £1, 10s 4d towards the cost of the illustration of Mr W. F Harper's paper in the *Proceedings*; to Dr A G Hutchison for £15 towards the cost of the illustration of his paper in the *Transactions*, and to the Royal Society of London for a sum of £250 from the Government Publication Grant in aid of the cost of publishing the Society's *Transactions* and *Proceedings* during the session 1932–1933

Treasurer's Report:—

The Treasurer stated that, as was the case last year, there was no outstanding feature with regard to finance. The Accounts of last year showed a surplus of £20, 14s 1½d, while this year they showed a deficiency of £62, 9s 9d

The President nominated as Scrutineers, Sir Harold Stiles and Dr Alexander Lauder.

The Ballot for the Election of Office-Bearers and Council was then taken

Dr J. E. Mackenzie moved the adoption of the Reports and the reappointment of Messrs Lindsay, Jamieson & Haldane, C A., as auditors for the ensuing session These motions were approved.

The Scrutineers reported that the Ballot Papers were in order, and the President declared that the following Office-Bearers and Council had been duly elected —

Emeritus Professor Sir E. A Sharpey-Schafer, M.D, D Sc., LL.D, F R S, President
J. B. Clark, M A., LL.D, J.P
Professor James Ritchie, M.A., D Sc
Principal Sir Thomas H. Holland, K C S I, K C.I.E, D.L, D Sc., LL.D, F.R.S.
The Hon. Lord Sands, Kt, LL D, D.D
Professor C. G. Darwin, M.A, F.R S.
Professor R. A Sampson, M A, D Sc., LL D., F.R S
} Vice-Presidents.
Professor J. H. Ashworth, D.Sc, F.R S, General Secretary.
Professor F. A. E. Crew, M.D., D.Sc, Ph D.
Professor James P. Kendall, M A, D Sc., F.R S
} Secretaries to Ordinary Meetings
James Watt, W.S, LL.D., Treasurer
Professor D'Arcy W. Thompson, C.B, D.Litt, LL.D, F.R S., Curator of Library and Museum.

ORDINARY MEMBERS OF COUNCIL.

Professor P. T. Herring, M.D., F.R.C.P.E.
Professor Thomas M. Macrobert, M A., D.Sc.
Professor Godfrey H. Thomson, D.Sc, Ph.D.
Malcolm Wilson, D.S., A R C.S.
Professor E. B. Bailey, M.C., M.A., F.R.S.
Professor J. C. Brash, M.C., M.A., M.D.
Professor A. J. Clark, M.C., B.A., M.D., F.R.S.
Professor A. G. Ogilvie, M.A., B.Sc.
Professor E. M. Wedderburn, M.A., D.Sc, LL.B., W.S.
Lt.-Col. A. G. M'Kendrick, M.B., D.Sc., F.R.C P.E.
Emeritus Professor James Mackinnon, M.A, Ph.D., LL.D
Professor William Peddie, D.Sc.

The **President** thanked the Scrutineers for their services.

The Public were then admitted.

The **President**, in the course of his remarks, referred to the losses of the Society by the deaths of Ordinary and Honorary Fellows, details of which may be found in the Obituary Notices, already published in the *Proceedings*.

In introducing Professor C. H. **Lander**, C.B.E., the **President** said:—

It is common knowledge that the production of oil from coal, by hydrogenation, is one of the most important industrial problems in this country at the present moment. The theory of the liquefaction of coal is well understood—the difficulty is so to adapt the process that it can be worked on a large scale and at a profit. This problem has been intensively studied by the Fuel Research Board of the Department of Scientific and Industrial Research during the past decade, and the promise of conversion of theory into practice recently announced has been rendered possible largely by the labours of this Board, of which Professor **Lander** was director from 1923 to 1931. His improvements in the construction of the stills used in low-temperature carbonisation may be particularly mentioned. Professor **Lander** has served with distinction on many other government committees connected with the coal industry, and since 1931 has held the chair of Mechanical Engineering at the Imperial College.

Professor C. H. **Lander**, C.B.E., at the request of the Council and in terms of the **Bruce-Preller Lecture Fund**, then delivered an address, with Lantern Illustrations, on "The Liquefaction of Coal."

The **President** cordially thanked Professor **Lander** for his address.

THE KEITH, MAKDOUGALL-BRISBANE, NEILL, GUNNING VICTORIA JUBILEE, JAMES SCOTT, BRUCE, AND DAVID ANDERSON-BERRY PRIZES, AND THE BRUCE-PRELLER LECTURE FUND.

The above Prizes will be awarded by the Council in the following manner:—

I. KEITH PRIZE.

The KEITH PRIZE, consisting of a Gold Medal and from £40 to £50 in Money, will be awarded in the Session 1935–1936 for the "best communication on a scientific subject, communicated,* in the first instance, to the Royal Society of Edinburgh during the Sessions 1933–1934 and 1934–1935." Preference will be given to a Paper containing a discovery. (See also Council's resolutions at the end of these regulations.)

II. MAKDOUGALL-BRISBANE PRIZE.

(*Amended June* 7, 1926.)

This Prize is to be awarded biennially by the Council of the Royal Society of Edinburgh to such person, for such purposes, for such objects, and in such manner as shall appear to them the most conducive to the promotion of the interests of science; with the *proviso* that the Council shall not be compelled to award the Prize unless there shall be some individual engaged in scientific pursuit, or some Paper written on a scientific subject, or some discovery in science made during the biennial period, of sufficient merit or importance in the opinion of the Council to be entitled to the Prize.

1. The Prize, consisting of a Gold Medal and a sum of Money, will be awarded before the close of the Session 1934–1935, for an Essay, Paper, or other work having reference to any branch of scientific inquiry, either material or mental.
2. It is open to all men of science.
3. The specific subjects taken into consideration in the current award are governed by the resolutions of the Council as stated at the end of these regulations.
4. For the current period the Committee is representative of Group B.
5. The Committee will consider Papers presented to the Society within the Sessions 1932–1933 and 1933–1934, and will make a recommendation.

It is empowered to recommend either.—

(*a*) An award to the Author of an Essay or Paper considered as above, or

(*b*) That no award be made on the ground that, within its group, no Paper of sufficient merit has been presented, or

---

* For the purposes of this award the word "communicated" shall be understood to mean the date on which the manuscript of a Paper is received in its final form for printing, as recorded by the General Secretary or other responsible official.

(*c*) That the Prize be awarded to some distinguished man of learning, who may not have presented a Paper to the Society within the period considered, but who is willing to deliver an address.

III. NEILL PRIZE.

The Council of the Royal Society of Edinburgh having received the bequest of the late Dr PATRICK NEILL of the sum of £500, for the purpose of "the interest thereof being applied in furnishing a Medal or other reward every second or third year to any distinguished Scottish Naturalist, according as such Medal or reward shall be voted by the Council of the said Society," hereby intimate:—

1. The NEILL PRIZE, consisting of a Gold Medal and a sum of Money, will be awarded during the Session 1935–1936.

2. The Prize will be given for a Paper of distinguished merit, on a subject of Natural History, by a Scottish Naturalist, which shall have been presented * to the Society during the two years preceding the fourth Monday in October 1935,—or failing presentation of a Paper sufficiently meritorious, it will be awarded for a work or publication by some distinguished Scottish Naturalist, on some branch of Natural History, bearing date within five years of the time of award. (See also Council's resolutions at the end of these regulations.)

IV. GUNNING VICTORIA JUBILEE PRIZE

This Prize, founded in the year 1887 by Dr R. H. GUNNING, is to be awarded quadrennially by the Council of the Royal Society of Edinburgh, in recognition of original work in Physics, Chemistry, or Pure or Applied Mathematics.

Evidence of such work may be afforded either by a Paper presented * to the Society, or by a Paper on one of the above subjects, or some discovery in them elsewhere communicated or made, which the Council may consider to be deserving of the Prize.

The Prize consists of a sum of money, and is open to men of science resident in or connected with Scotland. The first award was made in the year 1887. The next award will be made in Session 1936–1937

In accordance with the wish of the Donor, the Council of the Society may on fit occasions award the Prize for work of a definite kind to be undertaken during the three succeeding years by a scientific man of recognised ability.

V. JAMES SCOTT PRIZE.

This Prize, founded in the year 1918 by the Trustees of the JAMES SCOTT Bequest, is to be awarded triennially, or at such intervals as the Council of the Royal Society of Edinburgh may decide, "for a lecture or essay on the fundamental concepts of Natural Philosophy."

---

* For the purposes of this award the word "presented" shall be understood to mean the date on which the manuscript of a Paper is received in its final form for printing, as recorded by the General Secretary or other responsible official.

VI. BRUCE PRIZE.

The Society is trustee of a fund, instituted in 1923, to commemorate the work of Dr W. S. BRUCE as an explorer and scientific investigator in polar regions.

The Committee of Award is appointed jointly by the Royal Society of Edinburgh, the Royal Physical Society, and the Royal Scottish Geographical Society.

The Prize consists of a Bronze Medal and sum of Money. It is open to workers of all nationalities, with a preference, *ceteris paribus*, for those of Scottish birth or origin, and is to be awarded biennially for some notable contribution to Natural Sciences, such as Zoology, Botany, Geology, Meteorology, Oceanography, and Geography; the contribution to be in the nature of new knowledge, the outcome of a personal visit to polar regions on the part of the recipient. The recipient shall preferably be at the outset of his career as an investigator.

The next award will be made in 1934. Papers for the consideration of the Committee should be in the hands of the General Secretary of the Royal Society, 22 George Street, Edinburgh, not later than March 31 of that year.

VII. BRUCE-PRELLER LECTURE FUND.

The Council of the Royal Society of Edinburgh having received in 1929 the bequest of the late Dr CHARLES DU RICHE PRELLER of the sum of £500, decided that the income thereof be applied by the Council biennially as an honorarium for a special BRUCE-PRELLER Lecture or Address by an outstanding man of science, its subject to be Geology or Electrical or Physical Science, or in the discretion of the Council some other branch of science. The next award will be made in Session 1934–1935.

VIII. DAVID ANDERSON-BERRY FUND.

The Council of the Royal Society of Edinburgh having received in the year 1930, free of duty, the capital sum of one thousand pounds (£1000), to be used in terms of the will of the late Dr DAVID ANDERSON-BERRY, dated 23rd April 1926, decided that the income thereof be applied triennially, "in the first place in the presentation of a gold medal, and in the second place in the payment of a sum of money to the winner for the year of such gold medal, the winner being the person who, in the opinion of the Society, shall be the producer for the year of the best essay on the nature of X-rays and their therapeutical effect on human diseases."

The first award will be made in July 1935.

RESOLUTIONS OF COUNCIL IN REGARD TO THE MODE OF AWARDING PRIZES.

(*See Minutes of Meeting of January* 18, 1915.)

I. With regard to the Keith and Makdougall-Brisbane Prizes, which are open to all Sciences, the mode of award will be as follows:—

1. Papers or essays to be considered shall be arranged in two groups, A and B,—Group A to include Astronomy, Chemistry, Mathematics, Metallurgy, Meteorology, and Physics; Group B to include Anatomy, Anthropology, Botany, Geology, Pathology, Physiology, and Zoology.
2. These two prizes shall be awarded to each group in alternate biennial periods, provided Papers worthy of recommendation have been communicated to the Society.
3. Prior to the adjudication the Council shall appoint, in the first instance, a Committee composed of representatives of the group of Sciences which did not receive the award in the immediately preceding period. The Committee shall consider the Papers which come within their group of Sciences, and report in due course to the Council.
4. In the event of the aforesaid Committee reporting that within their group of subjects there is, in their opinion, no Paper worthy of being recommended for the award, the Council, on accepting this report, shall appoint a Committee representative of the alternate group to consider Papers coming within their group and to report accordingly.
5. Papers to be considered by the Committees shall fall within the period dating from the last award in groups A and B respectively.

II. With regard to the Neill Prize, the term "Naturalist" shall be understood to include any student in the Sciences composing group B, namely, Anatomy, Anthropology, Botany, Geology, Pathology, Physiology, Zoology.

AWARDS OF THE KEITH, MAKDOUGALL-BRISBANE, NEILL, GUNNING, JAMES SCOTT, BRUCE, AND DAVID ANDERSON-BERRY PRIZES, AND THE BRUCE-PRELLER LECTURE FUND.

I. KEITH PRIZE.

1st Biennial Period, 1827-29.—Dr Brewster, for his papers "on his Discovery of Two New Immiscible Fluids in the Cavities of certain Minerals," published in the Transactions of the Society.

2nd Biennial Period, 1829-31.—Dr Brewster, for his paper "on a New Analysis of Solar Light," published in the Transactions of the Society.

3rd Biennial Period, 1831-33.—Thomas Graham, Esq., for his paper "on the Law of the Diffusion of Gases," published in the Transactions of the Society.

4th Biennial Period, 1833-35.—Professor J. D. Forbes, for his paper "on the Refraction and Polarization of Heat," published in the Transactions of the Society.

5th Biennial Period, 1835-37.—John Scott Russell, Esq., for his researches "on Hydrodynamics," published in the Transactions of the Society.

6th Biennial Period, 1837-39.—Mr John Shaw, for his experiments "on the Development and Growth of the Salmon," published in the Transactions of the Society.

7th Biennial Period, 1839-41.—Not awarded.

8th Biennial Period, 1841-43.—Professor James David Forbes, for his papers "on Glaciers," published in the Proceedings of the Society.

9th Biennial Period, 1843-45.—Not awarded.

10th Biennial Period, 1845-47.—General Sir Thomas Brisbane, Bart., for the Makerstoun Observations on Magnetic Phenomena, made at his expense, and published in the Transactions of the Society.

11th Biennial Period, 1847-49.—Not awarded.

12th Biennial Period, 1849-51.—Professor Kelland, for his papers "on General Differentiation, including his more recent Communication on a process of the Differential Calculus, and its application to the solution of certain Differential Equations," published in the Transactions of the Society.

13th Biennial Period, 1851-53.—W. J. Macquorn Rankine, Esq., for his series of papers "on the Mechanical Action of Heat," published in the Transactions of the Society.

14th Biennial Period, 1853-55.—Dr Thomas Anderson, for his papers "on the Crystalline Constituents of Opium, and on the Products of the Destructive Distillation of Animal Substances," published in the Transactions of the Society.

15th Biennial Period, 1855-57.—Professor Boole, for his Memoir "on the Application of the Theory of Probabilities to Questions of the Combination of Testimonies and Judgments," published in the Transactions of the Society.

16th Biennial Period, 1857-59.—Not awarded.

17th Biennial Period, 1859-61.—John Allan Broun, Esq., F.R.S., Director of the Trevandrum Observatory, for his papers "on the Horizontal Force of the Earth's Magnetism, on the Correction of the Bifilar Magnetometer, and on Terrestrial Magnetism generally," published in the Transactions of the Society.

18th Biennial Period, 1861-63.—Professor William Thomson, of the University of Glasgow, for his Communication "on some Kinematical and Dynamical Theorems."

19th Biennial Period, 1863-65.—Principal Forbes, St Andrews, for his "Experimental Inquiry into the Laws of Conduction of Heat in Iron Bars," published in the Transactions of the Society.

20th Biennial Period, 1865-67.—Professor C. Piazzi Smyth, for his paper "on Recent Measures at the Great Pyramid," published in the Transactions of the Society.

21st Biennial Period, 1867–69.—Professor P. G. Tait, for his paper "on the Rotation of a Rigid Body about a Fixed Point," published in the Transactions of the Society.

22nd Biennial Period, 1869–71.—Professor Clerk Maxwell, for his paper "on Figures, Frames, and Diagrams of Forces," published in the Transactions of the Society.

23rd Biennial Period, 1871–73 —Professor P. G. Tait, for his paper entitled "First Approximation to a Thermo-electric Diagram," published in the Transactions of the Society.

24th Biennial Period, 1873–75.—Professor Crum Brown, for his Researches "on the Sense of Rotation, and on the Anatomical Relations of the Semicircular Canals of the Internal Ear."

25th Biennial Period, 1875–77.—Professor M. Forster Heddle, for his papers "on the Rhombohedral Carbonates," and "on the Felspars of Scotland," published in the Transactions of the Society.

26th Biennial Period, 1877–79.—Professor H. C Fleeming Jenkin, for his paper "on the Application of Graphic Methods to the Determination of the Efficiency of Machinery," published in the Transactions of the Society; Part II having appeared in the volume for 1877–78.

27th Biennial Period, 1879–81.—Professor George Chrystal, for his paper "on the Differential Telephone," published in the Transactions of the Society.

28th Biennial Period, 1881–83.—Thomas Muir, Esq , LL.D., for his "Researches into the Theory of Determinants and Continued Fractions," published in the Proceedings of the Society.

29th Biennial Period, 1883–85.—John Aitken, Esq , for his paper "on the Formation of Small Clear Spaces in Dusty Air," and for previous papers on Atmospheric Phenomena, published in the Transactions of the Society

30th Biennial Period, 1885–87.—John Young Buchanan, Esq , for a series of communications, extending over several years, on subjects connected with Ocean Circulation, Compressibility of Glass, etc., two of which, viz , "On Ice and Brines," and "On the Distribution of Temperature in the Antarctic Ocean," have been published in the Proceedings of the Society.

31st Biennial Period, 1887–89.—Professor E. A Letts, for his papers on the Organic Compounds of Phosphorus, published in the Transactions of the Society.

32nd Biennial Period, 1889–91 —R. T. Omond, Esq , for his contributions to Meteorological Science, many of which are contained in vol. xxxiv of the Society's Transactions.

33rd Biennial Period, 1891–93.—Professor Thomas R. Fraser, F.R.S., for his papers on *Strophanthus hispidus*, Strophanthin, and Strophanthidin, read to the Society in February and June 1889 and in December 1891, and printed in vols. xxxv, xxxvi, and xxxvii of the Society's Transactions

34th Biennial Period, 1893–95 —Dr Cargill G Knott, for his papers on the Strains produced by Magnetism in Iron and in Nickel, which have appeared in the Transactions and Proceedings of the Society.

35th Biennial Period, 1895–97.—Dr Thomas Muir, for his continued communications on Determinants and Allied Questions.

36th Biennial Period, 1897–99 —Dr James Burgess, for his paper "on the Definite Integral $\frac{2}{\sqrt{\pi}}\int_0^t e^{-t^2}dt$, with extended Tables of Values," printed in vol xxxix of the Transactions of the Society.

37th Biennial Period, 1899–1901.—Dr Hugh Marshall, for his discovery of the Persulphates, and for his Communications on the Properties and Reactions of these Salts, published in the Proceedings of the Society.

38th Biennial Period, 1901–03.—Sir William Turner, K.C.B , LL.D., F R.S., etc., for his memoirs entitled "A Contribution to the Craniology of the People of Scotland," published in the Transactions of the Society, and for his "Contributions to the Craniology of the People of the Empire of India," Parts I, II, likewise published in the Transactions of the Society.

39th Biennial Period, 1903–05.—Thomas H. Bryce, M.A., M.D., for his two papers on "The Histology of the Blood of the Larva of *Lepidosiren paradoxa*," published in the Transactions of the Society within the period.

40th Biennial Period, 1905–07.—Alexander Bruce, M.A , M.D., F.R.C.P.E., for his paper entitled "Distribution of the Cells in the Intermedio-Lateral Tract of the Spinal Cord," published in the Transactions of the Society within the period.

41st Biennial Period, 1907–09.—Wheelton Hind, M.D., B.S., F.R.C.S., F.G.S., for a paper published in the Transactions of the Society, "On the Lamellibranch and Gasteropod Fauna found in the Millstone Grit of Scotland."

42nd Biennial Period, 1909–11.—Professor Alexander Smith, B.Sc., Ph.D., of New York, for his researches upon "Sulphur" and upon "Vapour Pressure," appearing in the Proceedings of the Society.

43rd Biennial Period, 1911–13.—James Russell, Esq., for his series of investigations relating to magnetic phenomena in metals and the molecular theory of magnetism, the results of which have been published in the Proceedings and Transactions of the Society, the last paper having been issued within the period.

44th Biennial Period, 1913–15.—James Hartley Ashworth, D.Sc., for his papers on "Larvæ of Lingula and Pelagodiscus," and on "Sclerocheilus," published in the Transactions of the Society, and for other papers on the Morphology and Histology of Polychæta.

45th Biennial Period, 1915–17.—Robert C. Mossman, Esq., for his work on the Meteorology of the Antarctic Regions, which originated with the important series of observations made by him during the voyage of the "Scotia" (1902–1904), and includes his paper "On a Sea-Saw of Barometric Pressure, Temperature, and Wind Velocity between the Weddell Sea and the Ross Sea," published in the Proceedings of the Society.

46th Biennial Period, 1917–19.—John Stephenson, Lt.-Col., I.M.S., for his series of papers on the Oligochæta and other Annelida, several of which have been published in the Transactions of the Society.

47th Biennial Period, 1919–21.—Ralph Allen Sampson, F.R.S., for his Astronomical Researches, including the papers "Studies in Clocks and Time-keeping: No. 1, Theory of the Maintenance of Motion; No. 2, Tables of the Circular Equation," published in the Proceedings of the Society within the period of the award.

48th Biennial Period, 1921–23.—John Walter Gregory, F.R.S., for his papers published in the Transactions of the Society, and in recognition of his numerous contributions to Geology, extending over a period of thirty-six years.

49th Biennial Period, 1923–25.—Herbert Westren Turnbull, M.A., for the papers on "Hyper-Algebra," "Invariant Theory," and "Algebraic Geometry," three of which have been published in the Proceedings within the period of award.

50th Biennial Period, 1925–27.—Thomas John Jehu, M.A., M.D., F.G.S., and Robert Meldrum Craig, M.A., B.Sc., F.G.S., for the joint series of papers which have recently appeared in the Transactions of the Society on the "Geology of the Outer Hebrides."

51st Biennial Period, 1927–29.—Christina C. Miller, B.Sc., Ph.D., for her papers on the "Slow Oxidation of Phosphorus Trioxide," published in the Proceedings within the period of the award, and in consideration of subsequent developments on "Slow Oxidation of Phosphorus," published elsewhere.

52nd Biennial Period, 1929–31.—Alan William Greenwood, M.Sc., Ph.D., for his papers on the "Biology of the Fowl," several of which have appeared in the Proceedings within the period of award.

II. MAKDOUGALL-BRISBANE PRIZE.

1st Biennial Period, 1859.—Sir Roderick Impey Murchison, on account of his Contributions to the Geology of Scotland.

2nd Biennial Period, 1860–62.—William Seller, M.D., F.R.C.P.E., for his "Memoir of the Life and Writings of Dr Robert Whytt," published in the Transactions of the Society.

3rd Biennial Period, 1862–64.—John Denis Macdonald, Esq., R.N., F.R.S., Surgeon of H.M.S. "Icarus," for his paper "on the Representative Relationships of the Fixed and Free Tunicata, regarded as Two Sub-classes of equivalent value; with some General Remarks on their Morphology," published in the Transactions of the Society.

4th Biennial Period, 1864–66.—Not awarded.

5th Biennial Period, 1866–68.—Dr Alexander Crum Brown and Dr Thomas Richard Fraser, for their conjoint paper "on the Connection between Chemical Constitution and Physiological Action," published in the Transactions of the Society.

6th Biennial Period, 1868–70.—Not awarded.

7th Biennial Period, 1870–72.—George James Allman, M.D., F.R.S., Emeritus Professor of Natural History, for his paper "on the Homological Relations of the Cœlenterata," published in the Transactions, which forms a leading chapter of his Monograph of Gymnoblastic or Tubularian Hydroids—since published.

8TH BIENNIAL PERIOD, 1872–74.—Professor LISTER, for his paper "on the Germ Theory of Putrefaction and the Fermentive Changes," communicated to the Society, 7th April 1873.

9TH BIENNIAL PERIOD, 1874–76.—ALEXANDER BUCHAN, A.M., for his paper "on the Diurnal Oscillation of the Barometer," published in the Transactions of the Society.

10TH BIENNIAL PERIOD, 1876–78.—Professor ARCHIBALD GEIKIE, for his paper "on the Old Red Sandstone of Western Europe," published in the Transactions of the Society.

11TH BIENNIAL PERIOD, 1878–80.—Professor PIAZZI SMYTH, Astronomer-Royal for Scotland, for his paper "on the Solar Spectrum in 1877–78, with some Practical Idea of its probable Temperature of Origination," published in the Transactions of the Society.

12TH BIENNIAL PERIOD, 1880–82.—Professor JAMES GEIKIE, for his "Contributions to the Geology of the North-West of Europe," including his paper "on the Geology of the Faroes," published in the Transactions of the Society.

13TH BIENNIAL PERIOD, 1882–84.—EDWARD SANG, Esq., LL.D., for his paper "on the Need of Decimal Subdivisions in Astronomy and Navigation, and on Tables requisite therefor," and generally for his Recalculations of Logarithms both of Numbers and Trigonometrical Ratios—the former communication being published in the Proceedings of the Society.

14TH BIENNIAL PERIOD, 1884–86.—JOHN MURRAY, Esq., LL.D., for his papers "On the Drainage Areas of Continents and Ocean Deposits," "The Rainfall of the Globe, and Discharge of Rivers," "The Height of the Land and Depth of the Ocean," and "The Distribution of Temperature in the Scottish Lochs as affected by the Wind."

15TH BIENNIAL PERIOD, 1886–88.—ARCHIBALD GEIKIE, Esq., LL.D., for numerous Communications, especially that entitled "History of Volcanic Action during the Tertiary Period in the British Isles," published in the Transactions of the Society.

16TH BIENNIAL PERIOD, 1888–90.—Dr LUDWIG BECKER, for his paper on "The Solar Spectrum at Medium and Low Altitudes," printed in vol. xxxvi, Part I, of the Society's Transactions.

17TH BIENNIAL PERIOD, 1890–92.—HUGH ROBERT MILL, Esq., D.Sc., for his papers on "The Physical Conditions of the Clyde Sea Area," Part I being already published in vol. xxxvi of the Society's Transactions.

18TH BIENNIAL PERIOD, 1892–94.—Professor JAMES WALKER, D.Sc., Ph.D., for his work on Physical Chemistry, part of which has been published in the Proceedings of the Society, vol. xx, pp. 255–263. In making this award, the Council took into consideration the work done by Professor Walker along with Professor Crum Brown on the Electrolytic Synthesis of Dibasic Acids, published in the Transactions of the Society.

19TH BIENNIAL PERIOD, 1894–96.—Professor JOHN G. M'KENDRICK, for numerous Physiological papers, especially in connection with Sound, many of which have appeared in the Society's publications.

20TH BIENNIAL PERIOD, 1896–98.—Dr WILLIAM PEDDIE, for his papers on the Torsional Rigidity of Wires.

21ST BIENNIAL PERIOD, 1898–1900.—Dr RAMSAY H. TRAQUAIR, for his paper entitled "Report on Fossil Fishes collected by the Geological Survey in the Upper Silurian Rocks of Scotland," printed in vol. xxxix of the Transactions of the Society.

22ND BIENNIAL PERIOD, 1900–02.—Dr ARTHUR T. MASTERMAN, for his paper entitled "The Early Development of *Cribrella oculata* (Forbes), with remarks on Echinoderm Development," printed in vol. xl of the Transactions of the Society.

23RD BIENNIAL PERIOD, 1902–04.—JOHN DOUGALL, M.A., for his paper on "An Analytical Theory of the Equilibrium of an Isotropic Elastic Plate," published in vol. xli of the Transactions of the Society.

24TH BIENNIAL PERIOD, 1904–06.—JACOB E. HALM, Ph.D., for his two papers entitled "Spectroscopic Observations of the Rotation of the Sun," and "Some Further Results obtained with the Spectroheliometer," and for other astronomical and mathematical papers published in the Transactions and Proceedings of the Society within the period.

25TH BIENNIAL PERIOD, 1906–08.—D. T. GWYNNE-VAUGHAN, M.A., F.L.S., for his papers, 1st, "On the Fossil Osmundaceæ," and 2nd, "On the Origin of the Adaxially-curved Leaf-trace in the Filicales," communicated by him conjointly with Dr R. Kidston.

26TH BIENNIAL PERIOD, 1908–10.—ERNEST MACLAGAN WEDDERBURN, M.A., LL.B., for his series of papers bearing upon "The Temperature Distribution in Fresh-water Lochs," and especially upon "The Temperature Seiche."

27TH BIENNIAL PERIOD, 1910–12.—JOHN BROWNLEE, M.A., M.D., D.Sc., for his contributions to the Theory of Mendelian Distributions and cognate subjects, published in the Proceedings of the Society within and prior to the prescribed period.

28TH BIENNIAL PERIOD, 1912–14.—Professor C. R. MARSHALL, M.A., M.D., for his studies "On the Pharmacological Action of Tetra-alkyl-ammonium Compounds."

29TH BIENNIAL PERIOD, 1914–16.—ROBERT ALEXANDER HOUSTOUN, Ph D., D.Sc., for his series of papers on "The Absorption of Light by Inorganic Salts," published in the Proceedings of the Society.

30TH BIENNIAL PERIOD, 1916–18.—Professor A. ANSTRUTHER LAWSON, for his Memoirs on "The Prothalli of *Tmesipteris Tannensis* and of *Psilotum*," published in the Transactions of the Society, together with previous papers on Cytology and on The Gametophytes of various Gymnosperms

31ST BIENNIAL PERIOD, 1918–20.—Professor J. H MACLAGAN WEDDERBURN of Princeton University, for his Memoirs in Universal Algebra, etc., published in the Transactions and Proceedings of the Society, and elsewhere.

32ND BIENNIAL PERIOD, 1920–22.—Professor W. T. GORDON, M.A, D Sc., for his paper on "Cambrian Organic Remains from a Dredging in the Weddell Sea," published in the Transactions of the Society within the period, and for his investigations on the Fossil Flora of the Pettycur Limestone, previously published in the Transactions

33RD BIENNIAL PERIOD, 1922–24.—Professor H. STANLEY ALLEN, D Sc, for his papers on the "Quantum and Atomic Theory," published in the Society's Proceedings within the period

34TH BIENNIAL PERIOD, 1924–26.—CHARLES MORLEY WENYON, C.M G., C B.E, F.R S., for his distinguished work in Protozoology extending over a period of twenty-one years.

35TH BIENNIAL PERIOD, 1926–28.—W. O. KERMACK, M.A., D.Sc, for his contributions to Chemistry, published in the Society's Proceedings and elsewhere.

36TH BIENNIAL PERIOD, 1928–30.—NELLIE B EALES, D Sc., for her papers in the Society's Transactions on "The Anatomy of a Fœtal African Elephant.'

37TH BIENNIAL PERIOD, 1930–32.—A. C. AITKEN, M A., D.Sc., for various contributions to Mathematics, published in the Society's Proceedings and elsewhere

III. THE NEILL PRIZE.

1ST TRIENNIAL PERIOD, 1856–59.—Dr W. LAUDER LINDSAY, for his paper "on the Spermogones and Pycnides of Filamentous, Fruticulose, and Foliaceous Lichens," published in the Transactions of the Society

2ND TRIENNIAL PERIOD, 1859–62.—ROBERT KAYE GREVILLE, LL D., for his contributions to Scottish Natural History, more especially in the department of Cryptogamic Botany, including his recent papers on Diatomaceæ.

3RD TRIENNIAL PERIOD, 1862–65.—ANDREW CROMBIE RAMSAY, F.R.S., Professor of Geology in the Government School of Mines, and Local Director of the Geological Survey of Great Britain, for his various works and memoirs published during the last five years, in which he has applied the large experience acquired by him in the Direction of the arduous work of the Geological Survey of Great Britain to the elucidation of important questions bearing on Geological Science.

4TH TRIENNIAL PERIOD, 1865–68.—Dr WILLIAM CARMICHAEL M'INTOSH, for his paper "on the Structure of the British Nemerteans, and on some New British Annelids," published in the Transactions of the Society.

5TH TRIENNIAL PERIOD, 1868–71.—Professor WILLIAM TURNER, for his papers "on the Great Finner Whale; and on the Gravid Uterus, and the Arrangement of the Fœtal Membranes in the Cetacea," published in the Transactions of the Society.

6TH TRIENNIAL PERIOD, 1871–74.—CHARLES WILLIAM PEACH, Esq., for his Contributions to Scottish Zoology and Geology, and for his recent contributions to Fossil Botany.

7TH TRIENNIAL PERIOD, 1874–77.—Dr RAMSAY H. TRAQUAIR, for his paper "on the Structure and Affinities of *Tristichopterus alatus* (Egerton)," published in the Transactions of the Society, and also for his contributions to the Knowledge of the Structure of Recent and Fossil Fishes.

8TH TRIENNIAL PERIOD, 1877–80.—JOHN MURRAY, Esq., for his paper "on the Structure and Origin of Coral Reefs and Islands," published (in abstract) in the Proceedings of the Society.

9TH TRIENNIAL PERIOD, 1880–83.—Professor W. A. HERDMAN, for his papers "on the Tunicata," published in the Proceedings and Transactions of the Society.

10TH TRIENNIAL PERIOD, 1883–86.—B. N. PEACH, Esq., for his Contributions to the Geology and Palæontology of Scotland, published in the Transactions of the Society.

11TH TRIENNIAL PERIOD, 1886–89.—ROBERT KIDSTON, Esq., for his Researches in Fossil Botany, published in the Transactions of the Society.

12TH TRIENNIAL PERIOD, 1889–92.—JOHN HORNE, Esq., F.G S., for his Investigations into the Geological Structure and Petrology of the North-West Highlands.

13TH TRIENNIAL PERIOD, 1892–95.—ROBERT IRVINE, Esq., for his papers on the Action of Organisms in the Secretion of Carbonate of Lime and Silica, and on the solution of these substances in Organic Juices. These are printed in the Society's Transactions and Proceedings.

14TH TRIENNIAL PERIOD, 1895–98.—Professor J. COSSAR EWART, for his recent Investigations connected with Telegony.

15TH TRIENNIAL PERIOD, 1898–1901 —Dr JOHN S. FLETT, for his papers entitled "The Old Red Sandstone of the Orkneys" and "The Trap Dykes of the Orkneys," printed in vol xxxix of the Transactions of the Society

16TH TRIENNIAL PERIOD, 1901–04.—Professor J GRAHAM KERR, M.A., for his Researches on *Lepidosiren paradoxa*, published in the Philosophical Transactions of the Royal Society, London.

17TH TRIENNIAL PERIOD, 1904–07.—FRANK J. COLE, B Sc, for his paper entitled "A Monograph on the General Morphology of the Myxinoid Fishes, based on a Study of Myxine," published in the Transactions of the Society, regard being also paid to Mr Cole's other valuable contributions to the Anatomy and Morphology of Fishes.

1ST BIENNIAL PERIOD, 1907–09 —FRANCIS J. LEWIS, M.Sc, F L S., for his papers in the Society's Transactions "On the Plant Remains of the Scottish Peat Mosses"

2ND BIENNIAL PERIOD, 1909–11.—JAMES MURRAY, Esq, for his paper on "Scottish Rotifers collected by the Lake Survey (Supplement)," and other papers on the "Rotifera" and "Tardigrada," which appeared in the Transactions of the Society – (this Prize was awarded after consideration of the papers received within the five years prior to the time of award. see Neill Prize Regulations).

3RD BIENNIAL PERIOD, 1911–13 —W S. BRUCE, LL D, in recognition of the scientific results of his Arctic and Antarctic explorations.

4TH BIENNIAL PERIOD, 1913–15 —ROBERT CAMPBELL, D Sc, for his paper on "The Upper Cambrian Rocks at Craigeven Bay, Stonehaven," and "Downtonian and Old Red Sandstone Rocks of Kincardineshire," published in the Transactions of the Society.

5TH BIENNIAL PERIOD, 1915–17.—W. H. LANG, F.R S, M.B, D.Sc, for his paper in conjunction with Dr R KIDSTON, F.R.S, on *Rhynia Gwynne-Vaughani*, Kidston and Lang, published in the Transactions of the Society, and for his previous investigations on Pteridophytes and Cycads.

6TH BIENNIAL PERIOD, 1917–19 —JOHN TAIT, D.Sc., M D., for his work on Crustacea, published in the Proceedings of the Society, and for his papers on the blood of Crustacea.

7TH BIENNIAL PERIOD, 1919–21.—Sir EDWARD A. SHARPEY-SCHAFER, F.R S., for his recent contributions to our knowledge of Physiology, and in recognition of his published work extending over a period of fifty years.

8TH BIENNIAL PERIOD, 1921–23 —JOHN M'LEAN THOMPSON, M.A., D.Sc., University of Liverpool, for his series of Memoirs on Staminal Zygomorphy, and on the Anatomy of the Filicales.

9TH BIENNIAL PERIOD, 1923–25.—FREDERICK ORPEN BOWER, F.R.S., for his recent contributions to Botanical knowledge and in recognition of his published work extending over a period of forty-five years.

10TH BIENNIAL PERIOD, 1925–27.—ARTHUR ROBINSON, M.D., M.R.C.S., for his contributions to Comparative Anatomy and Embryology.

11TH BIENNIAL PERIOD, 1927–29 —EDWARD BATTERSBY BAILEY, M.C., F.R.S., in recognition of his valuable contributions to the Geology of Scotland, two of which have recently appeared in the Transactions of the Society.

12TH BIENNIAL PERIOD, 1929–31.—CHARLES HENRY O'DONOGHUE, D.Sc., for his papers on the "Blood Vascular System," and for his earlier work on the "Morphology of the *corpus luteum*."

IV. GUNNING VICTORIA JUBILEE PRIZE.

1ST TRIENNIAL PERIOD, 1884–87.—Sir WILLIAM THOMSON, Pres. R.S.E., F.R.S., for a remarkable series of papers "on Hydrokinetics," especially on Waves and Vortices which have been communicated to the Society.

2ND TRIENNIAL PERIOD, 1887–90.—Professor P. G. TAIT, Sec. R.S.E., for his work in connection with the "Challenger" Expedition, and his other Researches in Physical Science.

3RD TRIENNIAL PERIOD, 1890–93.—ALEXANDER BUCHAN, Esq., LL.D., for his varied, extensive, and extremely important Contributions to Meteorology, many of which have appeared in the Society's publications.

4TH TRIENNIAL PERIOD, 1893–96.—JOHN AITKEN, Esq., for his brilliant Investigations in Physics, especially in connection with the Formation and Condensation of Aqueous Vapour.

1ST QUADRENNIAL PERIOD, 1896–1900.—Dr T. D. ANDERSON, for his discoveries of New and Variable Stars.

2ND QUADRENNIAL PERIOD, 1900–04.—Sir JAMES DEWAR, LL.D., D.C.L., F.R.S., etc., for his researches on the Liquefaction of Gases, extending over the last quarter of a century, and on the Chemical and Physical Properties of Substances at Low Temperatures: his earliest papers being published in the Transactions and Proceedings of the Society.

3RD QUADRENNIAL PERIOD, 1904–08.—Professor GEORGE CHRYSTAL, M.A., LL.D., for a series of papers on "Seiches," including "The Hydrodynamical Theory and Experimental Investigations of the Seiche Phenomena of Certain Scottish Lakes."

4TH QUADRENNIAL PERIOD, 1908–12.—Professor J. NORMAN COLLIE, Ph.D., F.R.S., for his distinguished contributions to Chemistry, Organic and Inorganic, during twenty-seven years, including his work upon Neon and other rare gases. Professor Collie's early papers were contributed to the Transactions of the Society.

5TH QUADRENNIAL PERIOD, 1912–16.—Sir THOMAS MUIR, C.M.G., LL.D., F.R.S., for his series of Memoirs upon "The Theory and History of Determinants and Allied Forms," published in the Transactions and Proceedings of the Society between the years 1872 and 1915.

6TH QUADRENNIAL PERIOD, 1916–20.—C. T. R. WILSON, Esq., F.R.S., in recognition of his important discoveries in relation to Condensation Nuclei, Ionisation of Gases and Atmospheric Electricity.

7TH QUADRENNIAL PERIOD, 1920–24.—Sir J. J. THOMSON, O.M., F.R.S., in recognition of his great discoveries in Physical Science.

8TH QUADRENNIAL PERIOD, 1924–28.—Professor E. T. WHITTAKER, F.R.S., in recognition of his distinguished contributions to Mathematical Science, and of his promotion of Mathematical Research in Scotland.

9TH QUADRENNIAL PERIOD, 1928–32.—Emeritus Professor Sir J. WALKER, F.R.S., for numerous contributions to Physical and General Chemistry.

V. JAMES SCOTT PRIZE.

1ST AWARD, 1918–22.—Professor A. N. WHITEHEAD, F.R.S., for his lecture delivered on June 5, 1922, on "The Relatedness of Nature."

2ND AWARD, 1922–27.—Sir JOSEPH LARMOR, M.A., D.Sc., LL.D., F.R.S., for his lecture delivered on July 4, 1927, on "The Grasp of Mind on Nature."

3RD AWARD, 1927–30.—Professor NIELS BOHR, for his lecture delivered on May 26, 1930, on "Philosophical Aspects of Atomic Theory."

4TH AWARD, 1930–33.—Professor Dr ARNOLD SOMMERFELD, for his lecture delivered on May 1, 1933, on "Ways to the Knowledge of Nature."

VI. BRUCE PRIZE.

1ST AWARD, 1926.—JAMES MANN WORDIE, M.A., for his Oceanographical and Geological work in both Polar Regions.

2ND AWARD, 1928.—H. U. SVERDRUP, for his contributions to the knowledge of the Meteorology, Magnetism, and Tides of the Arctic, as an outcome of his travels with the Norwegian Expedition in the "Maud" from 1918 to 1925.

3RD AWARD, 1930.—N. A. MACKINTOSH, M.Sc., A.R.C.S., for his researches into the Biology of Whales in the Waters of the Falkland Islands Dependencies.

4TH AWARD, 1932.—HENRY GINO WATKINS, for important contributions to the topography of Spitsbergen, Labrador, and East Greenland, and investigation of the Ice Cap of Greenland.

VII. BRUCE-PRELLER LECTURE FUND.

1ST AWARD, 1931.—Professor E. T. WHITTAKER, F.R.S., for his lecture, "James Clerk Maxwell and Mechanical Descriptions of the Universe."

2ND AWARD, 1933 —Professor C. H. LANDER, C.B.E., for his lecture on October 23, 1933, on "The Liquefaction of Coal"

VIII. DAVID ANDERSON-BERRY FUND.

ABSTRACT

OF

THE ACCOUNTS

OF

THE ROYAL SOCIETY OF EDINBURGH,

SESSION—1st October 1932 to 30th September 1933.

JAMES WATT, LL.D., W.S.,
Treasurer.

I. GENERAL FUND

CHARGE.

1. Arrears of Contributions at 30th September 1932			£103 19 0
Less—Written off as irrecoverable			9 9 0
			£94 10 0
2. Contributions for present Session:—			
1. 413 Fellows at £3, 3s each		£1300 19 0	
2. Fees of Admission and Contributions of thirty-four new Fellows at £6, 6s. each		214 4 0	
3. Commutations		111 6 0	
			1626 9 0
3. Extra Contributions for 1932–33 under Amended Law, No. 6·—			
1. Voluntary Contributions		£51 9 0	
2. Commutations		10 10 0	
			61 19 0
4. Interest received—			
a. Interest on £445, 10s. 3½% War Loan, 1952 [originally 5%] (R. M. Smith Legacy), Untaxed		£18 18 8	
b. Interest on £751, 16s. 3½% War Loan, 1952 [originally 5%] (Special Subscription Fund), Untaxed		31 19 1	
c. General—			
Interest on £7830 3½% War Loan, 1952 [originally 5%], Untaxed	£332 15 6		
Interest on £2100 2½% Consolidated Stock, Untaxed.	52 10 0		
Interest on Deposit Receipts	7 18 7		
		393 4 1	
			444 1 10
5. Transactions and Proceedings sold			178 17 9
6. Grants—Annual Grant from Government		£600 0 0	
Grant from Royal Society, Government Publication Grant		250 0 0	
			850 0 0
Amount of the Charge			£3255 17 7

DISCHARGE.

1. TAXES, INSURANCE, COAL AND LIGHTING:—				
Insurance				£22 5 5
Coal, etc, to 6th May 1933				39 2 0
Gas to 1st Sept. 1933				2 11 2
Electric Light to 14th April 1933				12 16 4
Water, 1932–33				4 4 0
Equitable Life Assurance Co., Ltd., for Superannuation Scheme			£52 10 0	
Sun Life Assurance Society, for Superannuation Scheme			7 18 4	
Scottish Widows' Fund and Life Assurance Society, for Superannuation Scheme			18 6 8	
			£78 15 0	
Less—Received from Assistant Secretary		£17 10 0		
Received from Assistant Librarian		4 7 6		
			21 17 6	
				56 17 6
				£137 16 5
2. SALARIES:—				
General Secretary			£100 0 0	
Librarian and Assistant Secretary			350 0 0	
Assistant Librarian			165 0 0	
Office Keeper			151 12 0	
Treasurer's Clerk			35 0 0	
				801 12 0
3. EXPENSES OF TRANSACTIONS AND PROCEEDINGS:—				
a. TRANSACTIONS:—				
Neill & Co, Ltd, Printers			£572 16 5	
Hislop & Day, Ltd., Engravers			76 12 5	
The Zinco-Collotype Co			132 0 0	
Macfarlane & Erskine, Printers			8 7 6	
			£789 16 4	
Less Receipts:—				
Grants from Carnegie Trustees and others towards the Papers, etc., of Miss Case, Dr Amy Fleming, and Messrs C. H. O'Donoghue and A. G. Hutchison			46 1 10	
			£743 14 6	
b. PROCEEDINGS:—				
Neill & Co., Ltd., Printers		£674 17 5		
Hislop & Day, Ltd, Engravers		32 16 1		
		£707 13 6		
Less Receipts.—				
1. Revenue of Publication Fund	£85 11 9			
2. Grants from Carnegie Trustees and others towards the Papers, etc., of Miss E. Boyd, Sir E. A. Sharpey-Schafer, and Messrs E. L. Ince, Geo. Bond, W. F. Harper, and T. Nicol	125 7 5			
		210 19 2		
			496 14 4	
				1240 8 10
Carry forward				£2179 17 3

	£	s.	d.	£	s.	d.
Brought forward				£2179	17	3
4. BOOKS, PERIODICALS, NEWSPAPERS, ETC.:—						
James Thin, Bookseller	£309	12	4			
R. Grant & Son, Booksellers	4	11	7			
Robertson & Scott, News Agents	7	8	2			
Ray Society, Subscription	1	1	0			
Berwickshire Naturalists' Club, Do.	0	10	0			
Palæontographical Society, Do.	1	1	0			
History of Science Society, Do.	1	9	10			
Edinburgh and Leith Post-Office Directory, Ltd.	0	13	6			
Institution of Civil Engineers, for Abstracts	0	10	0			
Bombay Natural History Society, for *Journal*	1	5	6			
Scientific and Learned Societies' Year Book 1933	0	8	6			
Folk Lore Society, Sir J. G. Frazer's Bibliography	1	1	0			
				329	12	5
5. OTHER PAYMENTS:—						
Neill & Co., Ltd., Printers	£119	9	0			
A. Cowan & Sons, Ltd.	15	18	11			
S. Duncan & Sons, Tailors (uniform)	9	9	0			
Macvitties, Guest & Co., Ltd	42	11	9			
Andrew H. Baird.	6	13	0			
Lindsay, Jamieson & Haldane, C.A., Auditors	10	10	0			
The Window Cleaning Co., Ltd.	14	19	0			
Orrock & Son, Ltd., Bookbinders	150	3	3			
Federated Superannuation System for Universities	5	0	0			
G. Waterston & Sons, Ltd.	13	19	10			
Telephone Accounts	13	18	3			
Roneo, Ltd.—Shelving, etc	95	3	3			
W. S. Brown & Sons, Upholsterers, etc.	8	16	0			
J. & T. Scott, Joiners and Painters	33	9	6			
J. Sinclair & Son, Painters	5	3	0			
Miscellaneous Accounts under £5	20	9	9			
Charwoman	63	14	0			
Petty Expenses, Postages, Carriage, etc.	44	17	0			
Edward & Co., Electricians	8	4	11			
"Polar Year" Expenses	6	14	3			
				689	3	8
6. ARREARS OF CONTRIBUTIONS outstanding at 30th September 1933:—						
Present Session	£75	12	0			
Previous Sessions	44	2	0			
				119	14	0
Amount of the Discharge				£3318	7	4

	£	s.	d.
Amount of the Charge	£3255	17	7
Amount of the Discharge	3318	7	4
Excess of Discharge transferred to Special Subscription Fund	£62	9	9

SPECIAL SUBSCRIPTION FUND

To 30th September 1933.

CHARGE.

	£	s.	d.	£	s.	d.
Total Subscriptions towards Fund				£1128	17	9
Less—Written off War Loan Investment				7	12	0
				£1121	5	9
Less—Transfers to General Fund to meet Deficits up to 30th September 1932	£373	11	7			
Add—Deficit for year to 30th September 1933.	62	9	9			
				436	1	4
AMOUNT OF THE CHARGE				£685	4	5

DISCHARGE.

BALANCE OF FUND—	
£751, 16s. 3½% War Loan, 1952 (originally 5%)	£751 16 0
Due by Treasurer	10 15 0
Due by War Loan Suspense Account	0 17 0
	£763 8 0
Less—Due to Union Bank of Scotland, Ltd., on Current Account	78 3 7
AMOUNT OF THE DISCHARGE	£685 4 5

WAR LOAN SUSPENSE ACCOUNT.

To 30th September 1933.

CHARGE.

Due by Union Bank of Scotland, Ltd., on Current Account at 30th September 1932.		£110 4 6
Interest on Deposit Receipt (included in General Fund)	£0 17 0	
AMOUNT OF THE CHARGE		£110 4 6

DISCHARGE.

Due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1933.	£111 1 6
Less—Due to General Fund	0 17 0
AMOUNT OF THE DISCHARGE	£110 4 6

II. KEITH FUND

Year to 30th September 1933.

CHARGE.

1. **BALANCE** due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1932		£53 2 1
2. **INTEREST RECEIVED**:—		
On £650 3½% War Loan, 1952 (originally 5%), Untaxed.	£27 12 6	
On Deposit Receipts	0 13 1	
		28 5 7
		£81 7 8

DISCHARGE.

BALANCE due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1933	£81 7 8

III. NEILL FUND

Year to 30th September 1933.

CHARGE.

1. **BALANCE** due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1932		£23 8 5
2. **INTEREST RECEIVED**:—		
On £300 3½% War Loan, 1952 (originally 5%), Untaxed.	£12 15 0	
On Deposit Receipts	0 5 4	
		13 0 4
		£36 8 9

DISCHARGE.

BALANCE due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1933	£36 8 9

IV. MAKDOUGALL-BRISBANE FUND

Year to 30th September 1933.

CHARGE.

1. **Balance** due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1932		£74 15 9
2. **Interest Received**:—		
On £400 3½% War Loan, 1952 (originally 5%), Untaxed	£17 0 0	
On Deposit Receipts	0 13 9	
		17 13 9
		£92 9 6

DISCHARGE

1. Alex. Kirkwood & Son—Gold Medal	£20 0 0
Alex. Craig Aitken—1930-32 Award	20 0 0
	£40 0 0
2. **Balance** due by Union Bank of Scotland, Ltd, on Deposit Receipt at 30th September 1933	52 9 6
	£92 9 6

V. MAKERSTOUN MAGNETIC METEOROLOGICAL OBSERVATION FUND

Year to 30th September 1933.

CHARGE.

1. **Balance** due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1932		£95 2 9
2. **Interest Received**:—		
On £250 3½% War Loan, 1952 (originally 5%), Untaxed.	£10 12 6	
On Deposit Receipts	0 19 5	
		11 11 11
		£106 14 8

DISCHARGE.

Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1933	£106 14 8

VI. GUNNING VICTORIA JUBILEE PRIZE FUND

Year to 30th September 1933.

CHARGE.

1. **Balance** due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1932		£172 1 2
2. **Interest Received**:—		
On £599, 14s. 3½% War Loan, 1952 (originally 5%), Untaxed	£25 9 9	
On Deposit Receipts	1 10 9	
		27 0 6
		£199 1 8

DISCHARGE.

1. Sir James Walker—1928-32 Award	£105 0 0
2. **Balance** due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1933	94 1 8
	£199 1 8

VII. JAMES SCOTT PRIZE FUND

Year to 30th September 1933.

CHARGE.

1. BALANCE due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1932		£30 13 4
2. INTEREST RECEIVED:—		
On £247, 10s 3½% War Loan, 1952 (originally 5%), Untaxed	£10 10 4	
On Deposit Receipts	0 3 6	
		10 13 10
		£41 7 2

DISCHARGE.

1. Prof. Arnold Sommerfeld—1930–33 Award	£40 0 0
2. BALANCE due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1933	1 7 2
	£41 7 2

VIII. PUBLICATION FUND

(COMPRISING PETER GUTHRIE TAIT MEMORIAL FUND AND DR JOHN AITKEN FUND)

Year to 30th September 1933.

CHARGE.

1. PETER GUTHRIE TAIT MEMORIAL FUND:—		
Year's Interest on £1550 3½% War Loan, 1952 (originally 5%), Untaxed		£65 17 6
2. DR JOHN AITKEN FUND:—		
Year's Interest on £445, 10s 3½% War Loan, 1952 (originally 5%), Untaxed	£18 18 8	
Interest on Deposit Receipts	0 15 7	
		19 14 3
		£85 11 9

DISCHARGE

Transferred to General Fund to meet Cost of Publications (see General Fund Discharge, No. 3*b*)	£85 11 9

IX. DR W. S. BRUCE MEMORIAL FUND

Year to 30th September 1933.

CHARGE.

1. BALANCE due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1932		£13 17 1
2. INTEREST RECEIVED:—		
On £233 3½% Conversion Loan, 1961	£8 3 0	
On Deposit Receipts	0 3 2	
		8 6 2
		£22 3 3

DISCHARGE.

BALANCE due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1933	£22 3 3

X. BRUCE-PRELLER LECTURE FUND

Year to 30th September 1933.

CHARGE.

1. **Balance** due by Union Bank of Scotland, Ltd , on Deposit Receipt at 30th September 1932		£43 3 0
2. **Dividend and Interest Received**:—		
On £140, 9s. Royal Bank of Scotland Stock, less Tax £5, 19s. 4d.	£17 18 2	
On Deposit Receipts	0 9 8	
		18 7 10
3. **Repayment** of Income Tax for year to December 1932		4 19 4
		£66 10 2

DISCHARGE

Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1933	£66 10 2

XI. DR DAVID ANDERSON-BERRY FUND

Year to 30th September 1933

CHARGE.

1. **Balance** of Revenue due by Union Bank of Scotland, Ltd , on Deposit Receipt, at 30th September 1932		£83 11 10
2. **Interest Received**:—		
On £1528, 0s. 4d Local Loans 3% Stock, *less* Tax £11, 9s. 0d.	£34 7 8	
On Deposit Receipts	0 7 8	
		34 15 4
3. Income Tax repaid for year to 5th April 1933		11 9 0
		£129 16 2

DISCHARGE.

1. **Royal Mint**—Amount of Account for Striking Die for Medal	£120 0 0
2. **Balance** due by Union Bank of Scotland, Ltd., on Deposit Receipt, at 30th September 1933	9 16 2
	£129 16 2

STATE OF THE FUNDS BELONGING TO THE ROYAL SOCIETY OF EDINBURGH

As at 30th September 1933.

1. GENERAL FUND—

1. £7830 3½% War Loan, 1952		£7830 0 0
2. £2100 2½% Consolidated Stock at 53%		1113 0 0
3. £445, 10s. 3½% War Loan, 1952. Mr Robert Mackay Smith, Legacy		445 10 0
4. Arrears of Contributions, as per preceding Abstract of Accounts		119 14 0
5. Balance of Special Subscription Fund—		
£751, 16s. 3½% War Loan, 1952	£751 16 0	
Cash due by Treasurer	10 15 0	
Due by War Loan Suspense Account	0 17 0	
	£763 8 0	
Less—Cash due to Union Bank of Scotland, Ltd., on Current Account	78 3 7	
		685 4 5
6. War Loan Suspense Account—		
Due by Union Bank of Scotland, Ltd., on Deposit Receipt		110 4 6
Amount		£10,303 12 11

Exclusive of Library, Museum, Pictures, etc., and Furniture in the Society's Rooms at George Street, Edinburgh.

2. KEITH FUND—

1. £650 3½% War Loan, 1952	£650 0 0
2. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt	81 7 8
AMOUNT	£731 7 8

3. NEILL FUND—

1. £300 3½% War Loan, 1952	£300 0 0
2. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt	36 8 9
AMOUNT	£336 8 9

4. MAKDOUGALL-BRISBANE FUND—

1. £400 3½% War Loan, 1952	£400 0 0
2. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt	52 9 6
AMOUNT	£452 9 6

5. MAKERSTOUN MAGNETIC METEOROLOGICAL OBSERVATION FUND—

1. £250 3½% War Loan, 1952	£250 0 0
2. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt	106 14 8
AMOUNT	£356 14 8

6. GUNNING VICTORIA JUBILEE PRIZE FUND—Instituted by Dr Gunning of Edinburgh and Rio de Janeiro—

1. £599, 14s. 3½% War Loan, 1952	£599 14 0
2. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt	94 1 8
AMOUNT	£693 15 8

7. JAMES SCOTT PRIZE FUND—

1 £247, 10s. 3½% War Loan, 1952	£247 10 0
2. Balance due by Union Bank of Scotland, Ltd , on Deposit Receipt	1 7 2
AMOUNT	£248 17 2

8. PUBLICATION FUND—

(COMPRISING PETER GUTHRIE TAIT MEMORIAL FUND AND DR JOHN AITKEN FUND)

1. PETER GUTHRIE TAIT MEMORIAL FUND:—		
£1550 3½% War Loan, 1952		£1550 0 0
2. DR JOHN AITKEN FUND:—		
£445, 10s. 3½% War Loan, 1952	£445 10 0	
Deposit Receipt with Union Bank of Scotland, Ltd.	71 6 1	
		516 16 1
AMOUNT		£2066 16 1

9. DR W. S. BRUCE MEMORIAL FUND—

1. £233 3½% Conversion Loan, at 72⅞% (cost price)	£169 15 11
2. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt	22 3 3
AMOUNT	£191 19 2

10. BRUCE-PRELLER LECTURE FUND—

1. £140, 9s. Royal Bank of Scotland, Stock taken over at 350%	£491 11 6
2. Balance due by Union Bank of Scotland, Ltd , on Deposit Receipt	66 10 2
AMOUNT	£558 1 8

11. DR DAVID ANDERSON-BERRY FUND—

1. £1528, 0s 4d. Local Loans 3% Stock at 65$\frac{7}{16}$% (cost price)	£1000 0 0
2. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt	9 16 2
AMOUNT	£1009 16 2

Note 1.—As previously, 3½% War Loan, 1952 (originally 5%), has been uniformly valued at par in the above State of Funds.

2.—Under the Will of the late Prof. Charles Piazzi Smyth and his wife, the Society will, on the expiry of certain liferents, become entitled to payment of the residue to be applied as set out in the will.

EDINBURGH, 16*th October* 1933.—We have examined the preceding Accounts of the Treasurer of the Royal Society of Edinburgh for the Session 1932–1933, and have found them to be correct. The securities for the various Investments, as noted in the foregoing Statement of Funds, have been verified by us at 30th September 1933.

LINDSAY, JAMIESON & HALDANE, C.A.,
Auditors.

VOLUNTARY CONTRIBUTORS who have made a Single Payment under Law VI (end of para. 3), up to 30th September 1933.

Professor Edwin Bramwell, ...	£10	10	0

VOLUNTARY CONTRIBUTORS under Law VI (end of para. 3), to 30th September 1933.

Col. A. F. Appleton, . .	£1	1	0
Sir James Barr, . .	1	1	0
Sir T. Hudson Beare, . .	1	1	0
F. A. Black, Esq , . .	1	1	0
Professor F. O. Bower, F.R.S., .	1	1	0
Judge F. E. Bradley (deceased),	1	1	0
Principal O. C Bradley, . .	1	1	0
J. W. Butters, Esq , . .	1	1	0
Professor E. W. W Carlier, .	1	1	0
Col. David Carnegie, . .	1	1	0
Professor E. G. Coker, F.R.S., .	1	1	0
Professor J. N. Collie, F.R.S., .	1	1	0
Dr E. Oswald Fergus, . .	1	1	0
Dr J. Haig Ferguson, .	1	1	0
Dr R. A. Fleming, . .	1	1	0
J S. Ford, Esq , . . .	1	1	0
Joseph S. Goodall, Esq., .	1	1	0
Dr W. D. Henderson, .	1	1	0
Dr E. M. Horsburgh, .	1	1	0
Dr W. F. Hume, . .	1	1	0
A. W. Kerr, Esq., ...	1	1	0
Professor J. Graham Kerr, F R.S.,	1	1	0
Archibald King, Esq. (1931–32, 1932–33), ...	2	2	0
F. H Lightbody, Esq , . .	1	1	0
	£26	5	0

Carried forward, .	£26	5	0
A. Veitch Lothian, Esq , .	1	1	0
James A. Macdonald, Esq., .	1	1	0
Dr George MacGowan (1932–33, 1933–34), . . .	2	2	0
Sir W. L. MacKenzie, . .	1	1	0
Dr Devendranath Mallik, .	1	1	0
Dr Hugh R. Mill, .	1	1	0
Dr Alexander Morgan (1931–32), ...	1	1	0
R. C. Mossman, Esq , .	1	1	0
Sir Thomas Muir, F R S., .	1	1	0
J. T. Pearce, Esq (1931–32, 1932–33), . .	2	2	0
A G Ramage, Esq , . .	1	1	0
Professor R A Robertson, .	1	1	0
Edward Smart, Esq , . .	1	1	0
Dr H F. Stockdale, . .	1	1	0
Gilbert Thomson, Esq (1931–32), . .	1	1	0
G Sutherland Thomson, Esq.,	1	1	0
R. T Thomson, Esq , .	1	1	0
Dr A Logan Turner, . .	3	3	0
Sir James Walker, F.R.S , .	1	1	0
J. C. Wright, Esq , . . .	1	1	0
Total, .	£51	9	0

Single payment, ...	£10	10	0
Other payments, ...	51	9	0
Total, . .	£61	19	0

THE COUNCIL OF THE SOCIETY.

23rd October 1933.

PRESIDENT.

EMERITUS PROFESSOR SIR E. A. SHARPEY-SCHAFER, M.D., D.Sc., LL.D., F.R.S.

VICE-PRESIDENTS

J. B. CLARK, M.A., LL.D., J.P.
PROFESSOR JAMES RITCHIE, M.A., D.Sc.
PRINCIPAL SIR THOMAS H. HOLLAND, K.C.S.I., K.C.I.E., D.L., Hon. D.Sc., LL.D., F.R.S.
THE HON. LORD SANDS, Kt., LL.D., D.D.
PROFESSOR C. G. DARWIN, M.A., F.R.S.
PROFESSOR R. A. SAMPSON, M.A., D.Sc., LL.D., F.R.S.

GENERAL SECRETARY

PROFESSOR J. H. ASHWORTH, D.Sc., F.R.S.

SECRETARIES TO ORDINARY MEETINGS.

PROFESSOR F. A. E. CREW, M.D., D.Sc., Ph.D.
PROFESSOR J. P. KENDALL, M.A., D.Sc., F.R.S.

TREASURER

JAMES WATT, W.S., LL.D.

CURATOR OF LIBRARY AND MUSEUM.

PROFESSOR D'ARCY W. THOMPSON, C.B., Hon. D.Sc., D.Litt., LL.D., F.R.S.

COUNCILLORS.

PROFESSOR P. T. HERRING, M.D., F.R.C.P.E.

PROFESSOR THOMAS M. MACROBERT, M.A., D.Sc.

PROFESSOR GODFREY H. THOMSON, D.Sc., Ph.D.

MALCOLM WILSON, D.Sc., A.R.C.S., F.L.S.

PROFESSOR E. B. BAILEY, M.C., M.A., F.R.S.

PROFESSOR J. C. BRASH, M.C., M.A., M.D.

PROFESSOR A. J. CLARK, M.C., B.A., M.D., F.R.S.

PROFESSOR A. G. OGILVIE, M.A., B.Sc.

PROFESSOR E. M. WEDDERBURN, M.A., D.Sc., LL.B., W.S.

LT.-COL. A. G. M'KENDRICK, M.B., D.Sc., F.R.C.P.E.

EMERITUS PROFESSOR JAMES MACKINNON, M.A., Ph.D., LL.D.

PROFESSOR WILLIAM PEDDIE, D.Sc.

OFFICE STAFF.

Assistant Secretary and Librarian, G. A. STEWART.
Assistant Librarian, R. J. B. MUNRO.
Housekeeper, SAMUEL HEDDLE.

PATRON.

HIS MOST EXCELLENT MAJESTY THE KING.

FELLOWS OF THE SOCIETY,

Corrected to 23rd October 1933.

N.B.—*Those marked* * *are Annual Contributors.*
" " † *have commuted Voluntary Contribution* (*see 3rd Paragraph, Law VI*).

M·B. prefixed to a name indicates that the Fellow has received a Makdougall-Brisbane Medal.
K. " " " " Keith Medal.
N. " " " " Neill Medal
V. J. " " " " the Gunning Victoria Jubilee Prize.
B. " " " " Bruce Medal
B-P " " " " Bruce-Preller Lectureship.
C. " " " has contributed one or more Communications to the Society's TRANSACTIONS or PROCEEDINGS.

Date of Election			Service on Council, etc.
1922		* Abernethy, Charles Lawrence, M A., B.Sc , Research Physicist, "Exnaböe," Craiglockhart Avenue, Slateford, Edinburgh 11	
1925	M-B. C.	* Aitken, Alexander Craig, M.A., D Sc , Lecturer in Actuarial Science, University of Edinburgh (16 Chambers Street)	
1889		† Alison, John, M A., LL.D , formerly Head Master, George Watson's College. 126 Craiglea Drive, Edinburgh 10	
1927	C.	* Allan, Douglas Alexander, D.Sc., Director, City of Liverpool Public Museums, William Brown Street, Liverpool	
1920	C. M-B.	* Allen, Herbert Stanley, M.A. (Cantab), D.Sc (London), F.R S., Professor of Natural Philosophy, University of St Andrews	1921–24.
1920	C.	* Anderson, Ernest Masson, M A., B Sc , F.G.S , 50 Greenbank Crescent, Edinburgh	
1905		Anderson, William, M.A , Head Science Master, George Watson's College, Edinburgh 6 Lockharton Crescent, Edinburgh 11	
1905		Andrew, George, M A , B A , H M.I S , Royal Technical College, George Street, Glasgow. Hamewith, Kilmacolm, Renfrewshire	
1881	C.	Anglin, Arthur H , M A., LL.D , M R I A , formerly Professor of Mathematics, Queen's College, Cork	
1930		* Annan, William, M A , C A , Professor of Accounting and Business Method, University of Edinburgh (South Bridge). Tofthill, Ferry Road West, Edinburgh 5	
1915		Anthony, Charles, M.Inst.C.E., M.Am Soc C E , F.C S , Spring Croft, Les Croutes, St Peter Port, Guernsey, Channel Islands	
1906		Appleton, Colonel Arthur Frederick, F.R.C.V S , Nutwell, 34 Shortlands Road, Shortlands, Kent	
1910	C.	Archibald, E. H., B Sc., Professor of Chemistry, University of British Columbia, Vancouver, Canada	
1933		* Arnot, Frederick Latham, B.Sc. (Sydney), Ph.D (Cantab), Lecturer in Natural Philosophy, University of St Andrews Sunnybrae, Kinkell Terrace, St Andrews	
1921		* Arthur, William, M.A , Lecturer in Mathematics, University of Glasgow. 148 Carmunnock Road, Cathcart, Glasgow	
1911	C. K.	* Ashworth, James Hartley, D Sc , F.R S. (GENERAL SECRETARY), Professor of Natural History, University of Edinburgh (West Mains Road). "Hillbank," Grange Loan, Edinburgh 9	1912–14. 1915–18, 1927–30. Sec. 1918–23. V-P 1923–26, 1930–33. Gen Sec. 1933–
1920		* Bagnall, Richard Siddoway, Hon. D.Sc., F.R.E.S., c/o Messrs Milward & Co , 80 Regent Street, London, W. 1	
1920	C. N.	* Bailey, Edward Battersby, M C., M.A., F.R.S., F.G.S , Professor of Geology, University of Glasgow	1932–

Date of Election			Service on Council, etc.
1896	C.	† Baily, Francis Gibson, M A , M Inst.E E., Emeritus Professor of Electrical Engineering, Heriot-Watt College, Edinburgh. Newbury, Colinton, Midlothian	1909–12, 1920–23. V-P 1929–32.
1931	C.	* Bain, William Alexander, Ph.D , Lecturer in Biophysics, Department of Physiology, University of Edinburgh 9 Falcon Gardens, Edinburgh 10	
1931		* Baird, William Macdonald, Fellow and Past President of the Faculty of Surveyors of Scotland, F.S.A.Scot., J.P Dalveen, Barnton Avenue, Davidson's Mains, Edinburgh 4	
1921		* Baker, Bevan Braithwaite, M A , D.Sc , Professor of Mathematics, Royal Holloway College, Englefield Green, London	
1928	C.	* Baker, Edwin Arthur, D.Sc. (Edin.), Assistant at the Royal Observatory, Edinburgh 17 Ladysmith Road, Edinburgh 9	
1905	C.	Balfour-Browne, William Alexander Francis, M A , F.Z.S , F.L S., F R.E.S., F.R.M.S , Barrister-at-Law, formerly Professor of Entomology, Imperial College of Science, London. Winscombe Court, Winscombe, Somerset, and 17 Langham Mansions, Earls Court Square, London, S W 5	
1933		* Banerjee, Prabodh Chandra, L R.C P E , L R C S E., F.R.F P.S.G., F.A.C.S , Major, I M S C/o Lloyds Bank, Ltd , 101–1 Clive Street, Calcutta, India	
1923		* Bannerman, David Armitage, M B.E., M.A (Cantab.), F.R G S., Special Assistant (Ornithology), Department of Zoology, British Museum (Natural History), London. 7 Pembroke Gardens, Kensington, London, W. 8	
1928		Barbour, George Brown, M A (Edin), M.A (Cantab), Ph D., F G.S , c/o Dr R. L. Dickinson, 438 West 116th Street, New York City, N Y., U.S.A.	
1886		Barclay, A. J. Gunion, M A , 3 Chandos Avenue, Oakleigh Park, London, N	
1903		Bardswell, Noël Dean, M.V O , M D , M.R.C.P. (Ed. and Lond), New County Hall, Westminster Bridge Road, London, S.E 1	
1922		* Barger, George, M A , D.Sc., Dr *h c*. (Padua), Hon. D.Sc. (Liverp.), Hon. M D (Heidelberg), LL.D (Michigan), F.R S., Hon Mem. Nederl. Chem. Vereen , Corr. Mem Bayerische Akad d. Wissensch. München and Ges. d Wissensch. Göttingen, Professor of Chemistry (Medical), University of Edinburgh (Teviot Place). 48 St Alban's Road, Edinburgh 9	1925–28.
1929		* Barker, Sydney George, Ph D , D I C., F Inst.P , Director of Research, Wool Industries Research Association, Torridon, Headingley, Leeds	
1914	C.	* Barkla, Charles Glover, M.A., D.Sc , F R.S., Professor of Natural Philosophy, University of Edinburgh (Drummond Street), Nobel Laureate, Physics, 1917. The Hermitage of Braid, Edinburgh	1915–18 1924–27.
1925		* Barlow, Thomas William Naylor, O B.E., M.R.C S., D.P H , Barrister-at-Law, 23 North Drive, New Brighton, Cheshire	
1927		* Barnett, John, F F.A , C.A., Scottish Widows' Fund Life Assurance Society, 9 St Andrew Square, Edinburgh 2	
1904		Barr, Sir James, C B.E., M.D., LL.D , F.R.C.P., Hindhead Brae, Hindhead, Surrey	
1921		* Bartholomew, John, M.C , M A., F.R.G.S., Geographical Institute, Duncan Street, Edinburgh. 19 George Square, Edinburgh 8	1925–28.
1932	C.	* Barton-Wright, Eustace Cecil, M Sc. (Lond), Chief Assist., Scottish Society for Research in Plant Breeding 30 Hillview Road, Corstorphine, Edinburgh 12	
1927		* Bastow, Stephen Everard, M Inst E E , M.Inst.Min.E., Managing Director, Bruce Peebles & Co , Ltd., Edinburgh. Northwood, Russell Place, Trinity, Edinburgh 5	
1929		* Bath, Frederick, Ph.D., Lecturer in Mathematics, University of St Andrews, Assistant to the Professor of Mathematics, University College, Dundee	
1913		† Beard, Joseph, F R.C.S. (Edin), M.R.C S., L R.C.P , D.P.H. (Cantab.), formerly Medical Officer of Health, City of Carlisle. 8 Carlton Gardens, Carlisle	
1888		Beare, Sir Thomas Hudson, Kt., B A., B Sc., M.Inst.C.E., J.P., D.L., Professor of Engineering, University of Edinburgh (Sanderson Engineering Laboratory, Mayfield Road). 10 Regent Terrace, Edinburgh 7	1907–09. V-P 1909–15, 1923–26.
1897	C	Beattie, Sir John Carruthers, K.B., D.Sc., LL.D , Vice-Chancellor and Principal, The University, Cape Town	
1893	C. M-B.	Becker, Ludwig, Ph D., Regius Professor of Astronomy, University of Glasgow. The Observatory, Dowanhill, Glasgow	
1933		* Begg, James Livingstone, F.G.S. (Treasurer, Geological Society of Glasgow) Elms, Mount Vernon, Glasgow	

Date of Election			Service on Council, etc
1916		* Bell, Robert John Tainsh, M A , D Sc , LL D (Glas), Professor of Mathematics, University of Otago, Dunedin, New Zealand	
1929		* Bennet, George, A H W C , B Sc , A M I Mech E , Lecturer in Mechanical Engineering, Heriot-Watt College 68 Arden Street, Edinburgh 10	
1893	C	Berry, Sir George A , M B , C M , LL D , F R C S E , King's Knoll, North Berwick	1916–19. V-P 1919–22.
1897	C	Berry, Richard J A , M D , F R C S E , Director of Medical Services, Stoke Park Colony, Stapleton, Bristol Rufford Canford Lane, Westbury on Trym, Bristol	
1932		* Bhatia, Sohan Lal, M C , M A , M D (Cantab), M R C P , Major, I M S Professor of Physiology and Dean, Grant Medical College Bombay Two Gables, Mount Pleasant Road, Malabar Hill Bombay India	
1880	C	Birch, de Burgh, C B , M D , Emeritus Professor of Physiology, University of Leeds	
1907		Black, Frederick Alexander, Solicitor, 57 Academy Street Inverness	
1931	C	* Black, Thomas Purves, M A (Edin), B Sc (Lond) Ph D (Edin), Headmaster Bellevue Technical and Commercial School Edinburgh 32 Eastfield Joppa 6	
1918		* Blight, Francis James, formerly Chairman and Managing Director, Charles Griffin & Co , Ltd Publishers Belstone Tor 37 Parkside, Mill Hill London, N W 7	
1933		* Bolam, Thomas Robert, M Sc (Bristol), D Sc (Edin) Lecturer in Chemistry University of Edinburgh (West Mains Road) 126 Blackford Avenue Edinburgh 9	
1894		Bolton, Herbert, D Sc , F G S , F Z S , formerly Director, Bristol Museum and Art Gallery, Bristol 318 Tilehurst Road Reading, Berks	
1915		* Boon, Alfred Archibald, D Sc , B A F I C , Emeritus Professor of Chemistry, Heriot Watt College, Edinburgh	
1925		* Borthwick, Albert William O B E , D Sc , Professor of Forestry, University of Aberdeen	
1925		* Bose, Sahay Ram, M A , D Sc F L S , Professor of Botany, Carmichael Medical College Belgachia, Calcutta, India	
1886	C N	Bower, Frederick Orpen, M A , D Sc , LL D F R S F L S , Emeritus Regius Professor of Botany, University of Glasgow 2 The Crescent, Ripon, Yorks	1887–90, 1893–96, 1907–09, 1917–19. V P 1910–16 P 1919–24
1924		* Bowman, Alexander, D Sc Scientific Superintendent, Marine Laboratory, Fishery Board for Scotland, Torry Aberdeen	
1903	C	Bradley, O Charnock, M D D Sc Principal Royal (Dick) Veterinary College, Edinburgh 9	1907-10, 1915–17
1926		* Braid, Kenneth William M A (Cantab) B Sc Professor of Botany, West of Scotland Agricultural College, 6 Blythswood Square, Glasgow	
1907		† Bramwell, Edwin, M D , F R C P E F R C P , Professor of Clinical Medicine, University of Edinburgh (Royal Infirmary) 23 Drumsheugh Gardens Edinburgh 3	
1932		* Brash, James Couper, M C , M A , M B , Ch B (Edin) M D (Birm), Professor of Anatomy, University of Edinburgh (Teviot Place)	1932–
1918		* Bremner, Alexander M A , D Sc , Headmaster, Demonstration School Training Centre, Aberdeen 13 Belgrave Terrace, Aberdeen	
1916	C	* Briggs, Henry, O B E , D Sc , Ph D , A R S M , James A Hood Professor of Mining, University of Edinburgh (79 Grassmarket, Edinburgh 1) 12 Gordon Terrace, Edinburgh 9	1923–26.
1895		Bright, Sir Charles, M Inst C E , M Inst F E , F R Ae S I Inst Radio E Little Brewers', Hatfield Heath, Harlow, Essex	
1893		Brock, G Sandison, M D , F R C P E , 53 Cheniston Gardens, Kensington, London, W 8	
1901	C	Brodie, W Brodie, M D , Camden House, Bletchingley, Surrey	
1907		Brown, Alexander, M A , B Sc Professor of Applied Mathematics, University, Cape Town	
1928		* Brown, Hugh Wylie, F I A , F F.A , 1 Cobden Crescent, Edinburgh 9	

Date of Election			Service on Council, etc.
1885	C.	Brown, J. Macdonald, M.D., F.R.C.S., Oriental Club, Hanover Square, London, W. 1	
1924		* Brown, Thomas Arnold, M.A., B.Sc., Professor of Mathematics, University College, Exeter	
1923		* Brown, Walter, M.A., B.Sc., Professor of Mathematics, University, Hong Kong, China	
1921		* Bruce, Alexander, B.Sc. (Edin), Government Agricultural Chemist and City Analyst, The Laboratory, Turret Road S., Colombo, Ceylon	
1912		* Bruce, Alexander Ninian, D.Sc., M.D., 8 Ainslie Place, Edinburgh 3	
1927		* Bryce, David Lawrence, The Ascension Vicarage, Thornhill Avenue, Plumstead, London, S.E. 18	
1898	C. K	† Bryce, Thomas Hastie, M A., M D. (Edin.), F.R.S., Professor of Anatomy, University of Glasgow. 2 The College, Glasgow	1911–14, 1922–25. V-P 1925–28.
1887		† Burnet, Sir John James, R.A., R.S.A , LL.D., Killermont, Rowledge, Farnham, Surrey	
1888		Burns, Rev. Thomas, C.B E., D.D , J P., F S A Scot , Minister of Lady Glenorchy's Parish Church. Croston Lodge, Chalmers Crescent, Edinburgh 9	
1917		* Burnside, George Barnhill, M.Inst.Mech.E., Fairhill, Dullatur	
1930	C.	* Burt, David Raitt Robertson, B Sc (St Andrews), F.L.S , Lecturer in Zoology, Ceylon University College, Colombo	
1896		Butters, John W., M.A , B Sc , formerly Rector of Ardrossan Academy. 116 Comiston Drive, Edinburgh 10	
1887	C.	Cadell, Henry Moubray, of Grange, D L , B.Sc , M Inst M E , LL D. (Edin), Linlithgow	1919–22.
1929	C.	* Calder, Alexander, Ph D., Assistant Marketing Officer, Pig Marketing Board. C/o Watson, 4 Salisbury Place, Edinburgh 9	
1910		* Calderwood, Rev Robert Sibbald, D.D , Minister of Cambuslang. The Old Manse, Cambuslang, Lanarkshire	
1893	C.	Calderwood, W. L., I.S.O., formerly Inspector of Salmon Fisheries of Scotland. New Club, Princes Street, Edinburgh	1923–26.
1926	C.	* Cameron, Alfred E. Henderson, M A , D Sc (Aberd.), Lecturer in Entomology, Department of Zoology, University (West Mains Road), Edinburgh. 8 West Savile Road, Edinburgh 9	
1933		* Cameron, Finlay James, F F A , F.I A , General Manager, Caledonian Insurance Company. Beech Knowe, Cramond Bridge, near Edinburgh	
1905	C.	Cameron, John, M.D , D.Sc., M.R C S., formerly Professor of Anatomy, Dalhousie University, Halifax, Nova Scotia. Balmashanner, Grove Road, East Cliff, Bournemouth	
1921		* Campbell, Andrew, Advisory Chemist, c/o Burmah Oil Co., Ltd., Research Laboratory, Fairlawn, Honor Oak Road, Forrest Hill, London, S E 15 The Avenue, Beckenham, Kent	
1918		* Campbell, John Mensies, D.D S. (Toronto), L D S. (Glas), L.D.S. (Ontario), F I.C D , 14 Buckingham Terrace, Glasgow, W	
1915	C N.	* Campbell, Robert, M.A , D.Sc., F.G.S , Reader in Petrology, University of Edinburgh (Grant Institute of Geology, West Mains Road) Maryton, Colinton	1920–23.
1927	C.	* Cannon, Herbert Graham, M.A., Sc.D. (Cantab.) D.Sc. (Lond.), F.L.S., Beyer Professor of Zoology, University of Manchester	
1899	C	Carlier, Edmund W W , B. ès Sc., M Sc , M D , F R E.S , Emeritus Professor of Physiology, University of Birmingham. Morningside, Dorridge, near Birmingham	
1910		Carnegie, Col. David, C.B.E., M.Inst.C.E., J.P., The Haven, Seasalter, Whitstable	
1931		* Carroll, John Anthony, M.A., Ph.D. (Cantab.), Professor of Natural Philosophy, University of Aberdeen. Marischal College, Aberdeen	
1920	C.	* Carruthers, R. G., F.G.S., District Geologist, H.M. Geological Survey, High Barn, Stocksfield-on-Tyne	
1905	C.	Carse, George Alexander, M.A., D.Sc., Reader in Natural Philosophy, University of Edinburgh (Drummond Street). 3 Middleby Street, Edinburgh 9	
1901		Carslaw, Horatio Scott, M A., Sc.D. (Cantab.), D.Sc., LL.D. (Glas.), Professor of Mathematics, University of Sydney, New South Wales	
1933		* Carswell, John Irvine, B.Sc., A.M.Inst C.E , A M.Inst Mech.E., Lecturer in Engineering, University of Edinburgh (Sanderson Engineering Laboratory, Mayfield Road). 43 Mansionhouse Road, Edinburgh 9	

Date of Election			Service on Council, etc
1925		* Carter, George Stuart, M A , Ph D , Corpus Christi College, Cambridge	
1898		Carus Wilson, Cecil, J P , F G S , F R G S , Altmore, Waldegrave Park, Strawberry Hill, Twickenham, and Sandacres Lodge, Parkstone, Dorset	
1932		* Cathcart, Edward Provan, C B E , M D , D Sc , LL D , F R S , Professor of Physiology, University of Glasgow 28 Hillhead Street, Glasgow	
1899		Chatham, James, Actuary, Ladieside, Melrose	
1932	C	* Childe, Vere Gordon, B A , B Litt , F R A I , F S A , Professor of Prehistoric Archæology, University of Edinburgh (14 Chambers Street)	
1925	C	* Chumley, James, M A , Ph D , Lecturer in Oceanography, Department of Zoology, University of Glasgow Thalassa, Thorn Drive, Bearsden, Dumbartonshire	
1928	C	* Clark, Alfred Joseph, M C , B A , M D , F R S , Professor of Materia Medica University of Edinburgh (Teviot Place) 67 Braid Avenue Edinburgh 10	1932–
1933		* Clark, Arthur Melville, M A (Edin) D Phil (Oxon), Lecturer in English Literature, University of Edinburgh 7 Harrison Road, Edinburgh 11	
1891		Clark, John Brown, M A , LL D , J P (VICE PRESIDENT) formerly Head Master of George Heriot's School Garleffin 146 Craiglea Drive, Edinburgh 10	1928–1931 V-P 1931–
1932		* Clark, Sir Thomas, Bart , Publisher Head of T & T Clark, Ltd 6 Wester Coates Road Edinburgh 12	
1903		Clarke, William Eagle, I S O , LL D , F L S Honorary Supervisor of the Bird Collection and formerly Keeper of the Natural History Collections, Royal Scottish Museum, Edinburgh	
1909		Clayton, Thomas Morrison, M D , D Hy B Sc , D P H Medical Officer of Health, Greenesfield House, Gateshead on Tyne	
1932		* Clouston David, M A , B Sc (Agric), D Sc C I E formerly Director, Imperial Agricultural Research Institute Pusa Forthview, Boswall Road Edinburgh 5	
1904	C	Coker, Ernest George, M A (Cantab), D Sc (Edin) Hon D Sc (Sydney and Louvain), M Sc (McGill), F R S , M Inst C E , M I Mech E Kennedy Professor of Civil and Mechanical Engineering, and Director of the Engineering Laboratories, University College Gower Street London W C 1	
1904		Coles, Alfred Charles, M D , D Sc , York House Poole Road, Bournemouth, W	
1888	V J C	Collie, John Norman, Ph D , D Sc , LL D F R S F C S , F I C , Emeritus Professor of Organic Chemistry, University College Gower Street, London 20 Gower Street London W C 1	
1909	C	Comrie Peter, M A , B Sc , LL D , formerly Rector Leith Academy 19 Craighouse Terrace Edinburgh 10	
1924	C	* Copson Edward Thomas M A D Sc Lecturer in Applied Mathematics, University of St Andrews Mayfield, St Andrews	
1929		* Coull, George D Sc Pharmaceutical Chemist, Smith's Place House Leith 6	
1928		* Coutie, Rev Alexander, Ph D , 13 Mayfield Gardens Edinburgh 9	
1914		* Coutts, William Barron, M A , B Sc , Senior Lecturer in Range Finding and Optics, Military College of Science Red Barracks Woolwich S E 18 11 Coleraine Road, Blackheath S E 3	
1911		* Cowan Alexander, Papermaker Valleyfield Penicuik, Midlothian	
1931		* Cowan, John Macqueen M A D Sc (Edin) B A (Oxon), F L S Assistant Keeper, Royal Botanic Garden, Edinburgh 17 Inverleith Place, Edinburgh 4	
1916	C	† Craig, E H Cunningham, B A (Cantab), Geologist and Mining Engineer, The Dutch House Beaconsfield	
1908		Craig, James Ireland, M A , B A , Woolwich House The Drive, Sydenham London, S E 26 (At present—Turf Club, Cairo)	
1925	C K	* Craig, Robert Meldrum, M A D Sc F G S , Lecturer in Economic Geology, University of Edinburgh (Grant Institute of Geology, West Mains Road)	
1933		* Craig Bennett, Arthur Lancelot, M A Ph D (Cantab), Lecturer in Zoology, University of Edinburgh (West Mains Road) 27 Grange Loan, Edinburgh 9	
1903		Crawford, Lawrence, M A , D Sc , Professor of Pure Mathematics, University, Cape Town	
1922	C	* Crew, Francis Albert Eley, M D , D Sc , Ph D (SECRETARY TO ORDINARY MEETINGS), Professor of Animal Genetics and Director of the Institute of Animal Genetics, University of Edinburgh (West Mains Road) 10 Salisbury Road, Edinburgh 9	1928–31 Sec 1931–

Date of Election			Service on Council, etc.
1931		* Crichton, John, M.A., B.Sc. (Edin.), Assistant Superintendent, Meteorological Office, Edinburgh. 10 Laverockbank Road, Edinburgh 5	
1870		Crichton-Browne, Sir Jas., Kt , M.D., LL.D , D.Sc., F.R.S , Vice-President and Treasurer of the Royal Institution of Great Britain. 45 Hans Place, London, S W. 1	
1929		* Cruickshank, Ernest William Henderson, M.D., D.Sc , Ph.D., Professor of Physiology, Dalhousie University, Halifax, Nova Scotia	
1914		* Cumming, Alexander Charles, O.B E., D.Sc., Roselands, Crescent Road, Blundell Sands, Liverpool	
1928		* Cumming, William Murdoch, D.Sc. (Glas.), F.I.C , M.Inst.Chem.E., "Young" Professor of Technical Chemistry, Royal Technical College, Glasgow "Bonnieblink," 4 Newlands Road, Newlands, Glasgow, C. 1	
1917		* Cunningham, Brysson, D.Sc., B.E., M.Inst C E., Lecturer in Waterways, Harbours, and Docks, University College, London. 141 Copers Cope Road, Beckenham, Kent	
1930		* Cunningham, John, C I.E , B A., M.D , Lt.-Colonel, I M S. (retired). South Bank, Grange Loan, Edinburgh 10	
1885		Daniell, Alfred, M A , LL B , D Sc , Advocate, The Athenæum Club, London	
1924		* Darwin, Charles Galton, M A , F R.S (VICE-PRESIDENT). Tait Professor of Natural Philosophy, University of Edinburgh (Drummond Street). 4 Churchhill, Edinburgh 10	1925–28. Sec 1928–33 V-P 1933–
1932		* Davidson, Leybourne Stanley Patrick, B.A (Cantab), M D (Edin), F.R C.P E., Regius Professor of Medicine, University of Aberdeen 55 Queen's Road, Aberdeen	
1930	C.	* Davies, Lewis Merson, F G S., F.R A.I., Lt.-Colonel, Royal Artillery (retired) 8 Garscube Terrace, Murrayfield, Edinburgh 12	
1931		* Dawson, Shepherd, M A , D Sc , Principal Lecturer in Psychology, Training College, Glasgow. Hazel Bank, Bearsden, Dumbartonshire	
1928		Dawson, Warren Royal, F R S L , Honorary Librarian of Lloyd's. 28 Grange Road, Barnes, London, S W. 13	
1917		* Day, T. Cuthbert, F C S., 36 Hillside Crescent, Edinburgh 7	
1923		* Deane, Arthur, M.R I A , Curator, Public Art Gallery and Museum, Belfast. Threave, Saintfield Road, Newtownbreda, Belfast	
1894		† Denny, Sir Archibald, Bart., LL D., 5 St Helen's Place, London, E.C. 4	
1925		Dey, Alexander John, Managing Director of T. & H Smith, Ltd , Manufacturing Chemists, Edinburgh Rothiemay, Corstorphine, Edinburgh 12	
1924		* Dinham, C H , B A , H M Geological Survey Edgemoor, 19 Highfield Road, Northwood, Middlesex	
1885	C.	Dixon, James Main, M.A , Litt. Hum Doctor, Professor of English, University of Southern California. University Avenue, Los Angeles, California, U.S.A.	
1923		* Dixon, Ronald Audley Martineau, of Thearne, F.G S , F.S A Scot , F.R.G.S., Thearne Hall, near Beverley	
1897		Dobbie, James Bell, F.Z S , 12 South Inverleith Avenue, Edinburgh 4	
1881	C.	Dobbin, Leonard, Ph D , formerly Reader in Chemistry, University of Edinburgh Faladam, Blackshiels, Midlothian	1904–07, 1913–16.
1918		* Dodd, Alexander Scott, Ph D., F.I.C , F C S., City Analyst for Edinburgh. 20 Stafford Street, Edinburgh 3	
1925		* Donald, Alexander Graham, M A., F F.A , F.S.A Scot., Secretary of the Scottish Provident Institution, Edinburgh. 18 Carlton Terrace, Edinburgh 7	
1905		Donaldson, Rev Wm. Galloway, J P., F.R.G.S., F E.I.S , The Manse of Forfar, Forfar	
1882	C.	Dott, David B , F I.C , Memb Pharm. Soc , Ravenslea, Musselburgh	
1921	M-B C.	* Dougall, John, M.A., D Sc., 47 Airthrey Avenue, Glasgow, W. 4	
1901 & 1918		Douglas, Carstairs Cumming, M.D , D.Sc., Professor of Medical Jurisprudence and Hygiene, Anderson's College, Glasgow. 110 South Brae Drive, Jordanhill, Glasgow	
1910		* Douglas, Loudon MacQueen, Newpark, Mid-Calder, Midlothian	
1932		* Drennan, Alexander Murray, M.D. (Edin.), F R.C.P E., Professor of Pathology, University of Edinburgh (Teviot Place)	

Date of Election			Service on Council etc
1923	C	* Drever James M A B Sc D Phil Professor of Psychology University of Edinburgh (South Bridge) Ivybank Wardie Road Edinburgh 5	1929–32
1901		Drinkwater Thomas W L R C P E L R C S E Chemical Laboratory Surgeons Hall Edinburgh	
1923		* Drummond J Montagu F M A (Cantab) Harrison Professor of Botany, University of Manchester	1928–31
1925		* Dryerre Henry M R C S Ph D Professor of Physiology Royal (Dick) Veterinary College Physiological Biochemist Animal Diseases Research Association Kenmore Lasswade	
1921		* Drysdale Charles Vickery C B O B E D Sc (Lond) M I E E F Inst P Director of Scientific Research and Experiment to the Admiralty S R E Department Archway Block N Admiralty Whitehall London S W 1	
1904		Dunlop William Brown M A 44 St Andrew Square Edinburgh 2	
1892	C	Dunstan M J R M A F I C F C S formerly Principal Royal Agricultural College Cirencester Windyacre Wrotham Kent	
1933		* Dymond Edmund Gilbert M A Lecturer in Natural Philosophy University of Edinburgh (Drummond Street) 8 Cobden Crescent Edinburgh 9	
1906	C	Dyson Sir Frank Watson K B E M A D Sc LL D F R S formerly Astronomer Royal Royal Observatory Greenwich 27 Westcombe Park Road Blackheath S E 3	
1925		* Eastwood George Samuel B Sc Principal Teacher of Mathematics Beath Secondary School Cowdenbeath Fife Craigie Lea Cowdenbeath Fife	
1904		Edwards John LL D 4 Great Western Terrace Kelvinside Glasgow	
1931		* Eggleton Philip D Sc Lecturer in Biochemistry Department of Physiology University of Edinburgh (Teviot Place) 36 Gillespie Crescent Edinburgh 10	
1924		* Elliot Rt Hon Walter Elliot P C M C M B Ch B D Sc LL D M P Minister of Agriculture 14 Markham Square Chelsea London	
1906	C	Ellis David D Sc Professor of Botany and Bacteriology Royal Technical College Glasgow	
1933		* Erskine John Maxwell General Manager of the Commercial Bank of Scotland Ltd 10 Spylaw Road Edinburgh 10	
1924		* Evans Arthur Humble M A Sc D Lecturer in English History Cheviot House Crowthorne Berks	
1924		Evans William Edgar B Sc Assistant in charge of Herbarium Royal Botanic Garden Edinburgh 4	
1879	C N	Ewart James Cossar M D LL D F R C S E F R S F Z S, Emeritus Regius Professor of Natural History University of Edinburgh Craigy Field Penicuik Midlothian	1882–85 1904–07 V P 1907–12
1902		Ewen John Taylor O B E B Sc M I Mech E J P H M Inspector of Schools (Emeritus) Pitscandly Forfar	
1878	C	† Ewing Sir James Alfred K C B M A D Sc LL D F R S Hon Memb Inst C E formerly Principal University of Edinburgh and Director of Naval Education Admiralty 5 Herschel Road Cambridge	1888–91, 1919–20 V P 1920–23 P 1924–29
1900	C	Eyre John W H M D M S (Dunelm) D P H (Cantab) Professor of Bacteriology Guy s Hospital London S E 1	
1931		* Fairbairn William Ronald Dodds M A M D D Psych (Edin) Lecturer in Psychology University of Edinburgh (South Bridge) 18 Lansdown Crescent Edinburgh 12	
1910	C	* Fairgrieve Mungo M Callum M A (Cantab and Glas) Senior Science Master Edinburgh Academy 37 Queen s Crescent Edinburgh 9	
1907	C	Falconer John Downie M A D Sc F G S formerly Director of the Geological Survey of Nigeria The Cedars Hatton Road Harlington Hayes Middlesex	
1923		* Feldman William Moses M D B S M R C P M R C S Physician St Mary s Hospital for Women and Children Plaistow 851 Finchley Road London N W 11	
1928		* Fenton Edward Wyllie M A B Sc (Aberd) Head of Botany Department Edinburgh and East of Scotland College of Agriculture 13 George Square, Edinburgh 8	

Date of Election			Service on Council, etc.
1907		Fergus, Edward Oswald, c/o Messrs M'Kay & Boyd, Solicitors, 50 Wellington Street, Glasgow	
1904		Ferguson, James Haig, M.D., LL.D, F.R.C.P.E., F.R.C.S.E, 7 Coates Crescent, Edinburgh 3	
1933		* Ferguson, Thomas, M.D., D.P.H. (Edin.), F.R C.P.E., D.Sc., H M. Medical Inspector of Factories, Home Office. 4 Park Street, Falkirk	
1925	C.	* Ferrar, William Leonard, M.A., Fellow and Tutor of Hertford College, Oxford	
1932		* Findlay, Sir John Edmund Ritchie, Bart., B.A. (Oxon.), Proprietor of the *Scotsman* 18 Lauder Road, Edinburgh 9	
1927	C.	* Finlay, Thomas Matthew, M A., D Sc. (Edin.), Lecturer in Palæontology, University of Edinburgh 11 Dudley Terrace, Leith 6	
1911		† Fleming, John Arnold, F.C.S., Pottery Manufacturer, Locksley, Helensburgh, Dumbartonshire	
1906		Fleming, Robert Alexander, M A., M D, LL D, F.R.C.P E, Consulting Physician, Royal Infirmary. 10 Chester Street, Edinburgh 3	
1900	C. N.	Flett, Sir John S, K.B.E., M A, D.Sc, LL.D., F.R S, Director of the Geological Survey of Great Britain and of the Museum of Practical Geology, 28 Jermyn Street, London, S W. 1	1916–19.
1872		Forbes, George, M.A., LL D., F R.S, M.Inst C.E., M.Inst.E.E., F.R A.S., formerly Professor of Natural Philosophy, Anderson's College, Glasgow. 11 Little College Street, Westminster, S W	
1892		Ford, John Simpson, F I C, 7 Corrennie Drive, Edinburgh 10	
1921		* Forrest, George Topham, F.R I.B A, F.G S, Architect to the London County Council, County Hall, Westminster Bridge, London, S E. 1	
1928	C.	* Forrest, James, M A, B Sc. (Glas.), D.Sc. (St Andrews), Lecturer in Physics, University College, Dundee. "Cumbrae," Oxford Street, Blackness, Dundee	
1933		* Forrester, Charles, A H.-W.C, F.I C, F.Inst P, Vice-Principal and Professor of Chemistry, Indian School of Mines, Dhanbad, India	
1933		Fowler, William Hope, C.V O, M.B., Ch.B. (Edin), M.R.C P., F R.C.S.E. (Deceased, October 4, 1933)	
1920	C.	* Franklin, Thomas Bedford, B.A (Cantab.), 28 Kingshill Drive, Kenton, Middlesex	
1910		* Fraser, Alexander, Actuary, 15 S. Learmonth Gardens, Edinburgh 4	
1929		* Fraser, David Kennedy, M.A, B Sc, Psychologist to Glasgow Education Authority Edge o' the Moor, Milngavie, Dumbartonshire	
1928		* Fraser, John, M C., M.D., Ch.M., F.R.C.S E., Regius Professor of Clinical Surgery, University of Edinburgh (Royal Infirmary). 20 Moray Place, Edinburgh 3	
1928		* Fraser, Kenneth, M.D (Edin), D.P.H. (Cantab.), D T M. (Edin.), County Medical Officer of Health, Cumberland. The Croft, Scotby, near Carlisle	
1914		* Fraser, William, Managing Director, Neill & Co, Ltd., Printers, 212 Causewayside, Edinburgh 9	
1896	C	Fraser-Harris, David Fraser, B.Sc. (Lond.), D.Sc. (Birm.), M.D., formerly Professor of Physiology, Dalhousie University, Halifax, Nova Scotia. Grove Park Lodge (3), Chiswick, London, W. 4	
1907		Galbraith, Alexander, "Ravenswood," Dalmuir, Glasgow	
1933		* Galbraith, Augustus William de Rohan, M Inst.C E, M Inst C E.I., F S.E, City Engineer, Christchurch, New Zealand. The Spur, Sumner, New Zealand	
1888	C.	Galt, Alexander, D.Sc, formerly Keeper of the Department of Technology, Royal Scottish Museum, Edinburgh. C/o Clydesdale Bank, 1 Melville Place, Edinburgh 3	
1901		Ganguli, Sanjiban, M A, Principal, Maharaja's College, and Director of Public Instruction, Jaipur State, Jaipur, India	
1933		* Gardner, Alfred Charles, M Inst.C.E., M.Inst.Mech.E., M.Inst.E E., F.G.S, Chief Engineer, Clyde Navigation Trust, 16 Robertson Street, Glasgow. 324 Albert Drive, Pollokshields, Glasgow	
1926		* Gardner, John Davidson, B.Sc., A.M.Inst.C.E., Chief Assistant to Messrs D. & C. Stevenson, Civil Engineers, Edinburgh. 23 Ivy Terrace, Edinburgh 11	
1930	C.	* Geddes, Alexander Ebenezer M'Lean, O.B.E., M.A., D.Sc., Lecturer in Natural Philosophy, University of Aberdeen. 12 Louisville Avenue, Aberdeen	

Date of Election			Service on Council, etc.
1909	C.	Geddes, Rt. Hon. Sir Auckland C, P.C., G.C.M.G., K.C.B., M.D., LL.D., formerly British Ambassador to the U.S.A. Frensham, The Layne, Rolvenden, Kent	
1909		Gentle, William, B.Sc., Head Master, George Heriot's School. 10 West Savile Road, Edinburgh 9	
1920	C.	* Ghosh, Sudhamoy, M.Sc. (Cal.), D.Sc. (Edin.), F C S., Professor of Chemistry, School of Tropical Medicine and Hygiene, Central Avenue, Calcutta, India	
1914		* Gibb, Sir Alexander, G.B.E., C B., M.Inst.C.E, Queen Anne's Lodge, Westminster, London, S.W. 1	
1916		* Gibb, Alfred William, M.A, D Sc., Professor of Geology, University of Aberdeen. 1 Belvidere Street, Aberdeen	
1910	C.	* Gibb, David, M.A., B.Sc., Lecturer in Mathematics, University of Edinburgh (16 Chambers Street). 15 South Lauder Road, Edinburgh 9	
1917	C.	* Gibson, Alexander, M B., Ch.B., F.R.C.S., 620 Medical Arts Building, Winnipeg, Canada	
1921		* Gibson, Walcot, D.Sc, F R S, F.G.S, formerly Assistant Director, H.M. Geological Survey (Scotland). Pathways, Fairlight Road, Hythe, Kent	
1911		Gidney, Sir Henry A. J., Kt, F.R C S E, D.P.H, J.P, M.L.A., Lt.-Col., I.M.S. (retired), c/o The Allahabad Bank, Ltd, Calcutta, India	
1933		* Gillespie, Robert Pollock, M.A, B Sc., Ph D (Cantab), Lecturer in Mathematics, University of Glasgow. Ashcot, Kilbarchan Road, Johnstone, Renfrewshire	
1933		* Gillespie, Thomas Haining, Director-Secretary, Zoological Society of Scotland. Corstorphine Hill House, Murrayfield, Edinburgh 12	
1925		* Gillies, William King, M.A., B.A., F.E I.S, LL.D (Glas), Rector of the Royal High School, Edinburgh. Davaar, 12 Suffolk Road, Edinburgh 9	
1907		Gilruth, John Anderson, M R C.V S, D V.Sc (Melb), Clowes Street, South Yarra, Melbourne, Australia	
1909		Gladstone, Hugh Steuart, M.A, M.B O.U, F.Z S, Capenoch, Penpoint, Dumfries	
1911		Gladstone, Reginald John, M.D, F R.C S, Lecturer and Senior Demonstrator of Anatomy, King's College, University of London. 22 Court Lane Gardens, London, S.E 21	
1925	C.	* Goldie, Archibald Hayman Robertson, M.A, B A, Superintendent, Meteorological Office, Air Ministry, Edinburgh, 6 Drumsheugh Gardens, Edinburgh 3	1929–32.
1910		Goodall, Joseph Strickland, M B. (Lond), M R C P, F.R.C.S.E, M S.A. (Eng.), Professor of Physiology and Biology, City of London Hospital 136 Harley Street, London, W. 1	
1901		Goodwillie, James, M.A, B Sc., 239 Clifton Road, Aberdeen	
1913	C. M-B.	* Gordon, William Thomas, M A, D Sc. (Edin), M.A. (Cantab.), Professor of Geology, University of London, King's College, Strand, W.C	
1897		Gordon-Munn, John Gordon, M.D., Croys, Dalbeattie	
1923		* Grabham, George Walter, O.B E., M.A. (Cantab.), F.G S, Government Geologist, Anglo-Egyptian Sudan Box 178, Khartoum	
1924		Graham, Robert James Douglas, M.A., D.Sc, Lecturer in Plant Physiology, University of Edinburgh Royal Botanic Garden, Edinburgh 4	
1931		* Grant, Robert, Publisher (Oliver & Boyd), Edinburgh. 6 Kilgraston Road, Edinburgh 10	
1898	C.	Gray, Albert A, M.D., 5 Hammersmith Terrace, London, W. 6	
1909	C.	Gray, James Gordon, D.Sc, Professor of Applied Physics, University of Glasgow. 11 The University, Glasgow	1913–15.
1918		* Gray, Wm Forbes, F.S.A.Scot., 8 Mansionhouse Road, Edinburgh 9	
1927	C. K.	* Greenwood, Alan William, D Sc. (Melb.), Ph D. (Edin.), Lecturer in the Institute of Animal Genetics, University of Edinburgh (West Mains Road)	
1922		* Greenwood, William Osborne, M.D. (Leeds), B S. (Lond.), L.S.A., Woodroyd, 19 Ripon Road, Harrogate, Yorks	
1925		* Greig, David Middleton, M.B, C.M., F.R.C.S.E., LL.D., Conservator of the Museum of the Royal College of Surgeons of Edinburgh. 12 Abbotsford Crescent, Edinburgh 10	
1906		Greig, Edward David Wilson, C.I.E., M.D., D.Sc., Lt.-Col., I.M.S. (retired), 38 Coates Gardens, Edinburgh 12	
1931		* Greig, John Russell, Ph.D. (Edin.), Director, Moredun Institute, Animal Diseases Research Association. Wedderlie, Kirkbrae, Liberton 9	

Date of Election			Service on Council, etc.
1905		† Greig, Sir Robert Blyth, M.C., LL D, Secretary to the Department of Agriculture for Scotland, York Buildings, Queen Street, Edinburgh 2	1921–24. V-P 1924–27.
1910		* Grimshaw, Percy Hall, I.S.O., F R E S., Keeper, Natural History Department, Royal Scottish Museum 49 Lygon Road, Edinburgh 9	
1899		† Guest, Edward Graham, M.A, B.Sc, J P., 5 Newbattle Terrace, Edinburgh 10	
1927		* Gulland, John Masson, M.A. (Oxon.), D.Sc. (Edin.), Ph.D. (St Andrews), Reader in Biochemistry, University of London, Lister Institute, Chelsea Bridge Road, London, S.W. 1	
1907		Gulliver, Gilbert Henry, D.Sc., A.M I.Mech E., 99 Southwark Street, London, S E.	
1930		* Guthrie, Douglas, M.D, F.R C S., Lecturer in Diseases of the Ear, Nose, and Throat, School of Medicine of the Royal Colleges, Edinburgh. 4 Rothesay Place, Edinburgh 3	
1933	C	* Guthrie, William Gilmour, M.A. (Edin), B A (Cantab), Ph.D, Professor of Mathematics and Natural Philosophy, Magee College, Londonderry, Northern Ireland	
1911		* Guy, William, F R C S., L R C P., L D S Ed, LL D (Penn), Consulting Dental Surgeon, Edinburgh Royal Infirmary, Lecturer on Human and Comparative Dental Anatomy and Physiology. 11 Wemyss Place, Edinburgh 3	
1922		* Hannay, Robert Kerr, M A, LL D, H R.S.A, Fraser Professor of Scottish History and Palæography, University of Edinburgh (South Bridge). Historiographer-Royal for Scotland. 5 Royal Terrace, Edinburgh 7	
1923		* Hanneford-Smith, William, A M Inst C E, Hon A R I B A 1 The Avenue, Gravesend, Kent	
1918		* Hardie, Patrick Sinclair, M.A, B Sc., formerly Head of the Physics Department, Medical School, Cairo 10 Baldovan Road, Downfield, Dundee	
1928		* Harding, William Gerald, F R Hist S, F S A Scot, F.R E S, Peckwater House, Charing, Kent	
1923	C	* Harris, Robert Graham, M A, D.Sc (Edin), Lorraine, Manor Road, Farnborough, Hants	
1914		Harrison, Edward Philip, Ph D, F Inst P, Chief Scientist, H.M S "Vernon," Portsmouth	
1921		* Harrison, John William Heslop, D.Sc. (Durham), F.R S., Professor of Botany, Armstrong College, Newcastle. The Avenue, Birtley, Co. Durham	
1926	C.	* Harrower, John Gordon, M.B., Ch.M. (Glas), F R.C S.E., D Sc, (Edin), Professor of Anatomy, King Edward VII Medical College, and Surgeon, General Hospital, Singapore	
1926		* Harvey, William Frederick, C.I.E., M A, M.B, C M, D P.H, Lieut.-Col, I M S (retired), Histologist, Research Laboratory, Royal College of Physicians, Edinburgh. 56 Garscube Terrace, Edinburgh 12	
1893		Hehir, Sir Patrick, K C I E, C.B., C M G, F R C.P E, F.R.C S.E., D.P.H., (Cantab), D T.M (Liverpool), Maj.-General, I M S (retired). Ennisfarne, Westward Ho, Devon	
1931		* Henderson, John, F C I I, Manager and Secretary, Edinburgh Assurance Co, Ltd Seaforth Cottage, York Road, Trinity, Edinburgh 5	
1929		* Henderson, Thomas, C B E, J P., F.S.A.Scot., Actuary of the Savings Bank of Glasgow 5 Belmont Crescent, Glasgow, W. 2	
1908		Henderson, William Dawson, M.A, B Sc, Ph.D., Lecturer, Zoological Laboratories, University, Bristol	
1925		* Heron, Alexander Macmillan, D.Sc., Superintendent, Geological Survey of India, Calcutta, India	
1916		* Herring, Percy Theodore, M.D, F.R.C.P.E., Professor of Physiology, University of St Andrews. Linton, St Andrews	1917–20 1931–
1922		Hindle, Edward, M.A., Sc D. (Cantab.), Ph.D., A.R C.S, London School of Hygiene and Tropical Medicine 32 Belsize Avenue, Hampstead, London, N.W 3	
1902		Hinxman, Lionel W., B.A., formerly of the Geological Survey of Scotland. 4 Morant Gardens, Ringwood, Hants	
1904		Hobday, Sir Frederick T. G, Kt., C.M.G., Dr. Med. Vet. (Zurich), F.R.C.V.S., Principal, Royal Veterinary College, Camden Town, London, N.W. 1	
1928	C.	* Hobson, Alfred Dennis, M A (Cantab.), Professor of Zoology, Armstrong College, Newcastle-upon-Tyne	

Date of Election			Service on Council, etc.
1928		* Hodge, William Vallance Douglas, M.A (Edin), M.A (Cantab), University Lecturer. 8 Grange Gardens, Grange Road, Cambridge	
1885		Hodgkinson, W. R., C.B E , M.A., Ph D , F I C., F.C.S , formerly Professor of Chemistry and Metallurgy, Artillery College, Woolwich. 89 Shooters Hill Road, Blackheath, Kent	
1923	C.	* Hogben, Lancelot Thomas, M.A., D.Sc , Professor of Social Biology (London School of Economics), University of London, Houghton Street, W.C. 2	
1927		Holden, Henry Smith, D Sc , F L S., Professor of Botany, University College, Nottingham	
1930		* Holland, Sir Thomas Henry, K.C S I , K C I.E , D L , Hon. D.Sc , LL.D., F.R.S. (VICE-PRESIDENT), Principal of the University of Edinburgh. Blackford Brae, Edinburgh	1931–32. V.P 1932–
1929	C.	* Hora, Sunder Lal, D.Sc. (Punjab et Edin), F L.S., F.Z.S , F A S.B., Senior Assistant Superintendent, Zoological Survey of India. Indian Museum, Calcutta	
1920	C.	* Horne, Alexander Robert, O B.E , B Sc , M.I Mech E., A M Inst.C.E., Professor of Mechanical Engineering, Heriot-Watt College, Edinburgh. 31 Queen's Crescent, Edinburgh 9	
1896		Horne, J. Fletcher, M.D , F.R.C.S.E , Shelley Hall, Huddersfield	
1904	C.	Horsburgh, Ellice Martin, M A., D Sc , Reader in Technical Mathematics, University of Edinburgh (16 Chambers Street) 11 Granville Terrace, Edinburgh 10	
1897		Houston, Sir Alex. Cruikshanks, K B E , C V O , M B , C M , D Sc , LL D., F.R.S , 20 Nottingham Place, London, W 1 (Died October 29, 1933)	
1912	C M-B.	* Houstoun, Robert Alexander, M.A , Ph D , D Sc., Lecturer in Physical Optics, University of Glasgow. 45 Kirklee Road, Glasgow	1929–32.
1893		Howden, Robert, M A , M B , C M , D Sc , LL D , Emeritus Professor of Anatomy, University of Durham Broomfield, Crieff	
1933		* Hume, Edgar Erskine, D S M , B.A , M A , LL D , Lieut -Col , U S. Army, Librarian of the Army Medical Library, Washington The Magnolias, Frankfort, Kentucky	
1910		Hume, William Fraser, D.Sc. (Lond.), Director, Geological Survey of Egypt, Helwân, Egypt. The Laurels, Rustington, Sussex	
1927		* Hunt, Owen Duke, B Sc (Manch), "Corrofell," Newton Ferrers, South Devon	
1923		* Hunter, Rev. Adam Mitchell, M A , D.Litt , Librarian of New College, Edinburgh 3 Suffolk Road, Edinburgh 9	
1932		* Hunter, Andrew, M A , B.Sc , M D , F R S C , Professor of Biochemistry, University of Glasgow	
1928		* Hunter, Arthur, F F.A., LL.D. (Edin), Vice-President and Chief Actuary of the New York Life Insurance Co. 124 Lloyd Road, Montclair, N J , U S.A	
1916		* Hunter, Charles Stewart, L R.C P E , L R C S E , D P H., Cotswold, 36 Streatham Hill, London, S W. 2	
1911		Hunter, Gilbert Macintyre, M Inst C.E , M Inst E S , M Inst M.E. Auchraig, Cramond Brig, near Edinburgh	
1887	C.	Hunter, William, C B , M D., M R C P.L. and E , M R.C S , LL D., 103 Harley Street, London	
1927		* Hyslop, James, M.A. (Glas), Ph D (Cantab.), Lecturer in Mathematics, University of Glasgow Criffel View, Rotchell Park, Dumfries	
1923	C.	* Ince, Edward Lindsay, M.A (Cantab), D Sc (Edin), Lecturer in Mathematics, Imperial College of Science and Technology, South Kensington. 6 Rutland Gardens, West Ealing, London, W. 13	
1920		* Inglis, James Gall, Publisher. 36 Blacket Place, Edinburgh 9	
1927		* Inglis, John Alexander, of Auchindinny and Redhall, K C , M A (Oxon.), LL.B (Edin), King's and Lord Treasurer's Remembrancer. 13 Randolph Crescent, Edinburgh 3	
1912		* Inglis, Robert John Mathieson, M.Inst.C.E., Assistant Chief Engineer, L.N.E.R , Liverpool Street Station, London. Dixton, Hadley Common, Barnet	
1917		* Irvine, Sir James Colquhoun, Kt., C.B.E., Ph.D. (Leipzig), D.Sc (St Andrews), Hon. D.Sc. (Liverpool, Princeton), Hon Sc D. (Cantab., Yale, Pennsylvania), Hon. LL.D. (Glas., Aberd., Edin., and Toronto), Hon. D.C.L. (Durham), D.L , F R.S., Hon. F.E.I.S., Hon. Mem. American Chemical Society, Principal of the University of St Andrews	1920–22. V.P 1922–25.

Date of Election			Service on Council, etc
1930	C	* Jack, David, M A , B Sc (Edin), Ph D (St Andrews), Lecturer in Natural Philosophy, United College, University of St Andrews 22 Grange Road, St Andrews	
1923		* Jack, John Louttit, Solicitor, Assistant Secretary, Department of Health for Scotland, 121A Princes Street, Edinburgh 2	
1912	C	* Jeffrey, George Rutherford, M D (Glas), F R C P (Edin), Bootham Park Private Mental Hospital, York	
1906	C K	Jehu, Thomas John, M A , M D , F G S , Professor of Geology, University of Edinburgh (Grant Institute of Geology, West Mains Road) 35 Great King Street, Edinburgh 3	1917–20, 1923–26 V-P 1929–32
1900		† Jerdan, David Smiles, M A , D Sc , Ph D , 26 Avenue du Château d'Eau, Saventhem, Belgium	
1931		* Johnson, Thomas, D Sc (Lond), Emeritus Professor of Botany, Royal College of Science for Ireland Tomeg, Hillview Drive, Corstorphine 12	
1925		* Johnston, Christopher Nicholson, The Hon Lord Sands, Kt , LL D , D D , Senator of the College of Justice (VICE PRESIDENT) 4 Heriot Row, Edinburgh 3	1929–32 V-P 1932–
1895		Johnston, Col Henry Halcro, C B , C B E , D Sc , M D , C M (Edin), F L S late Administrative Staff, Army Stromness Hotel, Stromness, Orkney	
1928		* Johnston Saint, Percy Johnston, M A (Cantab), Secretary, Wellcome Historical Medical Museum, 183 Euston Road, London, N W 1 4 Wyndham Place Bryanston Square, London, W	
1928		* Johnstone, Robert William, C B E , M A , M D (Edin), F R C S E , M R C P E , Professor of Midwifery and Diseases of Women, University of Edinburgh 26 Palmerston Place, Edinburgh 12	
1927		* Jones Edward Taylor, D Sc (Lond) Professor of Natural Philosophy, University of Glasgow	1927–30
1930	C	* Jones, Samuel Griffith, D Sc (Univ Wales), Lecturer in Botany, University of Glasgow Broomfield, Kilmacolm, Renfrewshire	
1928	C	* Jones, Tudor Jenkyn, M B , Ch B (Glas), Lecturer in Anatomy, University of Liverpool	
1922		* Juritz, Charles Frederick, M A , D Sc , F I C , Chief of the Union Department of Chemistry Villa Marina, Three Anchor Bay, Cape Town, South Africa	
1925	C	* Kemp, Charles Norman, B Sc , Technical Radiologist, Secretary of the Royal Scottish Society of Arts Ivy Lodge, Laverockbank Road, Edinburgh 5	
1929		* Kendall, James Pickering, M A , D Sc , F R S (SECRETARY TO ORDINARY MEETINGS), Professor of Chemistry, University of Edinburgh (West Mains Road) 14 Mayfield Gardens, Edinburgh 9	1931–33 Sec 1933–
1912		† Kennedy, Robert Foster, M D (Belfast), M B , B Ch (R U I), Associate Professor of Neurology, Cornell University, New York 410 East 57th Street, New York, U S A	
1927		* Kennedy, Walter Phillips, Ph D (Edin), L R C P and S E , A I C , Physiology Department, University (Teviot Place), Edinburgh	
1909		Kenwood, Henry Richard, C M G , M B , C M , Emeritus Chadwick Professor of Hygiene, University of London "Wadhurst," Queen's Road, Finsbury Park, London, N	
1925	C M B	* Kermack, William Ogilvy, M A , D Sc , Chemist, Research Laboratory of the Royal College of Physicians, 2 Forrest Road, Edinburgh 1	
1908		Kerr, Andrew William, F S A Scot , 81 Great King Street, Edinburgh 3	
1903 & 1923	C N	Kerr, John Graham, M A , F R S , F L S , F Z S , Regius Professor of Zoology, University of Glasgow 9 The University, Glasgow	1904–07, 1913–16, 1924–27 V-P 1928–31.
1891		Kerr, Joshua Law, M D , J P , "Stonehaven," Sheffield, Tasmania	
1913		* Kerr, Walter Hume, M A , B Sc , formerly Lecturer in Engineering Drawing and Structural Design, University of Edinburgh Glenfriars, Jedburgh	
1926		* [illegible]gir, Satis Ranjan, M Sc (Calcutta), D Sc (Edin), Physics Department, University, Dacca, India	
1907		King, Archibald, M A , B Sc , H M Inspector of Schools, 10 Leslie Road, Pollokshields, Glasgow	
1925		* King, Leonard Augustus Lucas, M A , Professor of Zoology, West of Scotland Agricultural College, Glasgow 14 Bank Street, Glasgow, W. 2	

Date of Election			Service on Council, etc.
1918		* Kingon, Rev. John Robert Lewis, M.A., D.Sc., The Manse, Dundee, Natal Province, South Africa	
1901		Knight, Rev. G. A Frank, M.A., D.D., F.S.A.Scot , 10 Hillhead Street, Glasgow	
1907		Knight, James, M A , D Sc., F.C S., F.G S., J.P., Rector, Queen's Park High School, Langside, Glasgow	
1927		* Lambie, Charles George, M.C., M.D , F R C P.E , Bosch Professor of Medicine, University of Sydney, Australia	
1920	C.	* Lamont, John Charles, Lieut.-Col., I.M S (retired), C.I.E., M.B., C.M (Edin.), M.R C.S , 7 Merchiston Park, Edinburgh 10	
1925	C. N.	* Lang, William Henry, M B., C.M , D Sc , LL.D. (Glas.), F.R S., Barker Professor of Cryptogamic Botany, University of Manchester	
1931		* Langrishe, John du Plessis, D.S.O , M.B , B Ch. (Dub), D.P.H., Lt.-Col. R.A.M C (retired), Lecturer in Public Health, University of Edinburgh (Usher Institute of Public Health, Warrender Park Road). 2 South Gillsland Road, Edinburgh 10	
1910	C.	* Lauder, Alexander, D Sc , Head of Chemistry Department, Edinburgh and East of Scotland College of Agriculture, 13 George Square, Edinburgh 8. Lecturer in Agricultural Chemistry, University of Edinburgh	1917–20. Sec. 1923–28.
1885	C	Laurie, Arthur Pillans, M A., D Sc , LL D , formerly Principal, Heriot-Watt College, Edinburgh 38 Springfield Road, St John's Wood, London, N W. 8	1908–11, 1913–16.
1921		* Laurie, The Rev Albert Ernest, M C , C.F , D.D , Rector of Old St Paul's, and Canon of St Mary's Cathedral, Edinburgh. Lauder House, Jeffrey Street, Edinburgh 1	
1905		Lawson, David, M A , M D., L R.C P and S E , Drumdarroch, Banchory, Kincardineshire	
1933		* Lees, David, D.S.O , M A., M.B , Ch B., F R.C S E , F R C P E , D P H , Clinical Medical Officer, Corporation of Edinburgh; Surgeon in charge of Venereal Diseases, Royal Infirmary, Edinburgh. 21 Coates Gardens, Edinburgh 12	
1903		Leighton, Gerald Rowley, O B.E , M.D., D Sc., formerly Medical Officer (Foods), Department of Health for Scotland Sharston, near Ramsey, Isle of Man	
1930		* Lelean, Percy Samuel, C.B , C M.G., F R C S , L.R.C.P , D P H., Professor of Public Health, University of Edinburgh (Usher Institute of Public Health, Warrender Park Road). 2 Barnton Loan, Davidson's Mains, Edinburgh 4	
1910		Levie, Alexander, F R C.V.S , D V S.M , Balmae, Manor Road, Littleover, Derby	
1916	C.	* Levy, Hyman, M.A , D.Sc , Professor of Mathematics, Imperial College of Science and Technology, London. Kitelands, Micheldever Station, Hants	
1914	C. N.	Lewis, Francis John, D Sc., F.L.S., Professor of Biology, University of Alberta, Edmonton South, Alberta, Canada	
1918		* Lidstone, George James, F.F A., F.I A , LL D., formerly Manager and Actuary, Scottish Widows' Fund Life Assurance Society. Hermiston House, Hermiston, Currie, Midlothian	1919–22
1905		Lightbody, Forrest Hay, 53 Queen Street, Edinburgh 2	
1931		* Lightfoot, Nicholas Morpeth Hutchinson, M A. (Cantab), Lecturer in Mathematics, Heriot-Watt College, Edinburgh 3 Park Gardens, Liberton, Edinburgh 9	
1923		* Lim, Robert Kho Seng, M B., Ch.B., D.Sc , Peking Union Medical College, Department of Physiology, Peking, China	
1912		* Lindsay, John George, M.A., B.Sc. (Edin.), Rector of Dunfermline High School	
1920	C.	* Lindsay, Thomas A., M.A , B.Sc., Head Master, Higher Grade School, Bucksburn, Aberdeenshire	
1912		* Linlithgow, The Most Honourable the Marquis of, K.T., G.C.I.E , D.L., Hopetoun House, South Queensferry	
1903		† Liston, William Glen, C.I.E., M.D , Lt -Col , I M.S. (retired), Milburn Tower, Gogar, Corstorphine, Edinburgh 12	
1929		* Little, John Robert, F.C.I.I., F.C.I.S., Manager and Secretary, Century Insurance Co., Ltd 5 Dalrymple Crescent, Edinburgh 9	
1932		* Lockhart, James Balfour, M.A., B.Sc. (Edin), Mathematical Master, Edinburgh Academy. Dundas House, Kinnear Road, Edinburgh 4	

Date of Election			Service on Council, etc.
1926		* Lorraine, Norman Stanley Rees, M.D., D.P.H. (Edin. and Glas.), Medical Officer of Health, Shoeburyness Urban District. 1 Burlescoombe Leas, Burlescoombe Road, Thorpe Bay, Southend-on-Sea	
1898		Lothian, Alexander Veitch, M.A., B.Sc., Glendoune, 996 Great Western Road, Glasgow	
1930		* Low, James Wotherspoon, B.Sc. (Edin.), Ph.D. (Bristol), 14 Great Stuart Street, Edinburgh 3	
1923	C.	* Ludlam, Ernest Bowman, M.A., D.Sc., Lecturer in Chemistry, University of Edinburgh (West Mains Road)	
1923		* Lyford-Pike, James, M.A., B.Sc., Lecturer in Forestry, University of Edinburgh. Rosetta, Liberton, Edinburgh 9	
1924		* Lyon, David Murray, M.D., F.R.C.P.E., D.P.H., D.Sc., Professor of Therapeutics and Clinical Medicine, University of Edinburgh (Royal Infirmary). Drum, Colinton, Edinburgh 11	
1894		Mabbott, Walter John, M.A., formerly Rector of County High School, Duns, Berwickshire. The Hawthorn, Farnham Lane, Haslemere, Surrey	
1917		* Macalister, Sir Donald, Bt., K.C.B., M.D., M.A., B.Sc., Chancellor of the University of Glasgow. Barrmore, Lady Margaret Road, Cambridge	
1929		* M'Arthur, Donald Neil, D.Sc., F.I.C., Professor of Agricultural Chemistry, West of Scotland Agricultural College, Glasgow, C. 2. 35 Kersland Street, Glasgow, W. 2	
1921		* M'Arthur, Neil, M.A., B.Sc., Lecturer in Mathematics, University of Glasgow. 1 Holyrood Crescent, Glasgow	
1926		* M'Bride, James Alexander, B.A. (Roy. Univ., Ireland), B.Sc. (Lond.), formerly Rector of Queen's Park Secondary School, Glasgow. Scottish Liberal Club, Princes Street, Edinburgh 2	
1883		M'Bride, Peter, M.D., F.R.C.P.E., 3 St Peter's Grove, York	
1931	C.	* McCallien, William John, D.Sc. (Glas.), Lecturer in Geology, University of Glasgow. Glenorchy, Tarbert, Argyll	
1930		* M'Candlish, Andrew Corrie, B.Sc. (Glas.), M.S.A., Ph.D. (Iowa), Advisory Officer in Milk Production, West of Scotland Agricultural College. Claunch, Sorbie, Wigtownshire	
1923		* M'Cracken, William, J.P., F.S.I., Englesea House, Crewe	
1931	C.	* M'Crea, William Hunter, M.A., Ph.D. (Cantab.), B.Sc. (Lond.), F.R.A.S., Asst. Professor of Mathematics, University of London. Imperial College of Science and Technology, South Kensington, S.W. 7	
1918		* M'Culloch, Rev. James David, 3 Ardgowan Street, Greenock	
1905		Macdonald, Hector Munro, O.B.E., M.A., F.R.S., Professor of Mathematics, University of Aberdeen. 52 College Bounds, Aberdeen	1908–11.
1897	C.	Macdonald, James A., M.A., B.Sc., formerly H.M. Inspector of Schools. "Rothes," Frankscroft, Peebles	
1920		* M'Donald, Stuart, M.A., M.D., F.R.C.P.E., Professor of Pathology, University of Durham. College of Medicine, Newcastle-on-Tyne	
1928		* MacDonald, Thomas Logie, M.A., B.Sc. (Glas.), F.R.A.S., 9 Colebrooke Terrace, Glasgow, W. 2	
1904		Macdonald, William, M.S.Agr., Sc.D., D.Sc., Editor, *Agricultural Journal* of South Africa. Rand Club, Johannesburg, Transvaal	
1886		Macdonald, William J., M.A., LL.D., 15 Comiston Drive, Edinburgh 10	
1931		* M'Dougall, John Bowes, M.D. (Glas.), F.R.F.P.S.G., F.R.C.P.E., Medical Director, British Legion Village, Preston Hall, Kent. Preston Hall, Aylesford, Kent	
1901	C.	MacDougall, R. Stewart, M.A., D.Sc., LL.D. (Edin.), Emeritus Professor of Biology, Royal (Dick) Veterinary College, Edinburgh. Ivy Lodge, Gullane, East Lothian	1914–17.
1910		Macewen, Hugh Allen, O.B.E., M.B., Ch.B., D.P.H. (Lond. and Cantab.), Local Government Board, Ministry of Health, Whitehall, London, S.W.	
1888	C.	M'Fadyean, Sir John, Kt., M.B., B.Sc., LL.D., formerly Principal and Professor of Comparative Pathology, Royal Veterinary College, Camden Town, London. Highlands House, Leatherhead	
1885	C.	† Macfarlane, J. M., D.Sc., LL.D., Emeritus Professor of Botany. 427 West Hansberry Street, Germantown, Pa., U.S.A.	
1897		MacGillivray, Angus, M.D., C.M., D.Sc., 23 South Tay Street, Dundee	
1878		M'Gowan, George, F.I.C., Ph.D., 21 Montpelier Road, Ealing, London, W. 5	
1932		* MacGregor, Archibald Gordon, M.C., B.Sc., Geologist, H.M. Geological Survey (Scotland). 1 Greenbank Terrace, Edinburgh 10	

Date of Election			Service on Council, etc.
1922		* Macgregor, Murray, M.A., D.Sc., Assistant Director (Scotland), H.M. Geological Survey. 19 Grange Terrace, Edinburgh 9	1930–33.
1903		M'Intosh, Donald C., M.A., D.Sc., formerly Education Officer, Elgin. Tomlay, Tomintoul, Banffshire	
1911		M'Intosh, John William, M.R.C.V.S., Dollis Hill Farm, Cricklewood, London, N.W. 2	
1927	C.	* M'Intyre, Donald, M.B.E., M.D (Glas), F.R.C S.E., Assistant Physician, Glasgow Royal Maternity and Women's Hospital. 9 Park Circus, Glasgow, C 3	
1912	C.	M'Kendrick, Anderson Gray, M B , D.Sc., F.R.C.P.E , Lt.-Col , I M.S. (retired), Superintendent, Research Laboratory, Royal College of Physicians, 2 Forrest Road, Edinburgh 1	1924–27. 1933–
1914		* M'Kendrick, Archibald, F R.C S E , D P.H , L D S , J P , 12 Rothesay Place, Edinburgh 3	
1900	C.	M'Kendrick, John Soutar, M.D , F R F P S (Glas), 2 Buckingham Terrace, Hillhead, Glasgow	
1910	C.	* Mackenzie, Alister, M.A., M.D., D P.H , Principal Medical Officer and Lecturer in Hygiene, Training Centre, Jordanhill, Glasgow 22 Queen's Gate, Dowanhill, Glasgow	
1916	C.	* Mackenzie, John E , D Sc , Reader in Chemistry, University of Edinburgh (West Mains Road) 2 Ramsay Garden, Edinburgh 1	
1905		Mackenzie, Sir Wm. Colin, K B , M D , F.R.C S , Director, Australian Institute of Anatomy, Canberra, F C T., Australia	
1904	C.	Mackenzie, Sir W. Leslie, M A , M D., D P.H , LL D , F R.C.P E , formerly Medical Member of the Scottish Board of Health. 14 Belgrave Place, Edinburgh 4	
1929	C.	* Mackie, John, M.A , D Sc , Rector, Leith Academy. 19 Beresford Avenue, Trinity, Leith 5	
1928		* Mackie, Thomas Jones, M D., M R C P E , Professor of Bacteriology, University of Edinburgh (Teviot Place). 22 Mortonhall Road, Edinburgh 9	
1910		* MacKinnon, James, M.A , Ph D , LL D., Emeritus Professor of Ecclesiastical History, University of Edinburgh. 12 Lygon Road, Edinburgh 9	1933–
1904		Mackintosh, Donald James, C B , M V.O , D.L , M B , C M , LL D., Superintendent, Western Infirmary, Glasgow	
1899		Maclean, Sir Ewan John, M.D., D Sc. (Hon), LL D , F.R C P (Lond), D L., J P , Emeritus Professor of Obstetrics and Gynæcology, Welsh National Medical School 12 Park Place, Cardiff	
1888	C.	Maclean, Magnus, M.A , D.Sc , LL.D , M.Inst C E., M.I.E.E , formerly Professor of Electrical Engineering, Royal Technical College. 108 University Avenue, Glasgow, W. 2	1916–19.
1933		* Macleod, James, F I.C , Manager, Glasgow Corporation Chemical Works Department 16 Colebrooke Street, Glasgow, W 2	
1932		* Macleod, John James Rickard, M.B , Ch B (Aberd), D.Sc (Toronto), LL.D., F.R S , Professor of Physiology, University of Aberdeen	
1916	C.	* M'Lintock, William Francis Porter, D Sc (Edin), Museum of Practical Geology, 28 Jermyn Street, London, S W 1	
1923		* Macmillan, Rt Hon Lord, P.C , LL D , 44 Millbank, Westminster, S.W. 1	
1932		* M'Neil, Charles, M A., M D. (Edin), F R C P E , Professor of Child Life and Health, University of Edinburgh (Royal Edinburgh Hospital for Sick Children). 44 Heriot Row, Edinburgh 3	
1917		* Macpherson, Rev. Hector Copland, M A , Ph D., F.R A S., Guthrie Memorial U F. Church 7 Wardie Crescent, Edinburgh 5	
1921		* M'Quistan, Dougald Black, M.A , B.Sc , Associate-Professor of Natural Philosophy, Royal Technical College, Glasgow 29 Viewpark Drive, Rutherglen	
1921	C.	* MacRobert, Thomas Murray, M A , D.Sc , Professor of Mathematics, University of Glasgow. 10 The University, Glasgow	1931–
1921	C.	* M'Whan, John, M.A. (Glasgow), Ph.D. (Gott.), Lecturer in Mathematics, University of Glasgow. 84 Munro Road, Jordanhill, Glasgow, W. 3	
1927		* Madwar, Mohamed Reda, Ph.D. (Edin), Assistant, Helwân Observatory, Egypt	
1898	C.	† Mahâlanobis, S. C., B.Sc. (Edin.), Professor of Physiology, University of Calcutta. P. 45 New Park Street, Calcutta	
1913		Majumdar, Tarak Nath, D.P.H. (Cal.), L.M.S., F C.S., Health Officer, IV, Calcutta. P. 235 Russa Road, P.O. Tollygunge	

Date of Election			Service on Council, etc.
1933		* Malcolm, John, M.D. (Edin.), Professor of Physiology, University of Dunedin, New Zealand, Medical School, King Street, Dunedin	
1917		* Malcolm, L. W. Gordon, M.Sc. (Cantab.), Conservator, Wellcome Historical Medical Museum, 183 Euston Road, London, N.W. 1	
1908		Mallik, Devendranath, Sc.D., B.A., Principal, Carmichael College, Rungpur, Bengal, India	
1912		Maloney, William Joseph, M.B.E., M.C., M.D. (Edin.), LL.D., formerly Professor of Neurology, Fordham University. 420 Park Avenue, New York City, U.S.A.	
1913		Marchant, Rev. Sir James, K.B.E., LL.D., F.R.A.S., F.L.S., Director, National Council for Promotion of Race-Regeneration. Rhondda House, 60 Gower Street, London, W.C. 1	
1901	C	Marshall, Francis Hugh Adam, C.B.E., Sc.D., F.R.S., Reader in Agricultural Physiology, University of Cambridge. Christ's College, Cambridge	
1920	C	* Marshall, John, M.A., D.Sc. (St Andrews), B.A. (Cantab.), University Reader in Mathematics, Bedford College, London. Logan House, 123 Torrington Park, London, N. 12	
1931		* Mason, John Huxley, F.R.C.V.S., Government Veterinary Laboratory, Onderstepoort, Pretoria, South Africa	
1885	C	Masson, Sir David Orme, K.B.E., M.A., D.Sc., LL.D., F.R.S., Emeritus Professor of Chemistry, University of Melbourne	
1913		Masson, George Henry, M.D., D.Sc., F.R.C.P.E., Port of Spain, Trinidad, British West Indies	
1898	C M-B.	Masterman, Arthur Thomas, M.A., D.Sc., F.R.S., formerly Superintending Inspector, H.M. Board of Agriculture and Fisheries. 3 Kedale Road, Seaford	1902–04.
1911		† Mathews, Gregory Macalister, M.B.O.U., Meadway, St Cross, Winchester, Hants	
1921		* Mathieson, John, F.R.S.G.S., Division Superintendent, Ordnance Survey (retired), 42 East Claremont Street, Edinburgh 7	
1906		Mathieson, Robert, F.C.S., St Serf's, Innerleithen	
1928		* Matthai, George, M.A. (Cantab.), F.Z.S., F.L.S., Professor of Zoology, Government College, Lahore, India	
1924		* Matthews, James Robert, M.A., F.L.S., Professor of Botany, University of Reading	
1932		* Maxwell, William, Managing Director of R. & R. Clark, Ltd. 14 South Inverleith Avenue, Edinburgh 4	
1917		* Maylard, A. Ernest, M.B., B.Sc. (Lond.), F.R.F.P.S. (Glasgow), Kingsmuir, Peebles	
1922		* Meakins, Jonathan Campbell, M.D., LL.D., F.R.C.P.E., Professor of Medicine and Director of the Department of Medicine, McGill University, Montreal, Canada	
1931		* Mears, Frank Charles, F.R.I.B.A., 3 Forres Street, Edinburgh 3	
1926		* Mekie, David Clark Thomson, M.A., Ph.D., Head Master, Darroch Intermediate School. 11 Minto Street, Edinburgh 9	
1901	C.	Menzies, Alan W. C., M.A., Ph.D., F.C.S., Professor of Chemistry, Princeton University, Princeton, New Jersey, U.S.A.	
1927		* Menzies, Sir Frederick Norton Kay, K.B.E., M.D., LL.D. (Edin.), F.R.C.P.E., D.P.H. (Lond.), Medical Officer of Health and School Medical Officer, Administrative County of London, County Hall, London, S.E.	
1933	C.	* Menzies, William John Milne, Inspector of Salmon Fisheries of Scotland, Fishery Board for Scotland. Tighbeag, Whitehouse Road, Cramond Bridge, near Edinburgh	
1929		* Mercer, Walter, M.B., Ch.B., F.R.C.S.E., Lecturer in Clinical Surgery, University of Edinburgh (Royal Infirmary). 12 Rothesay Terrace, Edinburgh 3	
1917		* Merson, George Fowlie, Manufacturing Technical Chemist, St John's Hill Works, Edinburgh 8	
1902	C.	† Metzler, William H., A.B., D.Sc., Ph.D., formerly Dean of the New York State College for Teachers, Albany, N.Y., U.S.A.	
1885	C. M-B.	Mill, Hugh Robert, D.Sc., LL.D., Hill Crest, Dormans Park, E. Grinstead	
1910		* Miller, John, M.A., D.Sc., Professor of Mathematics, Royal Technical College. 2 Northbank Terrace, North Kelvinside, Glasgow	

Date of Election			Service on Council, etc.
1930		* Miller, William Christopher, M R.C.V S, Lecturer, Institute of Animal Genetics, West Mains Road, Edinburgh. Scotiswoode, Alnwickhill Road, Liberton, Edinburgh 9	
1905		Milne, Archibald, M.A, D.Sc., Deputy Director of Studies, Edinburgh Provincial Training College 38 Morningside Grove, Edinburgh 10	
1905		Milne, C. H., M.A., D Litt., Head Master, Daniel Stewart's College 19 Merchiston Gardens, Edinburgh 10	
1904	C.	Milne, James Robert, D.Sc, Lecturer in Natural Philosophy, University of Edinburgh (Drummond Street) 7 Grosvenor Crescent, Edinburgh 12	
1886		Milne, William, M.A, B Sc, 70 Beechgrove Terrace, Aberdeen	
1933	C.	Milne-Thomson, Louis Melville, M A (Cantab), F R.A S, Professor of Mathematics, Royal Naval College, Greenwich Gothic House, Maze Hill, London, S E 10	
1899		Milroy, Thomas Hugh, M D., B Sc., LL D, Professor of Physiology, Queen's University, Belfast	
1889	C.	Mitchell, A Crichton, D Sc, Hon. Doc Sc (Genève), formerly Director of Public Instruction in Travancore, India 246 Ferry Road, Edinburgh 5. (Society's Representative on Governing Body of Heriot-Watt College)	1915–16, 1930–33. Cur. 1916–26. V-P 1926–29.
1897		† Mitchell, George Arthur, M.A, 9 Lowther Terrace, Kelvinside, Glasgow	
1900		Mitchell, James, M A., B.Sc, Islay Lodge, Lochgilphead, Argyll	
1911		Modi, Edalji Manekji, D Sc, LL D, Litt D, F C S, Arthur Road Chemical Works, Meher Buildings, Tardeo, Bombay, India	
1906		† Moffat, Rev. Alexander, M.A, LL D, formerly Professor of Physics, Christian College, Madras C/o National Bank of India, Post Box No 64, Madras, India	
1929		Moir, Henry, F F A, F I A, President, United States Life Insurance Co, 156 Fifth Avenue, New York City Upper Montclair, New Jersey	
1890	C.	† Mond, Sir Robert Ludwig, Kt., M A (Cantab), LL D, F C S, 9 Cavendish Square, London, W 1	
1887	C.	Moos, N. A. F, D Sc, J P, Director of Bombay and Alibag Observatories (retired) Red Leaf, Pedder Road, Bombay, India	
1896		Morgan, Alexander, O B E, M A, D Sc, formerly Principal, Edinburgh Provincial Training College 1 Midmar Gardens, Edinburgh 10	
1930		* Morison, John Miller Woodburn, M D, F R.C.P E, D M.R and E, Professor of Radiology, University of London, and Director of the Radiological Department, Cancer Hospital, Fulham Road, London, S.W 3	
1926		* Morris, James Archibald, R.S A., F S A Scot, Savoy Croft, Ayr	
1919		* Morris, Robert Owen, M.A, M D, C M (Edin), D P.H. (Liverpool) King Edward VII Welsh National Memorial Association (Tuberculosis). Hafod-ar-For, Aberdovey, N Wales	
1892	C.	Morrison, J. T., M A, B Sc, Professor of Mathematical Physics, University, Stellenbosch, Cape Colony	
1930		* Morton, James, LL D, Governing Director, Scottish Dyes, Ltd. Tuethur, Carlisle	
1901		Moses, O. St John, M D, D Sc., F R C S, Lt -Col, I M S (retired), formerly Professor of Medical Jurisprudence, Medical College, Calcutta 18 Manstone Road, Cricklewood, London, N W 2	
1892	C. K.	Mossman, Robert Cockburn, Lacar 4332, Villa Devoto F C P, Buenos Aires, Argentina	
1916		* Muir, Robert, M A., M D, Sc D., LL D, F R.S., F.R.C.P.E., Professor of Pathology, University of Glasgow 30 Victoria Crescent Rd., Glasgow, W.2	
1874	C. K. V. J.	Muir, Sir Thomas, C M.G, M A, D.Sc, LL.D., F R.S, formerly Superintendent-General of Education for Cape Colony Elmcote, Sandown Road, Rondebosch, South Africa	1885–88. V-P 1888–91.
1907		Muirhead, James M. P., J P., F.R S.L, F.S.S, "Carmel," Rouwkoop Road, Rondebosch, South Africa	
1933		* Murray, John, M A. (Aberd.), Ph.D. (Edin.), Rector, The Academy, Annan. 9 Seaforth Avenue, Annan	
1932		* Murray, John Murdoch, B Sc (Edin), Divisional Officer, Forestry Commission 76 Hillview Terrace, Corstorphine, Edinburgh 12	
1907		† Musgrove, James, M D, F R.C S, F.R.C.S.E., LL.D, Emeritus Professor of Anatomy, University of St Andrews. The Swallowgate, St Andrews	

Date of Election			Service on Council, etc.
1931		* Nelson, Alexander, B.Sc. (Glas.), Ph D. (Edin), N.D.A., Lecturer in Plant Physiology and Agricultural Botany, University of Edinburgh. 11 Wardie Avenue, Edinburgh 5	
1924		* Nelson, Philip, M.A., M D., Ph.D., F.S.A , Beechwood, Calderstones, Liverpool	
1898		Newman, Sir George, K.C.B., M.D , D.C.L., LL.D., F.R.C.P , Chief Medical Officer, Ministry of Health and Board of Education, Whitehall, London, S W. 1	
1928		* Nichols, James Edward, M Sc. (Dunelm), Ph D. (Edin.), Wool Industries Research Association, Torridon, Headingley, Leeds. Moorfield Cottage, Arthington, Yorks	
1933	C.	* Nicol, Thomas, M B., Ch.B , F.R.C.S E , Lecturer in Anatomy, University of Glasgow 77 Fotheringay Road, Glasgow, S 1	
1927		* Noble, Thomas Paterson, M.D (Edin.), F R C S , Phya Thai Palace, Bangkok, Siam	
1928	C. N.	* O'Donoghue, Charles Henry, D Sc (Lond), Reader in Zoology, University of Edinburgh (West Mains Road) Appin Lodge, Eskbank, Midlothian	
1925		* Ogg, William Gammie, M A , Ph D , Director, Macaulay Institute for Soil Research, Craigiebuckler, Aberdeen, Research Lecturer in Soil Science in the University of Aberdeen	
1923	C.	* Ogilvie, Alan G , M.A , B Sc (Oxon), Professor of Geography, University of Edinburgh (High School Yards) 40 Fountainhall Road, Edinburgh 9	1932–
1929		* Ogilvie, Frederick Wolff, M A , Professor of Political Economy, University of Edinburgh (South Bridge). 20 Murrayfield Gardens, Edinburgh 12	
1886		Oliver, James, M D , F L S., Physician to the London Hospital for Women, 123 Harley Street, London, W	
1895	C.	† Oliver, Sir Thomas, Kt , D L , M D , LL D., F R C P , ex-Vice Chancellor of the University of Durham 7 Ellison Place, Newcastle-upon-Tyne	
1930		* Oliver, William, B Sc , A.M Inst C.E , Professor of Organisation of Industry and Commerce, University of Edinburgh (South Bridge). 70 Netherby Road, Trinity, Edinburgh 5	
1930		* O'Riordan, George Francis, B Sc (Eng.), M.I Mech E , Principal of Battersea Polytechnic, London, S W 7 Hesslewood, 3 Hazlewell Road, Putney, London, S W 15	
1924		* Orr, John Boyd, D S O , M.C , M.A , D.Sc., M.D , F R.S , Director of the Rowett Research Institute for Research in Animal Nutrition, Aberdeen	
1915		* Orr, Lewis P , F F A , General Manager of the Scottish Life Assurance Co. 3 Belgrave Place, Edinburgh 4	
1932		* Orr, Matthew Young, 3 Royston Terrace, Edinburgh 4	
1927		* Owen William John, Memb Roy Soc. Victoria, Director, Department of Histology, Australian Institute of Anatomy, Canberra, Australia	
1908		Page, William Davidge, 13 Yarrell Mansions, Queen's Club Gardens, London, W. 14	
1905		Pallin, Lt.-Col William Alfred, C.B E , D S.O , F R C V S , 5 Tower Gardens, Hythe, Kent	
1924		* Parker, Joseph, D Sc , Principal, Fife Mining School, Cowdenbeath. 128 Stenhouse Street, Cowdenbeath	
1933		Parsons, Charles Wynford, B A., Lecturer in Zoology, University of Glasgow. Westbank, 65 Partickhill Road, Glasgow	
1901		Paterson, David, F C S., Leewood, Rosslyn Castle, Midlothian	
1918		* Paterson, Rev William Paterson, D D , LL D., Professor of Divinity, University of Edinburgh (South Bridge) 39 George Square, Edinburgh 8	
1927		* Patterson, Charles, A M I.Mar.E., Lecturer in Mechanical Engineering Design, University of Edinburgh (Sanderson Engineering Laboratories, Mayfield Road). 22 Dudley Terrace, Trinity, Edinburgh 6	
1926		* Patton, Donald, M A , Ph D., Lecturer in Botany, Glasgow Provincial College for the Training of Teachers. 15 Jordanhill Drive, Glasgow, W. 3	
1923	C.	* Peacock, Alexander David, D Sc , Professor of Zoology, University College, Dundee	
1907		Pearce, John Thomson, B.A., B.Sc., Bolton Manse, Haddington, East Lothian	
1914		Pearson, Joseph, D.Sc., F L.S , Director of the Colombo Museum, and Marine Biologist to the Ceylon Government, Colombo Museum, Ceylon	
1904		Peck, James Wallace, C B., M A., Second Secretary, Scottish Education Department, Dover House, Whitehall, London, S.W. 1	1926–28.

Date of Election			Service on Council, etc.
1887	C. M-B.	Peddie, Wm., D.Sc., Professor of Natural Philosophy, University College, Dundee The Weisha, Ninewells, Dundee	1904–07, 1908–11, 1933– V-P 1919–22.
1925		* Penman, David, D.Sc , M.Inst.M E , Principal, Indian School of Mines, Dhanbad, India	
1893		Perkin, Arthur George, Hon. D.Sc. (Leeds), F.R S , F.I C , Emeritus Professor in the University of Leeds. Grosvenor Lodge, Grosvenor Road, Headingley, Leeds	
1931	C.	* Phemister, James, M.A., D.Sc. (Glas.), Senior Geologist, H M. Geological Survey (Scotland) Everland, 2 Denham Green Terrace, Edinburgh 5	
1889		† Philip, Sir Robert William, Kt , M A., M.D , LL D , F R.C P.E., Professor of Tuberculosis, University of Edinburgh 45 Charlotte Square, Edinburgh 2	V-P 1927–30.
1907	C.	† Phillips, Charles E. S , O B E., Castle House, Shooters Hill, Woolwich, S.E 18	
1929	C.	* Phillips, John Frederick Vicars, D.Sc , F.L S., Professor of Botany, University of the Witwatersrand, Johannesburg, Union of South Africa	
1932		* Pickard, James Nichol, B.A (Cantab), Ph.D (Edin), The Cottage, Carlops, by Penicuik, Midlothian	
1928		* Pilcher, Robert Stuart, General Manager, Manchester Corporation Tramways 55 Piccadilly, Manchester	
1914		* Pilkington, Basil Alexander, "Kambla," Davidson's Mains, Edinburgh 4	
1933		* Pillers, Alfred William Noel, F R C V S., D.V S M., Chief Veterinary Officer to the Corporation of the City of Liverpool; Lecturer in Veterinary Parasitology, University of Liverpool. Hawkstone, Queen's Drive, Walton, Liverpool 4	
1908	C.	Pirie, James Hunter Harvey, B Sc., M.D., F R.C P.E , Research Pathologist and Bacteriologist, South African Institute for Medical Research. P.O Box 1038, Johannesburg, South Africa	
1911		* Pirie, James Simpson, M Inst.C E., 25 Grange Road, Edinburgh 9	
1906		Pitchford, Herbert Watkins, C M G., F R C V S , Victoria Club, Pietermaritzburg, South Africa	
1924		* Ponder, Eric, M D., D.Sc , Professor of General Physiology, Washington Square College, New York University, New York, U.S.A	
1919		* Porritt, B. D., M Sc. (Lond.), F.I C , Director of Research, Research Association of British Rubber Manufacturers, 105–7 Lansdowne Road, Croydon, Surrey	
1888		† Prain, Sir David, Kt , Lt -Col , I.M.S (retired), C.M G , C.I E , M.A , M.B., LL D , F R S , F L S., For. Memb. K. Svensk. Vetensk Akad ; Hon Memb. Soc Lett. ed Arti d Zelanti, Acireale; Pharm. Soc Gt. Britain; Corr. Memb. Bayer Akad. Wiss , etc ; formerly Director, Royal Botanic Gardens, Kew, Surrey The Well Farm, Warlingham, Surrey	
1932		* Prasad, Gorakh, D.Sc (Edin), Reader in Mathematics, University of Allahabad. Beli Road, Allahabad, India	
1926	C.	* Prashad, Baini, D Sc , Superintendent, Zoological Survey of India, Indian Museum, Calcutta	
1892	C.	Pressland, Arthur J , M A. (Cantab.), 28 Carlyle Road, Cambridge	
1933		* Preston, Frank Anderson Baillie, L.R.I.B A., F S A.Scot., Lecturer in Municipal Engineering, Royal Technical College, Glasgow. 27 Ferguson Avenue, Milngavie	
1928		Price, Charles Edward, J P , formerly M.P for Central Edinburgh, Hon. Freeman of the City of Edinburgh Westlands, Weston Rhyn, Shropshire	
1915		† Price, Frederick William, M D. (Edin), Consulting Physician to the Royal Northern Hospital, London; Senior Physician to the National Hospital for Diseases of the Heart. 133 Harley Street, London, W.	
1932		* Price, Thomas Slater, O.B E , D.Sc., F R.S., Professor of Chemistry, Heriot-Watt College, Edinburgh. 2 Cluny Drive, Edinburgh 10	
1932		* Pringle, John, Hon D.Sc., F.G.S., Senior Geologist, Geological Survey (Scotland), 19 Grange Terrace, Edinburgh 9	
1911		Purdy, John Smith, D.S O., M.D., C.M. (Aberd.), D P.H. (Cantab.), F.R.G.S., Town Hall, Sydney, N.S.W , Australia	
1920	C.	* Purser, George Leslie, M A. (Cantab.), F.Z.S., Lecturer in Embryology, University of Aberdeen	

Date of Election			Service on Council etc
1898		Purves John Archibald D Sc Chilliswood Trull Taunton	
1899	C	Ramage, Alexander G Lochcote Linlithgowshire	
1904		Ratcliffe, Joseph Riley, M B C M c/o The Librarian University Birmingham	
1900		Raw Nathan C M G M D 30 Clarendon Court Maida Vale London W 9	
1927	C	* Read Herbert Harold D Sc (Lond) A R C S F G S George Herdman Professor of Geology University of Liverpool	
1929		* Read Selwyn B A Schoolmaster Edinburgh Academy 2 Oxford Terrace Edinburgh 4	
1902		Rees Roberts John Vernon M D D Sc D P H 90 Fitzjohns Avenue Hampstead London N W 3	
1913		Reid Harry Avery O B E F R C V S D V H Bacteriologist and Pathologist Department of Agriculture Wellington New Zealand C/o Bank of New Zealand 1 Queen Victoria Street London E C	
1924		* Reid William Carstairs Civil Engineer 23 Saxe Coburg Place Edinburgh 3	
1914		Renshaw Graham M D M R C S L R C P L S A Editor of the *Avicultural Magazine* Sale Bridge House Sale Manchester	
1913		* Richardson Harry M Inst E E M Inst M E 16 Stratford Place London W 1	
1908		Richardson Linsdall F G S 104 Greenfield Road Harborne Birmingham	
1927		* Richey James Ernest B A B A I (Trinity College Dublin) F G S District Geologist H M Geological Survey (Scotland) 19 Grange Terrace Edinburgh 9	
1930		* Ritchie Allan Watt Chief Sanitary Inspector City of Edinburgh 2 Queensferry Terrace Edinburgh 4	
1916	C	* Ritchie James M A D Sc (VICE PRESIDENT) Regius Professor of Natural History University of Aberdeen	1921–24, 1926–28 Sec 1928–31 V P 1931–
1914	C	* Ritchie James Bonnyman D Sc Rector The Academy Ayr 28 Carrick Road Ayr	
1906	C	Ritchie William Thomas M D F R C P E Professor of Medicine University of Edinburgh (Teviot Place) 10 Douglas Crescent Edinburgh 12	
1929		* Robb Richard Alexander M A B Sc M Sc Lecturer in Mathematics University of Glasgow 19 Blairhall Avenue Langside Glasgow S 1	
1931		* Robb William N D A Director of Research Scottish Society for Research in Plant Breeding Craigs House Corstorphine Midlothian	
1898	C	Roberts Hon Alexander William D Sc F R A S Lovedale South Africa	
1919		* Roberts Alfred Henry O B E M Inst C E Superintendent and Engineer Leith Docks 40 Buckingham Terrace Edinburgh 4	
1926		* Roberts John Alexander Fraser M A (Cantab) D Sc Stoke Park Colony Stapleton Bristol	
1928	C	* Roberts Owen Fiennes Temple M C M A (Cantab) Lecturer in Astronomy and Meteorology University of Aberdeen 20 Belgrave Terrace Aberdeen	
1902	C	Robertson Robert A M A B Sc Professor of Botany University of St Andrews	
1919		* Robertson William Alexander F F A Century Insurance Co Ltd 18 Charlotte Square Mardale 3 Buckstone Park Edinburgh 10	
1896	C	Robertson W G Aitchison D Sc M D F R C P E Barrister at Law Lincoln s Inn St Margarets St Valerie Road Bournemouth	
1932	C	* Robson John Michael M D B Sc Beit Fellow Institute of Animal Genetics University of Edinburgh (West Mains Road) Tayinloan Loanhead	
1926		* Romanis William Hugh Cowie M A M B M C (Cantab) F R C S Surgeon to St Thomas s Hospital London 120 Harley Street London W 1	
1916		* Ronald David M Inst C E Chief Engineer Scottish Board of Health 125 George Street Edinburgh 2	
1909	C	Ross Alex David M A D Sc F Inst P F R A S Professor of Physics University of Western Australia Perth Western Australia	
1921		* Ross Edward Burns M A lately Professor of Mathematics Madras Christian College Madras 41 Liberton Brae Edinburgh 9	
1931	C	* Ruse Harold Stanley M A (Oxon) D Sc Lecturer in Mathematics University of Edinburgh 12 North Street St Leonards on Sea Sussex	
1906		Russell Alexander Durie B Sc Mathematical Master Falkirk High School 14 Heugh Street Falkirk	

Date of Election			Service on Council, etc.
1930		Russell, David, LL.D., Paper Manufacturer Silverburn, Leven, Fife	
1902	C. K.	Russell, James, 22 Glenorchy Terrace, Edinburgh 9	
1925	C.	* Saddler, William, M.A , B A., Professor of Mathematics, Canterbury College, Christchurch, N Z.	
1906		Saleeby, Caleb Williams, M.D., 13 Greville Place, Hampstead, London, N.W. 6	
1916	C.	* Salvesen, The Rt Hon. Lord, P C , K.C , LL D., Judge of the Court of Session (retired), Dean Park House, Edinburgh 4	1920–22. V-P 1922–25.
1914		* Salvesen, Theodore Emile, F.R S A , F S A Scot , Chevalier de la Légion d'Honneur. 37 Inverleith Place, Edinburgh 4	
1912	C. K.	* Sampson, Ralph Allen, M.A., D.Sc , LL D , F R S (VICE-PRESIDENT), Astronomer Royal for Scotland, Professor of Astronomy, University of Edinburgh Royal Observatory, Edinburgh	1912–15, 1919–21. V-P 1915–18, 1933– Sec 1922–23. Gen Sec. 1923–33.
1903		Samuel, Sir John S., K.B E , D.L , J.P , F S A Scot , 13 Park Circus, Glasgow	
1927	C.	* Sandeman, Ian, M.A , B Sc., Ph.D. (St Andrews), Acting Chief Inspector of Schools, Education Department, Colombo, Ceylon	
1930		* Sansome, Frederick Whalley, Ph D., F L S , Assistant, John Innes Horticultural Institution, Merton, London Old Garden, Church Lane, Merton Park	
1922		* Sarkar, Bijali Behari, M.Sc , D Sc (Edin), Lecturer in Physiology, University, Calcutta. 33/3 Lansdowne Road, Calcutta	
1903		Sarolea, Charles, Ph D., D Litt , formerly Professor of French, University of Edinburgh. 21 Royal Terrace, Edinburgh 7	
1927		* Schlapp, Robert, M A (Edin), Ph D. (Cantab), Lecturer in Applied Mathematics, University of Edinburgh (16 Chambers Street) 1 Peel Terrace, Edinburgh 9	
1885	C.	† Scott, Alexander, M A , D.Sc , F R S , Director of Scientific Research at the British Museum. 34 Upper Hamilton Terrace, London, N W 8	
1919		* Scott, Alexander, M A., D.Sc , 3 Winton Terrace, Stoke-on-Trent	
1919		* Scott, Alexander Ritchie, B Sc. (Edin), D Sc (Lond), Principal, London County Council, Beaufoy Institute, Prince's Road, London, S E 11	
1917		* Scott, Henry Harold, M.D , F R C P , M R C S , D P H., Assistant Director, Bureau of Hygiene and Tropical Diseases, Keppel Street, Gower Street, London, W C. 1. 23 Gordon Mansions, London, W.C 1	
1928		* Senior-White, Ronald, F R.E S , Malariologist, Bengal-Nagpur Railway, Kidderpore, P O., Calcutta, India	
1930		* Shankland, Ernest Claud, F R Met S , River Superintendent, Port of London Authority Mariners, Balfour Gardens, Folkestone	
1900	C. N.	Sharpey-Schafer, Sir Edward Albert, Kt , M D , D Sc , LL D , F.R S. (PRESIDENT), Corresponding Member of the French Academy of Medicine, Emeritus Professor of Physiology, University of Edinburgh. Park End, North Berwick	1900–03, 1906–09, 1918–19. V-P 1913–17. P 1929–
1927		* Sharpley, Forbes Wilmot, B Sc. (Eng) (Lond), M Inst E E., Professor of Electrical and Mechanical Engineering, Indian School of Mines, Dhanbad, India	
1931		* Shaw, John James M'Intosh, M.A , M.D , F.R C S , Lecturer in Surgery and Clinical Surgery, University of Edinburgh. Greenaway, Kinnear Road, Edinburgh 4	
1927		* Shearer, Ernest, M.A., B.Sc. (Edin.), Professor of Agriculture and Rural Economy, Edinburgh University, and Principal, Edinburgh and East of Scotland College of Agriculture, 13 George Square, Edinburgh 8	
1931		* Shearer, James Fleming, M.A , B.Sc., Ph.D., Lecturer in Natural Philosophy, University of Glasgow. 79 Queen's Drive, Wavertree, Liverpool 15	
1932		* Sheppard, William Fleetwood, Sc.D., LL.M., formerly a Senior Examiner to the Board of Education. Cardrona, Berkhamsted, Herts	

Date of Election			Service on Council, etc.
1932		* Simpson, Alexander Rudolf Barbour, B Sc (Edin.), M.A. (Cantab.), F.R.G.S., Senior Geography Master, Canford School. Beaufort House, Canford School, Wimborne, Dorset	
1908		Simpson, George Freeland Barbour, M D., F.R.C.P E., F.R.C.S.E., J.P., 43 Manor Place, Edinburgh 3	
1900	C.	† Simpson, James Young, M.A , D.Sc., D.Jur., Professor of Natural Science, New College, Edinburgh. 25 Chester Street, Edinburgh 3	1922–25.
1932	C	* Simpson, John Baird, B.Sc. (Aberd.), Senior Geologist, H.M. Geological Survey (Scotland), 19 Grange Terrace, Edinburgh 9	
1900		Sinhjee, Sir Bhagvat, G.C I.E., M D., LL.D. (Edin), H.H. the Thakur Sahib of Gondal, Kathiawar, Bombay, India	
1903		† Skinner, Robert Taylor, M.A., J.P., House Governor, Donaldson's Hospital, Edinburgh 12	
1930	C	* Slater, Robert Henry, D Sc., Ph D. (Edin), Department of Chemical Pathology, St Mary's Hospital, London, W. 2	
1929		* Small, James Cameron, A Inst.E.E., Principal, Heriot-Watt College, Edinburgh 1 Grange Terrace, Edinburgh 9	
1926		* Small, James, D Sc , Professor of Botany, Queen's University, Belfast. Ardcolm, Knock, Belfast	
1901		Smart, Edward, B A , B.Sc., Tillyloss, Tullylumb Terrace, Perth	
1920		* Smellie, William Robert, M.A , D Sc , Geologist on the Staff of the Anglo-Persian Oil Company Baron Cliff, Cove, Dumbartonshire	
1928		Smith, Alick Drummond Buchanan, M.A , B Sc (Agric.) (Aberd.), M.S A. (Iowa), Lecturer, Institute of Animal Genetics, University of Edinburgh (West Mains Road)	
1921		* Smith, Norman Kemp, M.A , D Phil., D Litt , LL D , Professor of Logic and Metaphysics, University of Edinburgh (South Bridge). Ellerton, Grange Loan, Edinburgh 9	
1923		* Smith, Percy James Lancelot, M A (Oxon), F I C , F C.S , Science Master, Loretto School 47 Dalrymple Loan, Musselburgh	
1911		* Smith, Stephen, B.Sc , 34 Craigmillar Park, Edinburgh 9	
1929		* Smith, Sydney, M D , F R C P , D P.H , Professor of Forensic Medicine, University of Edinburgh (Teviot Place) 10 Oswald Road, Edinburgh 9	
1907	C.	† Smith, William Ramsay, D Sc , M D , C M , Permanent Head of the Health Department, South Australia Belair, South Australia	
1919		* Smith, Sir William Wright, Kt , M A., D ès Sc , Regius Professor of Botany, University of Edinburgh, Regius Keeper of the Royal Botanic Garden, and King's Botanist in Scotland Inverleith House, Edinburgh 4	Sec. 1923–28. V.P 1928–31.
1932		* Sneeden, Jean-Baptiste Octave, B.Sc , Ph D (Glas.), Lecturer on Heat Engines, Royal Technical College, Glasgow 39 Kingshouse Avenue, Cathcart, Glasgow	
1899		Snell, Ernest Hugh, M D , B.Sc., D P H (Cantab), Barrister-at-Law, late Medical Officer of Health, Coventry. 3 Eaton Road, Coventry	
1880		Sollas, William Johnson, M A , D Sc , LL D , F.R.S., Professor of Geology and Palæontology, University of Oxford 104 Banbury Road, Oxford	
1933		* Somerville, John Livingston, C.A , Auditor to the University of Edinburgh. Beechcroft, Ravelston Dykes, Edinburgh	
1910		* Somerville, Robert, B.Sc., Lindrum, Queensferry Road, Dunfermline	
1911	C	* Sommerville, Duncan M'Laren Young, M.A , D Sc., Professor of Pure and Applied Mathematics, Victoria College, Wellington, New Zealand	
1929		* Southwell, Thomas, D Sc , A R.C S., Lecturer in Helminthology, School of Tropical Medicine, Liverpool 8 Waverley Road, Sefton Park, Liverpool	
1925		* Staig, Robert Arnot, M.A , Ph.D , Lecturer in Zoology, University of Glasgow. Glenlea, Lasswade, Midlothian	
1891		Stanfield, Richard, A.R.S.M., M Inst.C.E., Emeritus Professor of Mechanics and Engineering, Heriot-Watt College, Edinburgh. 24 Mayfield Gardens, Edinburgh 9	1926–29.
1923		* Stebbing, Edward Percy, M.A., Professor of Forestry, University of Edinburgh (George Square)	
1885 & 1915		Steggall, John Edward Aloysius, M.A., Hon. A R.I.B.A., LL.D., Emeritus Professor of Mathematics, University College, Dundee (St Andrews University). Woodend, Perth Road, Dundee	
1923		* Stenhouse, Andrew G., F.G.S., 191 Newhaven Road, Edinburgh 6	

Date of Election			Service on Council, etc.
1929	C.	* Stephen, Alexander Charles, B.Sc., Assistant, Natural History Department, Royal Scottish Museum, Edinburgh. "Eastcroft," Cramond Bridge, Edinburgh 4	
1910		* Stephenson, Thomas, D.Sc., F.C.S., 13 Glencairn Crescent, Edinburgh 12	
1931		* Steven, George Alexander, B.Sc. (Edin.), Assistant Naturalist, Marine Laboratory, Plymouth. 1 Lipson Terrace, Plymouth, Devon	
1925		* Stevens, Alexander, M.A., B.Sc., Lecturer in Geography, University of Glasgow	
1886	C.	Stevenson, Charles A., B.Sc., M.Inst.C.E., Radella, North Berwick	
1884		† Stevenson, David Alan, B.Sc., M.Inst.C.E., Troqueer, Kingsknowe, Colinton, Midlothian	1928–31.
1919		* Stevenson, David Alan, B.Sc., M.Inst.C.E., 22 Glencairn Crescent, Edinburgh 12	
1931		* Stewart, Corbet Page, Ph.D. (Edin.), Lecturer in General Biochemistry, University of Edinburgh. 150 Craigleith Road, Edinburgh 4	
1925		* Stewart, David Smith, Ph.D., M.Inst.C.E., Lecturer on Structural Engineering Drawing, University of Edinburgh (Sanderson Engineering Laboratories, Mayfield Road). 82 Lasswade Road, Liberton, Edinburgh 9	
1924		* Stiles, Sir Harold Jalland, K.B.E., M.B., F.R.C.S.E., LL.D., Emeritus Professor of Clinical Surgery, University of Edinburgh. Whatton Lodge, Gullane, E. Lothian	
1902		Stockdale, Herbert Fitton, LL.D., formerly Director of the Royal Technical College, Glasgow. Clairinch, Upper Helensburgh, Dumbartonshire	
1889	C.	Stockman, Ralph, M.D., LL.D., F.R.C.P.E., F.F.P.S.G., Professor of Materia Medica and Therapeutics, University of Glasgow	1903–05.
1926		* Stokoe, William Norman, B.Sc., Ph.D. (Lond.), Chief Chemist, Craigmillar Creamery Co., Ltd. 8 Cobden Road, Edinburgh 9	
1906		Story, Fraser, O.B.E., formerly Professor of Forestry, University College, Bangor, North Wales. 4K Artillery Mansions, Victoria Street, London, S.W. 1	
1907		Strong, John, C.B.E., M.A., LL.D., Emeritus Professor of Education, University of Leeds. Devonshire Hall, Headingley, Leeds	
1930	C.	* Stump, Claude Witherington, M.D., D.Sc., Professor of Embryology and Histology, University of Sydney	
1903		Sutherland, David W., C.I.E., M.D., M.R.C.P., Lt.-Col., I.M.S. (retired), Braeside, Belhaven, Dunbar	
1930		* Sutherland, John Donald, C.B.E., LL.D., F.S.I., Chevalier of the Legion of Honour, Assistant Commissioner, Forestry, Scotland. 11 Inverleith Row, Edinburgh 4	
1925		Sutton, Richard L., M.D., D.Sc., LL.D., Professor of Diseases of the Skin, School of Medicine, University of Kansas, U.S.A.	
1932		* Swinton, William Elgin, B.Sc., Ph.D. (Glas.), F.L.S., F.Z.S., F.G.S., Curator of Fossil Reptiles and Amphibia, British Museum (Natural History), South Kensington, London, S.W. 7	
1917	C. N.	* Tait, John, D.Sc., M.D., Professor of Physiology, McGill University, Montreal, Canada	
1933		* Tait, John Barclay, B.Sc. (Edin.), Ph.D., A.H.-W.C., Edinburgh, Junior Naturalist (Hydrographer), Marine Laboratory, Fishery Board for Scotland, Aberdeen. 23 Cromwell Road, Aberdeen	
1890	C.	† Tanakadate, Aikitu, Hon. Professor of Natural Philosophy, Imperial University of Japan. Koisikawa, Zôsigayamati, 144, Tokyo, Japan	
1870		Tatlock, Robert R., F.C.S., City Analyst's Office, 156 Bath Street, Glasgow	
1899		Taylor, James, M.A., formerly Mathematical Master, Edinburgh Academy. 18 Hillview, Blackhall, Edinburgh 4	
1933		* Taylor, George, B.Sc. (Edin.), Assistant Keeper, Department of Botany, British Museum. Ballochmyle, Loudwater, Rickmansworth, Herts	
1885	C.	Thompson, D'Arcy Wentworth, C.B., M.A., LL.D., Hon. D.Sc. (Witwatersrand), D.Litt., F.R.S. (CURATOR OF LIBRARY AND MUSEUM), Professor of Natural History, University, St Andrews. 44 South Street, St Andrews	1892–95, 1896–99, 1907–10, 1912–15, 1922–25. V.-P. 1916–19. Curator 1926–

Date of Election			Service on Council etc
1932		Thompson Harold William D Litt (Edin) A M Ph D (Harvard) F S A Scot Professor of English N Y State College for Teachers 544 Providence Street Albany N Y U S A	
1917	C N	* Thompson John M Lean M A D Sc F L S Professor of Botany University of Liverpool	
1896		Thomson George Ritchie C M G M B C M formerly Professor of Surgery University of the Witwatersrand Johannesburg Transvaal Hordle Grange Hordle Hants	
1903		Thomson George S Kenmore Farm Whelpley Hill Chesham Bucks	
1906		Thomson Gilbert M A M Inst C E 164 Bath Street Glasgow C 2	
1926		* Thomson Godfrey Hilton D Sc Ph D Professor of Education University of Edinburgh (Moray House)	1931–
1926	C	* Thomson John M A Ph D (Glas) Lecturer in Plant Physiology University of Glasgow 2 Chartwell Terrace Bearsden Glasgow	
1899		Thomson R Tatlock F C S 156 Bath Street Glasgow	
1912	C	Thomson Robert Black M B (Edin) Aliwal North Cape Province S A	
1882		Thomson Sir William Kt M A B Sc LL D formerly Principal University of the Witwatersrand Dunedin Glencairn Simonstown South Africa	
1917		* Thorneycroft Wallace J P Strete Ralegh Whimple Exeter Devon	
1933		* Timms Geoffrey Ph D (Cantab) Lecturer in Mathematics University of St Andrews Deanscourt St Andrews	
1920		* Todd John Barber B Sc Ph D M I Mech E Reader in Engineering University of Edinburgh 98 Findhorn Place Edinburgh 9	
1917		* Tovey Donald Francis B A (Oxon) M Mus (Hon) Birmingham Professor of Music University of Edinburgh (Reid School of Music) 39 Royal Terrace Edinburgh 7	
1914		† Tredgold Alfred Frank M D (Durham) F R C P (Lond) Lecturer on Mental Deficiency University of London St Martins Guildford	
1915		* Trotter George Clark M D (Edin) D P H (Aberdeen) Medical Officer of Health Metropolitan Borough Islington Braemar 17 Haslemere Road Crouch End London N 8	
1922	C K	* Turnbull Herbert Westren M A F R S Professor of Mathematics University of St Andrews 2 Queens Terrace St Andrews	1928–31
1905		Turner Arthur Logan M D LL D F R C S E 27 Walker Street Edinburgh 3	1926–29 V P 1930–33
1925		* Turner Harry Moreton Stanley M B E M D M R C S L R C P D T M and H Chevalier de l Ordre Royale du Sauveur de Grèce Wing Commander R A F (retired) 65 Stert Street Abingdon on Thames near Oxford	
1924		* Turner Richard O B E M B C M Hotel Hydropathic Peebles	
1895		Turton Albert H M Inst M M 233 George Road Erdington Birmingham	
1918	C	* Tyrrell G W A R C S D Sc F G S Lecturer in Petrology Geological Department University of Glasgow	1926–29
1910		Vincent Swale M D (Lond) D Sc (Edin) Professor of Physiology University of London 15 Fishpool Street St Albans Herts	
1930	C	* Voge Cecil Innes Bothwell Ph D (Edin) London School of Hygiene and Tropical Medicine Keppel Street (Gower Street) London W C 1	
1932		* Wade Henry C M G D S O M D Senior Lecturer in Clinical Surgery University of Edinburgh (Royal Infirmary) 6 Manor Place Edinburgh 3	
1926		* Wakeley Cecil Pembrey Grey F R C S Surgeon to King s College Hospital London Lecturer in Anatomy King s College London 24 Queen Anne Street Cavendish Square London W 1	
1925	C	* Walker Frederick M A Ph D D Sc Lecturer in Geology University of St Andrews	
1891	C M B V J	Walker Sir James Kt D Sc Ph D LL D F R S Emeritus Professor of Chemistry University of Edinburgh 5 Wester Coates Road Edinburgh 12	1903–05 1910–13 1922–25 1928–31 V P 1916–19
1931		* Walker William James Ph D (Edin) Research Chemist H M Fuel Research Station East Greenwich London S E 10 C/o Harrison 64 Sandtoft Road Charlton London S E 7	
1902		Wallace Alexander G M A 56 Fonthill Road Aberdeen	

Date of Election			Service on Council etc
1886	C	Wallace Robert M A LL D F L S Emeritus Professor of Agriculture and Rural Economy University of Edinburgh C/o Mrs M Call 11 Bruntsfield Crescent Edinburgh 10	
1898		Wallace Wm M A Campsie Alta Canada	
1920		* Walmsley Thomas M D (Glas) Professor of Anatomy Queen s University Belfast	
1931	C	* Walton John M A (Cantab) D Sc (Manchester) Regius Professor of Botany University of Glasgow 23 Lilybank Gardens Glasgow W 2	
1927	C	* Wardlaw Claude Wilson D Sc (Glas) Imperial College of Tropical Agriculture Trinidad B W I	
1923		* Warren John Alexander M Inst C E 74 Balshagray Avenue Partick	
1901	C	Waterston David M A M D F R C S E Professor of Anatomy University of St Andrews	1916–19 1925 28
1927		* Watson Charles Brodie Boog F S A Scot 24 Carscube Terrace Edinburgh 12	
1923		* Watson H Ferguson M D F R F P S Ph D D P H (Glas) H M Senior Deputy Commissioner General Board of Control for Scotland 25 Palmerston Place Edinburgh 12	
1923	C	* Watson William M A B Sc (Edin) Lecturer in Physics Heriot Watt College Edinburgh 17 Braidburn Crescent Edinburgh 10	
1911		† Watt James W S F F A LL D (TREASURER) 7 Blackford Road Edinburgh 10	1924 26 Treasurer 1926–
1933		* Watt John Mitchell M B Ch B (Edin) Professor of Pharmacology University of the Witwatersrand Johannesburg South Africa	
1911		* Watt Right Rev Lauchlan MacLean D D LL D The Cathedral Glasgow 1 Athole Gardens Glasgow W 2	
1928		* Watters Alexander Marshall M A B Sc (Glas) Rector of Hawick High School High School House Hawick	
1896		† Webster John Clarence B A M D F R C P E Professor of Obstetrics and Gynæcology Rush Medical College Shediac N B Canada	
1907	M B C	† Wedderburn Ernest Maclagan M A D Sc LL B W S Professor of Conveyancing University of Edinburgh (South Bridge) 6 Succoth Gardens, Edinburgh 12	1913 16 1921 4 1932
1903	M B C	† Wedderburn J H Maclagan M A D Sc F R S Professor of Mathematics Princeton University Fine Hall Princeton N J U S A	
1904		Wedderspoon William Gibson M A LL D	
1930		* White Adam Cairns M B Ch B Ph D Assistant Pharmacologist Wellcome Physiological Research Laboratory Beckenham Kent	
1933		* Whitley William Frederic James M D (Edin) D P H (Oxon) Medical Officer of Health Northumberland County Council Westfield Cramlington Northumberland	
1931		* Whitson Sir Thomas Barnby D L C A LL D Lord Provost of the City of Edinburgh (1929–32) 27 Eglinton Crescent Edinburgh 12	
1911		* Whittaker Charles Richard F R C S E F S A Scot Lynwood Hatton Place Edinburgh 9	
1912	C V J B P	* Whittaker Edmund Taylor M A Hon Sc D (Dubl) LL D F R S Foreign Member of the R Accademia dei Lincei Rome Professor of Mathematics University of Edinburgh (16 Chambers Street) 48 George Square Edinburgh 8	1912–15 1922 25 Sec 1916–22 V P 1925 28
1928	C	* Whittaker John Macnaghten M A (Edin) M A (Cantab) D Sc Professor of Pure Mathematics University of Liverpool	
1918		* Whyte Rev Charles M A LL D F R A S U F Church Manse Kingswells Aberdeen	
1929	C	* Wiesner Bertold Paul D Sc Macaulay Lecturer Institute of Animal Genetics University of Edinburgh (West Mains Road)	
1918		* Wight John Thomas M I Mech E M I Mar E Joint Managing Director Messrs MacTaggart Scott & Co Ltd Loanhead Calderwood Villa Lasswade	
1925		* Wilkie David Percival Dalbreck O B E M D Ch M F R C S Professor of Surgery University of Edinburgh (Royal Infirmary) 9 Ainslie Place Edinburgh 3	
1926	C	* Williams Samuel Ph D Lecturer in Plant Morphology University of Glasgow 27 Lindsay Place Kelvindale Glasgow	

Date of Election			Service on Council, etc.
1908		Williamson, Henry Charles, M.A., D Sc, formerly Naturalist to the Fishery Board for Scotland. 11 St Mary's Road, Downfield, Dundee	
1928	C.	* Williamson, John, M.A. (Edin.), Ph.D. (Chicago), Associate Professor of Mathematics, Johns Hopkins University, Baltimore, U.S.A.	
1910	C	* Williamson, William, F.L.S., 7 Ventnor Terrace, Edinburgh 9	
1927	C	* Williamson, William Turner Horace, B.Sc. (Aberd.), Ph.D. (Edin.), Chief Chemist, Egyptian Ministry of Agriculture, Cotton Research Board, Giza, Egypt	
1900		Wilson, Alfred C, Bloomfield House, Sadberge, near Darlington	
1911		* Wilson, Andrew, O.B.E., D.L., M.Inst C.E, Master of the Merchant Company, 66 Netherby Road, Edinburgh 5	
1902	V J	† Wilson, Charles T. R, M A., LL D, F.R S., Nobel Prize, Physics, 1927, Jacksonian Professor of Natural Philosophy, University of Cambridge. Glencorse, Storey's Way, Cambridge	
1922		* Wilson, John, F R I.B.A., Chief Architect, Scottish Department of Health 20 Lomond Road, Edinburgh 5	
1920	C.	* Wilson, Malcolm, D Sc (London), A R C.S, F L S, Reader in Mycology and Bacteriology, University of Edinburgh (Royal Botanic Garden) Brent Knoll, Kinnear Road, Edinburgh 4	1931–
1924		* Wilson, William, M.A, LL.B., Advocate, Regius Professor of Public Law, University of Edinburgh (South Bridge). 38 Moray Place, Edinburgh 3	
1895		Wilson-Barker, Sir David, Kt, R D, R N.R., F R.G.S., formerly Captain-Superintendent, Thames Nautical Training College, H.M.S "Worcester." 12 Bolan Street, London, S.W 11	
1931	C.	* Wishart, John, M A, B Sc (Edin.), M.A. (Cantab.), D.Sc. (Lond.), Reader in Statistics, University of Cambridge Astraea, 18 Storey's Way, Cambridge	
1922	C. B.	* Wordie, James Mann, M A (Cantab), B.Sc. (Glas), 52 Montgomery Drive, Glasgow	
1933		* Wright, James, F.G S., Bellevue, Beveridge Road, Kirkcaldy	
1890		Wright, Johnstone Christie, Conservative Club, Edinburgh 2	
1890		† Wright, Sir Robert Patrick, LL D, formerly Chairman of the Board of Agriculture for Scotland The Heugh, North Berwick, East Lothian	
1911	C.	* Wrigley, Runic Whitehead, M A (Cantab), Assistant Astronomer, Royal Observatory, Edinburgh	
1882		Young, Frank W, C.B.E, F.C.S, H M. Inspector of Schools (Emeritus). 35 Pentland Terrace, Edinburgh 10	
1904		Young, R B, M A., D.Sc, F.G.S., Professor of Geology and Mineralogy, South African School of Mines and Technology, Johannesburg, Transvaal	

Number of Fellows, 722.

HONORARY FELLOWS OF THE SOCIETY.

(*At 23rd October* 1933.)

HIS ROYAL HIGHNESS THE PRINCE OF WALES.

Foreigners (limited to thirty-six by Law I).

Elected

1933 John Jacob Abel, Professor of Pharmacology, Johns Hopkins University, Baltimore.
1916 Charles Eugène Barrois, formerly Professor of Geology and Mineralogy, Université, Lille, France 41, Rue Pascal, Lille.
1923 Henri Bergson, Honorary Professor, College of France, Paris.
1930 Vilhelm Frimann Koren Bjerknes, Professor of Physics, Geophysical Institute, Bergen.
1927 Niels Bohr, Nobel Laureate, Physics, 1922, Professor of Physics, University of Copenhagen.
1927 Jules Bordet, Nobel Laureate, Medicine, 1919, Professor of Bacteriology, University of Brussels
1933 Filippo Bottazzi, Professor of Experimental Physiology, Royal Institute of Physiology, S Andrea delle Dame, 21, Naples.
1923 Marcellin Boule, Professor at the National Museum of Natural History, Laboratory of Palæontology, 3 Place Valhubert, Paris 5e.
1905 Waldemar Christofer Brogger, Professor of Mineralogy and Geology, K. Frederiks Universitet, Oslo
1916 Douglas Houghton Campbell, Emeritus Professor of Botany, Stanford University, California.
1920 William Wallace Campbell, President-Emeritus of the University of California, Berkeley, Director-Emeritus of the Lick Observatory, Mt Hamilton, California, and President of the National Academy of Sciences.
1930 Walter Bradford Cannon, Professor of Physiology, Harvard University, Cambridge, Mass
1930 Maurice Caullery, Professor of Zoology in the University of Paris Laboratoire d'Évolution des Êtres organisés, 105 Bould. Raspail, Paris, VIe.
1933 Edwin Grant Conklin, Professor of Biology, Princeton University, N.J.
1921 Reginald Aldworth Daly, Professor of Geology, Harvard University, Cambridge, Mass.
1927 Albert Einstein, Nobel Laureate, Physics, 1921, Princeton University, N J.
1913 George Ellery Hale, Honorary Director of Mount Wilson Observatory (Carnegie Institution of Washington), Pasadena, California.
1921 Johan Hjort, Professor of Marine Biology, University, Oslo.
1923 Arnold Frederik Holleman, Professor of Organic Chemistry, University, Amsterdam. Boekensteijn Parkweg 7, Bloemendaal.
1933 Nikolaj Konstantinovic Koltzoff, formerly Professor of Zoology, State University, Moscow; Director of the Research Institute of Experimental Biology Moscow 64, Voronzovo Pole 6.
1923 Tullio Levi-Civita, Professor of Mathematics, Regia Università, Rome
1927 Hans Horst Meyer, Emeritus Professor of Pharmacology, University of Vienna.
1923 Arthur Amos Noyes, Institute of Technology, Pasadena, California.
1908 Henry Fairfield Osborn, Research Professor of Zoology, Columbia University, Honorary President, American Museum of Natural History, New York, and Senior Geologist, U.S. Geological Survey.
1908 Ivan Petrovitch Pavlov, Emeritus Professor of Physiology, Academy of Sciences, Inst. Exper. Med, Nobel Laureate, Physiology and Medicine, 1904. V.O., 7 ligne No. 2, Leningrad.
1933 Albrecht Penck, Geheimrat Professor (Emeritus) of Geography, Friedrich-Wilhelms-Universität, Berlin.
1920 Charles Émile Picard, Perpetual Secretary, Academy of Sciences, Paris.

Elected

1921 Salvatore Pincherle, Professor of Mathematics, University of Bologna.

1913 Santiago Ramón y Cajal, Nobel Laureate, Medicine, 1906, formerly Professor of Histology and Pathological Anatomy, University, Madrid.

1920 Charles Richet, Nobel Laureate, Medicine, 1913, Professor of Physiology, Faculty of Medicine, Paris.

1930 Erik Helge Oswald Stensiö, Professor, Royal Natural History Museum, Stockholm.

1913 Vito Volterra, Professor of Mathematical Physics, Regia Università, Rome.

1910 Hugo de Vries, Professor of Plant Anatomy and Physiology, Lunteren.

1927 Richard Willstätter, Nobel Laureate, Chemistry, 1915, Professor of Chemistry, University of Munich. Munich 27, Moehlstrasse 29.

1923 Edmund Beecher Wilson, Professor of Zoology, Columbia University, New York.

1933 Pieter Zeeman, Nobel Laureate, Physics, 1902, Professor of Physics, University, Amsterdam.

Total, 36.

BRITISH SUBJECTS (LIMITED TO TWENTY BY LAW I).

1927 Sir William Henry Bragg, O M , K.B.E , M.A., D Sc , LL.D., F.R.S., Nobel Laureate, Physics, 1915, Fullerian Professor of Chemistry, Royal Institution, London.

1930 Sir Arthur Stanley Eddington, Kt , M.A , Hon. D Sc , F R.S , Plumian Professor of Astronomy and Experimental Philosophy, University of Cambridge.

1927 Sir John Bretland Farmer, Kt., M A., D Sc , LL D., F R S , formerly Professor of Botany, Imperial College of Science and Technology, London

1900 Andrew Russell Forsyth, M.A , Sc D , LL.D , Hon. Math D , F.R S , Emeritus Professor of Mathematics, Imperial College of Science and Technology, London; formerly Sadleirian Professor of Pure Mathematics, University of Cambridge.

1910 Sir James George Frazer, O M , Kt , D C.L , LL D , Litt D., F R.S., Commandeur de la Légion d'Honneur. Trinity College, Cambridge.

1930 Sir William Bate Hardy, Kt , M A , F.R.S , Director of Food Investigation, Department of Scientific and Industrial Research 5 Grange Road, Cambridge.

1927 Sir Frederick Gowland Hopkins, Kt , M A , M.B., D.Sc , LL D., Pres. R S., Joint Nobel Laureate, Medicine, 1929, Professor of Biochemistry, University of Cambridge

1930 Sir Arthur Keith, Kt , M D., LL.D , F.R.S , Master, Buckston Browne Research Farm, Downe, Farnborough.

1913 Sir Horace Lamb, Kt , M.A., Sc.D., D.Sc , LL D., F.R.S , formerly Professor of Mathematics, University of Manchester. 6 Selwyn Gardens, Cambridge.

1910 Sir Joseph Larmor, Kt , M.A , D.Sc., LL D , D C.L., F.R.S., formerly Lucasian Professor of Mathematics, University of Cambridge St John's College, Cambridge.

1933 Sir George Macdonald, K.C B , LL D., formerly Secretary, Scottish Education Department 17 Learmonth Gardens, Edinburgh 4.

1930 Robert Robinson, D Sc , F.R S , Waynflete Professor of Chemistry, University of Oxford.

1921 Rt. Hon. Lord Rutherford of Nelson, O.M., M A., D Sc., B.A , LL D., Past Pres R.S., Nobel Laureate, Chemistry, 1908, Cavendish Professor of Experimental Physics, University of Cambridge.

1916 Sir Arthur Schuster, Kt., Sc.D., D.Sc , Dr. ès Sci (Geneva), LL.D , F.R.S., Honorary Professor of Physics, University of Manchester. Yeldall, Twyford, Berks.

1930 Dukinfield Henry Scott, M.A., D Sc., LL D., Ph D., F.R.S., formerly Honorary Keeper of the Jodrell Laboratory, Royal Botanic Gardens, Kew East Oakley House, Basingstoke, Hants

1933 Sir William Napier Shaw, Kt , Sc.D (Cantab.), LL.D., F.R S., formerly Director Meteorological Office. 10 Moreton Gardens, London, S.W. 5.

1908 Sir Charles Scott Sherrington, O M , G B E., M.A., D.Sc., M D., LL.D., Past Pres. R.S , Joint Laureate, Nobel Prize, Medicine, 1932, Waynflete Professor of Physiology, University of Oxford.

1905 Sir Joseph John Thomson, O M., Kt., D.Sc , LL.D , Past Pres. R.S., Nobel Laureate, Physics, 1906, formerly Cavendish Professor of Experimental Physics, University of Cambridge, Master of Trinity College, Cambridge.

Total, 18.

CHANGES IN FELLOWSHIP DURING SESSION 1932–1933.

FELLOWS OF THE SOCIETY ELECTED.

FREDERICK LATHAM ARNOT.
PRABODH CHANDRA BANERJEE.
JAMES LIVINGSTONE BEGG.
THOMAS ROBERT BOLAM.
FINLAY JAMES CAMERON.
JOHN IRVINE CARSWELL.
ARTHUR MELVILLE CLARK.
ARTHUR LANCELOT CRAIG-BENNETT.
EDMUND GILBERT DYMOND.
JOHN MAXWELL ERSKINE.
THOMAS FERGUSON.
CHARLES FORRESTER.
WILLIAM HOPE FOWLER.
AUGUSTUS WM DE ROHAN GALBRAITH.
ALFRED CHARLES GARDNER.
ROBERT POLLOCK GILLESPIE.
THOMAS HAINING GILLESPIE.
WILLIAM GILMOUR GUTHRIE.
EDGAR ERSKINE HUME.
DAVID LEES.
JAMES MACLEOD.
JOHN MALCOLM
WILLIAM JOHN MILNE MENZIES.
LOUIS MELVILLE MILNE-THOMSON.
JOHN MURRAY.
THOMAS NICOL.
CHARLES WYNFORD PARSONS.
ALFRED WILLIAM NOEL PILLERS.
FRANK ANDERSON BAILLIE PRESTON.
JOHN LIVINGSTON SOMERVILLE.
JOHN BARCLAY TAIT.
GEORGE TAYLOR
GEOFFREY TIMMS
JOHN MITCHELL WATT
WILLIAM FREDERIC JAMES WHITLEY.
JAMES WRIGHT.

HONORARY FELLOWS ELECTED.

FOREIGN.

JOHN JACOB ABEL.
FILIPPO BOTTAZZI.
EDWIN GRANT CONKLIN.
NIKOLAJ KONSTANTINOVIC KOLTZOFF
ALBRECHT PENCK
PIETER ZEEMAN.

BRITISH.

SIR GEORGE MACDONALD.
SIR WILLIAM NAPIER SHAW.

FELLOWS DECEASED.

FRANCIS ERNEST BRADLEY.
BANAWARI LAL CHAUDHURI (died 1931).
WILLIAM INGLIS CLARK.
SIR DUGALD CLERK.
WILLIAM GRANT CRAIB.
JOHN CUTHBERTSON.
WILLIAM HOPE FOWLER.
JOSEPH ROBERT FRASER.
FREDERICK GARDINER.
JOHN GLAISTER.
JOHN HENDERSON.
WILLIAM JACOB HOLLAND.
THEOPHILUS BULKELEY HYSLOP.
ROBERT T. A. INNES
ROBERT JARDINE
JOHN ALFRED JONES (died 1923).
SPENCER MORT.
RALPH RICHARDSON
JOHN STEPHENSON
JOHN W. TAIT.
JAMES EDWARD TALMAGE.
SIR J. ARTHUR THOMSON
JOHN MILLAR THOMSON
WILLIAM ARTHUR WILLIAMS.

FOREIGN HONORARY FELLOW DECEASED.

JOHANNES SCHMIDT.

BRITISH HONORARY FELLOW DECEASED.

JOHN EDWARD MARR.

FELLOWS RESIGNED.

GEORGE HECTOR PERCIVAL.
D. CLEGHORN THOMSON.
DUGALD M'LELLAN.

LAWS OF THE SOCIETY.

Adopted July 3, 1916; *amended December* 18, 1916.

LAW I, amended February 5, 1934.
,, VI, ,, ,, 7, 1921.
,, ,, ,, July 2, 1928.
,, VIII, ,, May 3, 1920.

LAW IX, amended May 3, 1920.
,, XIII, ,, May 3, 1920.
,, XIX, ,, June 16, 1924.

I.

THE ROYAL SOCIETY OF EDINBURGH, which was instituted by Royal Charter in 1783 for the promotion of Science and Literature, shall consist of Ordinary Fellows (hereinafter to be termed Fellows) and Honorary Fellows. The number of Honorary Fellows shall not exceed sixty-six, of whom not more than twenty-two may be British subjects, and not more than forty-four subjects of Foreign States.

Fellows only shall be eligible to hold office or to vote at any Meeting of the Society.

ELECTION OF FELLOWS.

II.

Each Candidate for admission as a Fellow shall be proposed by at least four Fellows, two of whom must certify from personal knowledge. The Official Certificate shall specify the name, rank, profession, place of residence, and the qualifications of the Candidate. The Certificate shall be delivered to the General Secretary before the 30th of November, and, subject to the approval of the Council, shall be exhibited in the Society's House during the month of January following. All Certificates so exhibited shall be considered by the Council at its first meeting in February, and a list of the Candidates approved by the Council for election shall be issued to the Fellows not later than the 21st of February.

III.

The election of Fellows shall be by Ballot, and shall take place at the first Ordinary Meeting in March. Only Candidates approved by the Council shall be eligible for election. A Candidate shall be held not elected, unless he is supported by a majority of two-thirds of the Fellows present and voting.

IV.

On the day of election of Fellows two scrutineers, nominated by the President, shall examine the votes and hand their report to the President, who shall declare the result.

V.

Each Fellow, after his election, is expected to attend an Ordinary Meeting, and sign the Roll of Fellows, he having first made the payments required by Law VI. He shall be introduced to the President, who shall address him in these words:—

> *In the name and by the authority of THE ROYAL SOCIETY OF EDINBURGH, I admit you a Fellow thereof.*

PAYMENTS BY FELLOWS.

VI.

Each Fellow shall, before he is admitted to the privileges of Fellowship, pay an admission fee of three guineas, and a subscription of three guineas for the year of election. He shall continue to pay a subscription of three guineas at the beginning of each session so long as he remains a Fellow.

Each Fellow who was elected subsequent to December 1916 and previous to December 1920 shall also pay a subscription of three guineas at the beginning of each session so long as he remains a Fellow.

Each Fellow who was elected previous to December 1916, and who has not completed his twenty-five annual payments, shall, at the beginning of each session, pay three guineas until his twenty-five annual payments are made. Each Fellow who has completed or shall complete his payments shall be invited to contribute one guinea per annum or to pay a single sum of ten guineas.

A Fellow may compound for the annual subscriptions by a single payment of fifty guineas, or on such other terms as the Council may from time to time fix.

VII.

A Fellow who, after application made by the Treasurer, fails to pay any contribution due by him, shall be reported to the Council, and, if the Council see fit, shall be declared no longer a Fellow. Notwithstanding such declaration all arrears of contributions shall remain exigible.

ELECTION OF HONORARY FELLOWS.

VIII.

Honorary Fellows shall be persons eminently distinguished in Science or Literature. They shall not be liable to contribute to the Society's Funds. Personages of the Blood Royal may be elected Honorary Fellows at any time on the nomination of the Council, and without regard to the limitation of numbers specified in Law I.

IX.

Honorary Fellows shall be proposed by the Council. The nominations shall be announced from the Chair at the First Ordinary Meeting after their selection. The names shall be printed in the circular for the last Ordinary Meeting of the Session, when the election shall be by Ballot, after the manner prescribed in Laws III and IV for the Election of Fellows.

EXPULSION OF FELLOWS.

X.

If, in the opinion of the Council, the conduct of any Fellow is injurious to the character or interests of the Society, the Council may, by registered letter, request him to resign. If he fail to do so within one month of such request, the Council shall call a Special Meeting of the Society to consider the matter. If a majority consisting of not less than two-thirds of the Fellows present and voting decide for expulsion, he shall be expelled by declaration from the Chair, his name shall be erased from the Roll, and he shall forfeit all right or claim in or to the property of the Society.

XI.

It shall be competent for the Council to remove any person from the Roll of Honorary Fellows if, in their opinion, his remaining on the Roll would be injurious to the character or interests of the Society. Reasonable notice of such proposal shall be given to each member of the Council, and, if possible, to the Honorary Fellow himself. Thereafter the decision on the question shall not be taken until the matter has been discussed at two Meetings of Council, separated by an interval of not less than fourteen days. A majority of two-thirds of the members present and voting shall be required for such removal.

MEETINGS OF THE SOCIETY.

XII.

A Statutory Meeting for the election of Council and Office-Bearers, for the presentation of the Annual Reports, and for such other business as may be arranged by the Council, shall be held on the fourth Monday of October. Each Session of the Society shall begin at the date of the Statutory Meeting.

XIII.

Meetings for reading and discussing communications and for general business, herein termed Ordinary Meetings, shall be held, when convenient, on the first and third Mondays of each month from November to July inclusive, with the exception that in January the meetings shall be held on the second and fourth Mondays.

XIV.

A Special Meeting of the Society may be called at any time by direction of the Council, or on a requisition to the Council signed by not fewer than six Fellows. The date and hour of such Meeting shall be determined by the Council, who shall give not less than seven days' notice of such Meeting. The notice shall state the purpose for which the Special Meeting is summoned; no other business shall be transacted.

PUBLICATION OF PAPERS.

XV.

The Society shall publish Transactions and Proceedings. The consideration of the acceptance, reading, and publication of papers is vested in the Council, whose decision shall be final. Acceptance for reading shall not necessarily imply acceptance for publication.

DISTRIBUTION OF PUBLICATIONS.

XVI.

Fellows who are not in arrear with their Annual Subscriptions and all Honorary Fellows shall be entitled gratis to copies of the Parts of the Transactions and the Proceedings published subsequently to their admission.

Copies of the Parts of the Proceedings shall be distributed by post or otherwise, as soon as may be convenient after publication; copies of the Transactions or Parts thereof shall be obtainable upon application, either personally or by an authorised agent, to the Librarian, provided the application is made within five years after the date of publication.

CONSTITUTION OF COUNCIL.

XVII.

The Council shall consist of a President, six Vice-Presidents, a Treasurer, a General Secretary, two Secretaries to the Ordinary Meetings (the one representing the Biological group and the other the Physical group of Sciences),* a Curator of the Library and Museum, and twelve ordinary members of Council.

ELECTION OF COUNCIL.

XVIII.

The election of the Council and Office-Bearers for the ensuing Session shall be held at the Statutory Meeting on the fourth Monday of October. The list of the names recommended by the Council shall be issued to the Fellows not less than one week before the Meeting. The election shall be by Ballot, and shall be determined by a majority of the Fellows present and voting. Scrutineers shall be nominated as in Law IV.

XIX.

The President may hold office for a period not exceeding five consecutive years; the Vice-Presidents, not exceeding three; the Secretaries to the Ordinary Meetings, not exceeding five; the General Secretary, the Treasurer, and the Curator of the Library and Museum, not exceeding ten; and ordinary members of Council, not exceeding three consecutive years; provided that the Treasurer may be re-elected for more than ten successive years in cases where the Council declares to the Society that an emergency exists.

XX.

In the event of a vacancy arising in the Council or in any of the Offices enumerated in Law XVII, the Council shall proceed, as soon as convenient, to elect a Fellow to fill such vacancy for the period up to the next Statutory Meeting.

---

* The Biological group includes Anatomy, Anthropology, Botany, Geology, Pathology, Physiology, Zoology; the Physical group includes Astronomy, Chemistry, Mathematics, Metallurgy, Meteorology, Physics.

POWERS OF THE COUNCIL.

XXI.

The Council shall have the following powers:—(1) To manage all business concerning the affairs of the Society. (2) To decide what papers shall be accepted for communication to the Society, and what papers shall be printed in whole or in part in the Transactions and Proceedings. (3) To appoint Committees. (4) To appoint employees and determine their remuneration. (5) To award the various prizes vested in the Society, in accordance with the terms of the respective deeds of gift, provided that no member of the existing Council shall be eligible for any such award. (6) To make from time to time Standing Orders for the regulation of the affairs of the Society. (7) To control the investment or expenditure of the Funds of the Society.

At Meetings of the Council the President or Chairman shall have a casting as well as a deliberative vote.

DUTIES OF PRESIDENT AND VICE-PRESIDENTS.

XXII.

The President shall take the Chair at Meetings of Council and of the Fellows. It shall be his duty to see that the business is conducted in accordance with the Charter and Laws of the Society. When unable to be present at any Meetings or attend to current business, he shall give notice to the General Secretary, in order that his place may be supplied. In the absence of the President his duties shall be discharged by one of the Vice-Presidents.

DUTIES OF THE TREASURER.

XXIII.

The Treasurer shall receive the monies due to the Society and shall make payments authorised by the Council. He shall lay before the Council a list of arrears in accordance with Rule VII. He shall keep accounts of all receipts and payments, and at the Statutory Meeting shall present the accounts for the preceding Session, balanced to the 30th of September, and audited by a professional accountant appointed annually by the Society.

DUTIES OF THE GENERAL SECRETARY.

XXIV.

The General Secretary shall be responsible to the Council for the conduct of the Society's correspondence, publications, and all other business except that which relates to finance. He shall keep Minutes of the Statutory and Special Meetings of the Society and Minutes of the Meetings of Council. He shall superintend, with the aid of the Assistant Secretary, the publication of the Transactions and Proceedings. He shall supervise the employees in the discharge of their duties.

DUTIES OF SECRETARIES TO ORDINARY MEETINGS.

XXV.

The Secretaries to Ordinary Meetings shall keep Minutes of the Ordinary Meetings. They shall assist the General Secretary, when necessary, in superintending the publication of the Transactions and Proceedings. In his absence, one of them shall perform his duties.

DUTIES OF CURATOR OF LIBRARY AND MUSEUM.

XXVI.

The Curator of the Library and Museum shall have charge of the Books, Manuscripts, Maps, and other articles belonging to the Society. He shall keep the Card Catalogue up to date. He shall purchase Books sanctioned by the Council.

ASSISTANT SECRETARY AND LIBRARIAN.

XXVII.

The Council shall appoint an Assistant Secretary and Librarian, who shall hold office during the pleasure of the Council. He shall give all his time, during prescribed hours, to the work of the Society, and shall be paid according to the determination of the Council. When necessary he shall act under the Treasurer in receiving subscriptions, giving out receipts, and paying employees.

ALTERATION OF LAWS.

XXVIII.

Any proposed alteration in the Laws shall be considered by the Council, due notice having been given to each member of Council. Such alteration, if approved by the Council, shall be proposed from the Chair at the next Ordinary Meeting of the Society, and, in accordance with the Charter, shall be considered and voted upon at a Meeting held at least one month after that at which the motion for alteration shall have been proposed.

Additions to the Library—Presentations, etc.—1932–1933.

Andrews, R. C. Natural History of Central Asia: The New Conquest of Central Asia. Vol. I. 4to. New York, 1933. (*Presented by the American Museum of Natural History.*)

Barton-Wright, E. Recent Advances in Plant Physiology. 8vo. London, 1930. (*Presented by Author.*)

Bibliographie de J. J. Berzelius. Par Arne Holmberg. Part 1– . 8vo. Stockholm, 1933. (*Presented.*)

British Journal of Physical Medicine. April 1933– . 4to. London, 1933. (*Presented.*)

Bulletin mensuel de la Société Linnéenne de Lyon. Année 1– . 8vo. Lyon, 1932. (*Exchange.*)

Carnegie Institution of Washington:—

No. 401. Paullin, Charles O. Atlas of the Historical Geography of the United States. By Charles O. Paullin. Edited by John K. Wright. Fol. Washington and New York, 1932.

No. 418. Contributions to Palæontology: Papers Concerning the Palæontology of California, Oregon, and the Northern Great Basin Province. 8vo. Washington, 1932.

No. 422. Merriam, J. C., and Chester Stock. The Felidæ of Rancho La Brea. 4to. Washington, 1932.

No. 426. Bingham, Harold C. Gorillas in a Native Habitat. 8vo. Washington, 1932.

No. 427. Castle, W. E., and Paul B. Sawin. Contributions to the Genetics of the Domestic Rabbit. 8vo. Washington, 1932.

No. 428. Davenport, Charles B. The Genetical Factor in Endemic Goiter. 8vo. Washington, 1932.

No. 429. Howard, Hildegarde. Contributions to Palæontology: Eagles and Eagle-like Vultures of the Pleistocene of Rancho La Brea. 8vo. Washington, 1932.

No. 430. Gray, L. C., and E. K. Thompson. History of Agriculture in the Southern United States to 1860. Vols. I and II. La. 8vo. Washington, 1933.

No. 431. Shattuck, G. C. The Peninsula of Yucatan: Medical, Biological, Meteorological, and Sociological Studies. 4to. Washington, 1933.

No. 432. Jochelson, W. History, Ethnology, and Anthropology of the Aleut. 4to. Washington, 1933.

No. 433. Contributions to Embryology. Vol. XXIII, Nos. 134–138. 4to. Washington, 1932.

No. 434. Steggerda, Morris. Anthropometry of Adult Maya Indians: A Study of their Physical and Physiological Characteristics. 8vo. Washington, 1932.

No. 439. Chaney, R. W., and E. I. Sanborn. The Goshen Flora of West Central Oregon. 4to. Washington, 1933.

No. 441. MacDougal, D. T., and Earl B. Working. The Pneumatic System of Plants. 8vo. Washington, 1933.

No. 442. Heidel, W. A. The Heroic Age of Science: Conception, Ideals, and Methods of Science among the Ancient Greeks. 8vo. Washington, 1933.

Supplementary Publications No. 5: The Magnetic Field of the Earth and its Atmosphere. Three lectures by J. A. Fleming, A. E. Kennelly, and J. Bartels. La. 8vo. Washington, 1932.

(*Exchange.*)

Christopher Columbus: Documents and Proofs of his Genoese Origin. English-German Edition. Fol. Genoa, 1932. (*Presented.*)

Congrès National d'Astronomie des 21–22–23 Juillet 1932. (Comité National Français d'Astronomie.) 8vo. Paris. (*Presented.*)

Études des Gîtes Minéraux de la France: Bassin Houiller de la Sarre et de la Lorraine. Par MM Émile Siviard et Edmond Friedel. Atlas. Fol. Paris, 1932. (*Presented.*)

Fraser, Lt.-Col. F. C. Odonata. Vol. I. 8vo. London, 1933. (Fauna of British India.) (*Presented by Secretary of State for India.*)

Harada, J. English Catalogue of Treasures in the Imperial Repository, Shōsoin. 8vo. Tokyo. (*Exchange: Imperial Household Museum, Tokyo.*)

Head, Barclay V. Guide to the Principal Coins of the Greeks from *c.* 700 B.C to A.D. 270. Based on the work of Barclay V. Head. (British Museum.) 8vo. London, 1932.

Huygens, Christiaan. Œuvres complètes. Tome 17. 4to. La Haye, 1932. (*Presented.*)

Index to the Proceedings (1905–1930) and to the Philosophical Transactions (1901–1930) of the Royal Society of London. La. 8vo. London, 1932. (*Purchased.*)

The Indian Forester. Vol. LIX, No. 1– , 1933. La. 8vo. Dehra Dun, 1933. (*Presented.*)

Journal de Biologie. Tome 1, Nos. 1–4, 1932–1933. 8vo. Moscow/Leningrad, 1932–1933. (*Presented by Professor N. K. Koltzoff.*)

Journal de Biologie Expérimentale. Tomes 2–7, 1926–1931. 8vo. Moscow/Leningrad, 1926–1931. (*Presented by Professor N. K. Koltzoff.*)

Journal of Science of the Hiroshima University. Series A: Mathematics, Physics, Chemistry. Vol. I– . Series B: Zoology. Vol. I– . La. 8vo. Hiroshima, 1930. (*Exchange.*)

Macpherson, Hector. Makers of Astronomy. 8vo. Oxford, 1933. (*Presented by Author.*)

Metron. International Review of Statistics. Vol. VI– . La. 8vo. Rome, 1926. (*Purchased.*)

Oriental Studies Freer Gallery of Art. Published by the Smithsonian Institution. No. 1– . 4to. Washington, 1933. (*Exchange.*)

Prévost, Georges. Tables de Fonctions Sphériques et de leurs Intégrals. 4to. Paris, 1933. (*Presented by Author.*)

Publications du Muséum National d'Histoire Naturelle. No. 1– . 4to. Paris, 1933. (*Exchange.*)

Publications of the Tashkent Astronomical Observatory. Vol. I– . 8vo. Tashkent, 1928. (*Exchange.*)

Publications: Visindafélag Islendinga (Societas Scientiarum Islandica.) 1923– . 8vo. Reykjavék. (*Exchange.*)

Records of the Auckland Institute and Museum. Vol. I– . 8vo. Auckland, 1930. (*Exchange.*)

Report of the British Association, 1932. York Meeting. 8vo. London, 1932. (*Presented by Dr James Watt.*)

Reports of the Institute of Labour. No. 1– . La. 8vo. Kurashiki, 1931. (*Exchange.*)

The Russian Journal of Eugenics. Tomes 1–8, 1922–1929. 8vo. Moscow/Leningrad, 1922–1929. (*Presented by Professor N. K. Koltsoff.*)

Scott, Sir Walter. The Letters of Sir Walter Scott. Edited by H. J. C. Grierson, Davidson Cook, W. M. Parker, and Others. Centenary Edition, 1932. Vols. I–IV, 1787–1817. 8vo. Constable & Co., London, 1932–33. (*Presented anonymously through Messrs Constable, London.*)

Shorter Oxford English Dictionary. Revised and Edited by C. T. Onions. 2 vols. 4to. Oxford, 1933. (*Purchased.*)

Terrestrial Magnetism. Vol. XXXVII, No. 1– . La. 8vo. Washington, 1932. (*Exchange.*)

Turner, A. Logan. History of the University of Edinburgh, 1883–1933. 4to. Edinburgh, 1933. (*Presented by the University of Edinburgh.*)

Verslag van de Herdenking van het derde eeuwfeest van de Universeteit van Amsterdam, 1932. Fol. Amsterdam, 1933. (*Presented.*)

Watt, R. A. Watson, J. F. Herd, and L. H. Bainbridge-Bell. Applications of the Cathode Ray Oscillograph in Radio Research. 8vo. London, 1933. (*Presented by the Department of Scientific and Industrial Research.*)

INDEX.

Accounts of the Society, 1932–33, 411.
Additions to Library, 458
Aitken (A. C) On the Graduation of Data by the Orthogonal Polynomials of Least Squares, 54–78.
——— Awarded Makdougall-Brisbane Prize, 392, 394, 396, 407
Anderson-Berry Prize *See* Prizes
Auditor, Re-appointment of, 397
Awards of Prizes, 403.

Bailey (E. B.). Help from America in Reading Scottish Tectonics (*Title only.*) 392
Bartlett (M S). On the Theory of Statistical Regression, 260–283
Birth-rank and Intelligence, by J L. Gray and Pearl Moshinsky, 188–207.
Blaikley (Nellie M.) The Structure of the Foot in certain Mosses and in *Anthoceros lævis* (*Title only*: published in *Trans* , vol lvii) 395
Bradley (F. E). Obituary Notice of, 383
Briggs (H) and Sinha (R P.). Expansion and Contraction of Coal caused respectively by the Sorption and Discharge of Gas, 48–53
Briggs (H) *See* Kendall (P F.)
Bromine Diffusion Coefficients, by J E Mackenzie and H. W. Melville, 255–259.
Brough (J). Evolution of the Catopterid Fishes. (*Title only*) 390.
Bruce Prize. *See* Prizes
Bruce-Preller Lecture Fund. *See* Prizes
Bryce (T. H.). *See* Maclaren (N. H W.).

Calder (Mary G). Notes on the Kidston Collection of Fossil Plant Slides No I The Anatomy of the Axis of *Lepidodendron Brownii* Unger sp., with special reference to the Relationship between the Stem and *Lepidostrobus Brownii* Unger, sp. (*Title only*: published in *Trans.*, vol. lvii) 390.
——— Notes on the Kidston Collection of Fossil Plant Slides. No. II The Anatomy of the Axis of *Bothrodendron mundum* Williamson sp. (*Title only*: published in *Trans.*, vol. lvii.) 393.
Cannon (H. G.). On the Feeding Mechanism of certain Marine Ostracods. (*Title only*. published in *Trans* , vol. lvii.) 393.
Carnegie Trust, Thanks for Grants, 397.
Clark (W. I.). Obituary Notice of, 354.
Cleat of Coal, Formation of Rock Joints and the, by P. F. Kendall and H. Briggs, 164–187.
Clerk (Sir D.), Obituary Notice of, 355–356
Coal, Expansion and Contraction of, caused by Sorption and Discharge of Gas, by H. Briggs and R P Sinha, 48–53.
Consanguineous Parentage, Effect of, on Metrical Characters of Offspring, by L T Hogben, 239–251
Continued Fractions, Matrix Treatment of, by H W Turnbull, 151–163
——— Method of Matrices, by H W Turnbull, 208–219
Contributions, Voluntary, 419
Council, 1932–33, 389, 1933–34, 397, 420.
Cousins, Correlation for the Intelligence of, by J L Gray and Pearl Moshinsky, 188–207
Craib (W G), Obituary Notice of, 357–358
Craig (R M) *See* Jehu (T J).
Crew (F A E) A case of Non-disjunction in the Fowl, 89–104.
Crossing-over, The Cytological Basis of Suppression of, in Spermatogenesis in *Drosophila obscura*, by P Ch Koller and Th Townson, 130–146
Cuthbertson (J.), Obituary Notice of, 359

David Anderson-Berry Prize *See* Prizes
Diffusion Coefficients, Br_2–A, Br_2–CH_4, Br_2–HCl, Br_2–N_2O, by J E Mackenzie and H. W Melville, 255–259
Distance, Spatial, in a Curved Space-time, by H S. Ruse, 79–88.
Dix (Emily) The Sequence of Floras in the Upper Carboniferous, with special reference to South Wales (*Title only* published in *Trans.*, vol lvii.) 393.
Dolerites, Spitsbergen, Geology and Petrology of, by G. W. Tyrrell and K S Sandford, 284–321
Drosophila obscura, The Cytological Basis of Suppression of Crossing-over in Spermatogenesis in, by P. Ch. Koller and Th Townson, 130–146

Election of Fellows, 391
——— Honorary Fellows, 393.
Expansion and Contraction of Coal caused respectively by the Sorption and Discharge of Gas, by H Briggs and R. P. Sinha, 48–53

Fellows, Lists of Honorary, 447.
——— List of, 421
——— Obituary Notices, 354–384.
——— Elected, 391, 449. Honorary, 393, 449

Fellows, Deceased and Resigned, 449.
Fins, Supernumerary Pectoral in *Raia circularis* Loudon, by W. F. Harper, 26–30.
Fleming (Amy M.) The Innervation of the Uterus. (*Title only*: published in *Trans.*, vol. lvii.) 390.
Fowl, Non-disjunction in, by F. A. E. Crew, 89–104.
Fowler (W. H.), Obituary Notice of, 360–361.
Fraser (J. R.), Obituary Notice of, 383.
Fullarton (Margaret H.). On the Development of the Olfactory Organ in *Protopterus*, 1–6.

Gardiner (F.), Obituary Notice of, 362.
Gas, Expansion and Contraction of Coal caused by, by H. Briggs and R. P. Sinha, 48–53.
Geodesics, Null, and the Theory of Radiation, by W. O. Kermack, W. H. M'Crea, and E. T. Whittaker, 31–47.
German Periodicals, four discontinued, 397.
Glaister (J.), Obituary Notice of, 363–364.
Grants towards Publication, 397.
Gray (J. G.) Self-erecting Gyrostats. (*Title only* Address.) 391.
Gray (J. L.) and Moshinsky (Pearl) Studies in Genetic Psychology. The Intellectual Resemblance of Collateral Relatives, 188–207.
Gresson (R. A. R.) Studies on the Gametogenesis of *Stenophylax stellatus*, Curt (Trichoptera) Oogenesis, 322–346.
Guinea-Pig—Variations in the Œstrous Cycle in Virgin Animals, after Parturition, and during Pregnancy, by T. Nicol, 220–238.
Gunning-Victoria Jubilee Prize. *See* Prizes.

Hamilton (W. J.) The Restoration and Regeneration of the Epithelium and Endometrium of the Uterus of *Cavia* Post Partum in Non-pregnant Animals (*Title only*: published in *Trans.*, vol. lvii.) 390.
Harper (W. F.) Supernumerary Pectoral Fins in *Raia circularis* Loudon, 26–30.
Henderson (J.), Obituary Notice of, 383.
Herrman (L.) and Hogben (L.) The Intellectual Resemblance of Twins, 105–129.
Hippa asiatica, The Fæcal Pellets of, by H. B. Moore, 252–254.
Hogben (L. T.) A Matrix Notation for Mendelian Populations, 7–25.
——— The Effect of Consanguineous Parentage upon Metrical Characters of the Offspring, 239–251.
——— *See* Herrman (L.).
Holland (W. J.), Obituary Notice of, 365–366.
Honorary Fellows, List of, 447.
——— Elected, 393, 449; deceased, 449.
Hora (S. L.). Development and Probable Evolution of the Suctorial-disc in the Tadpoles of *Rana afghana*, Günther. (*Title only*: published in *Trans.*, vol. lvii.) 390.
Hutchison (A. G.) The Metamorphism of the Deeside Limestone. (*Title only*: published in *Trans.*, vol. lvii.) 390.
Hydrogen, Mathematical Representation of the Energy Levels of the Secondary Spectrum of, by Ian Sandeman, 347–353.
Hyslop (T. B.), Obituary Notice of, 383.

Innes (R. T. A.), Obituary Notice of, 367.
Intellectual Resemblance of Twins, by L. Herrman and L. Hogben, 105–129.

Jardine (R.), Obituary Notice of, 383.
Jehu (T. J.) and Craig (R. M.) Geology of the Outer Hebrides. Part V.—North Harris and Lewis (*Title only*: published in *Trans.*, vol. lvii.) 394.

Keith Prize. *See* Prizes.
Kendall (P. F.) and Briggs (H.) The Formation of Rock Joints and the Cleat of Coal, 164–187.
Kermack (W. O.), M'Crea (W. H.) and Whittaker (E. T.) On Properties of Null Geodesics, and their Application to the Theory of Radiation, 31–47.
Kerr (T.) On the Pituitary in *Lepidosiren* and its Development, 147–150.
Koller (P. Ch.) and Townson (Th.) Spermatogenesis in *Drosophila obscura* Fallen. I. The Cytological Basis of Suppression of Crossing over, 130–146.

Lander (C. H.) Bruce-Preller Lecturer, 392, 396, 398, 410.
Lang (W. H.) Contributions to the Study of the Old Red Sandstone Flora of Scotland. Part VIII. On *Arthrostigma*, *Psilophyton*, and some Associated Plant-remains from the Strathmore Beds of the Caledonian Old Red Sandstone (*Title only*: published in *Trans.*, vol. lvii.) 390.
Laws, 450.
Least Squares, Graduation of Data by, by A. C. Aitken, 54–78.
Lepidosiren, Structure and Development of Pituitary in, by T. Kerr, 147–150.
Library, Additions, 458.
——— German Periodicals, four discontinued, 397.
——— Shelving, 397.

M'Crea (W. H.). *See* Kermack (W. O.) and Whittaker (E. T.).
Mackenzie (J. E.) and Melville (H. W.) The Diffusion Coefficients of Bromine-Argon, Bromine-Methane, Bromine-Hydrogen Chloride, Bromine-Nitrous Oxide, 255–259.
——— Experimental Demonstration of the Measurements of the Diffusion Coefficients of Bromine-Hydrogen and Bromine-Carbon Dioxide. (*Title only*.) 391.
Maclaren (N. H. W.) and Bryce (T. H.). The Early Stages in Development of *Cavia* (*Title only*. published in *Trans.*, vol. lvii.) 392.
Makdougall-Brisbane Prize. *See* Prizes.
Marr (J. E.), Obituary Notice of, 368.

Matrices and Continued Fractions, by H. W. Turnbull, 151–163.
——— II., by H. W. Turnbull, 208–219
Matrix Notation for Mendelian Populations, by L. T. Hogben, 7–25
Meetings, Proceedings of Ordinary, 390.
——— Proceedings of Statutory, 387, 396.
Melville (H W.). *See* Mackenzie (J. E)
Mendelian Populations, Matrix Notation for, by L T Hogben, 7–25.
Metrical Characters, Effect of Consanguineous Parentage on, by L T. Hogben, 239–251
Mitchell (A. C). The Diurnal Incidence of Disturbance in the Terrestrial Magnetic Field (*Title only*. published in *Trans*, vol lvii.) 391.
Moore (H. B) The Fæcal Pellets of *Hippa asiatica*, 252–254.
Mort (S), Obituary Notice of, 384
Moshinsky (Pearl). *See* Gray (J L.).

Neill Prize *See* Prizes
Nicol (T.). Studies on the Reproductive System in the Guinea-Pig Variations in the Œstrous Cycle in the Virgin Animal, after Parturition, and during Pregnancy, 220–238.
Non-disjunction in the Fowl, by F A E Crew, 89–104.
Null Geodesics, and the Theory of Radiation, by W. O Kermack, W. H. M'Crea, and E. T Whittaker, 31–47.

Obituary Notices.—Francis Ernest Bradley, William Inglis Clark, Sir Dugald Clerk, William Grant Craib, John Cuthbertson, William Hope Fowler, Joseph Robert Fraser, Frederick Gardiner, John Glaister, John Henderson, William Jacob Holland, Theophilus Bulkeley Hyslop, Robert T A. Innes, Robert Jardine, John Edward Marr, Spencer Mort, Ralph Richardson, Johannes Schmidt, John Stephenson, John W Tait, James Edward Talmage, Sir J. Arthur Thomson, John Millar Thomson, and William Arthur Williams, 354–384
Œstrous Cycle Variations in the Guinea-Pig in Virgin Animals, after Parturition, and during Pregnancy, by T Nicol, 220–238
Olfactory Organ. Its Development in *Protopterus*, by Margaret H. Fullarton, 1–6
Oogenesis· Studies on the Gametogenesis of *Stenophylax stellatus*, Curt., by R. A. R Gresson, 322–346.

Papers read, 1932–33, 390.
Pellets, Fæcal, of *Hippa asiatica*, by H. B. Moore, 252–254.
Pituitary, Its Structure and Development in *Lepidosiren*, by T. Kerr, 147–150.
Polar Year, 396.
Polynomials, Orthogonal, Fitting of Data by, by A C. Aitken, 54–78.
Prizes, Bruce, 401, 409.
——— Bruce-Preller Lecture Fund, 392, 396, 398, 410.
Prizes, David Anderson-Berry, 396, 401, 410.
——— Gunning Victoria Jubilee, 392, 394, 396, 409
——— Keith, 399, 403.
——— Makdougall-Brisbane, 392, 394, 396, 407
——— Neill, 400, 407.
——— Scott, 392, 396, 400, 409.
——— Rules and Awards, 399, 403.
——— *See* in Accounts, 414–417
Proceedings of Ordinary Meetings, 390
——— Statutory Meetings, 387, 396.
Protopterus —On the Development of the Olfactory Organ of, by Margaret H Fullarton, 1–6

Radiation, Null Geodesics and the Theory of, by W O Kermack, W H M'Crea, and E T. Whittaker, 31–47
Raia circularis, Loudon, Supernumerary Pectoral Fins in, by W F Harper, 26–30.
Regression Coefficients, Distribution and Statistical Significance of, by M S Bartlett, 260–283
Relativity, Measurement of Spatial Distance in, by H S Ruse, 79–88
Reports, General Secretary's and Treasurer's, 396, 397
Representatives (R S E), Committees, Congresses, etc , 396.
Richardson (R), Obituary Notice of, 369.
Rock Joints, Formation of, and the Cleat of Coal, by P. F Kendall and H Briggs, 164–187
Royal Society, Government Publication Grant, 397
Ruse (H S) On the Measurement of Spatial Distance in a Curved Space-time, 79–88.

Sandeman (I). The Mathematical Representation of the Energy Levels of the Secondary Spectrum of Hydrogen, 347–353
Sandford (K S) *See* Tyrrell (G W.)
Schmidt (J), Obituary Notice of, 370–373.
Scott Prize *See* Prizes
Sex-linked Genes and Intelligence, by J L. Gray and Pearl Moshinsky, 188–207
Shelving added to Library, 397
Sibs, Correlation for Intelligence of, by J. L Gray and Pearl Moshinsky, 188–207
Simpson (J B) The Late-glacial Readvance Moraines of the Highland Border west of the River Tay (*Title only* published in *Trans*, vol. lvii) 392
Sinha (R P) *See* Briggs (H.)
Smith (W. W) and Tait (T). The Abnormal Atomic Weight of Calcium contained in Two very old Potassium-rich Deposits at Rhiconich, Sutherlandshire, and Portsoy, Banffshire (*Title only*) 394
Sommerfeld (A) Awarded James Scott Prize, 392, 396, 409
——— Ways to the Knowledge of Nature. (*Title only*. Address.) 392
Spectrum of Hydrogen, Mathematical Representation of the Energy Levels of, by Ian Sandeman, 347–353.

Spitsbergen, Geology and Petrology of Dolerites of, by G. W. Tyrrell and K. S. Sandford, 284–321.
Statistical Regression, On the Theory of, by M. S. Bartlett, 260–283.
Statutory General Meetings, 387, 396.
Stenophylax stellatus, Curt.—Studies on the Gametogenesis of: Oogenesis, by R. A. R Gresson, 322–346.
Stephen (A. C.). Studies on the Scottish Marine Fauna: the Natural Faunistic Divisions of the North Sea, illustrated by the Quantitative Distribution of the Mollusca. (*Title only*: published in *Trans*, vol. lvii.) 391.
——— Studies in the Scottish Marine Fauna. —A The Quantitative Distribution of the Echinoderms in the North Sea B The Validity of Petersen's Community Hypothesis in the North Sea (*Title only*· published in *Trans*, vol lvii) 393.
Stephenson (J), Obituary Notice of, 374–378.

Tait (J. W), Obituary Notice of, 384
Tait (T). *See* Smith (W W).
Talmage (J E), Obituary Notice of, 384.

Thomson (Sir J. A.), Obituary Notice of, 379–380.
Thomson (J. M.), Obituary Notice of, 384.
Townson (Th) *See* Koller (P. Ch.).
Transactions, *R.S.E.*, Index of Papers, 464
Turnbull (H. W). Matrices and Continued Fractions, 151–163.
——— Matrices and Continued Fractions, II., 208–219.
Twins, Intellectual Resemblance of, by L. Herrman and L. Hogben, 105–129.
Tyrrell (G. W) and Sandford (K. S.) The Geology and Petrology of the Dolerites of Spitsbergen, 284–321.

Voluntary Contributors, 419.

Walker (Sir J). Awarded Gunning Victoria Jubilee Prize, 392, 394, 396, 409
Whittaker (E. T) *See* Kermack (W. O.) and M'Crea (W. H)
Williams (S). A Contribution to the Experimental Morphology of *Lycopodium Selago*, with special reference to the Development of Adventitious Shoots. (*Title only* published in *Trans*, vol lvii) 395.
Williams (W A), Obituary Notice of, 381

Index of Papers published in the "Transactions" during Session 1932–33.

Bailey (E. B.) and Weir (J.). Submarine Faulting in Kimmeridgian Times· East Sutherland, vol lvii, No 14, p 429
Blaikley (Nellie M) The Structure of the Foot in certain Mosses and in *Anthoceros lævis*, vol. lvii, No. 28, p 699
Bryce (T H). *See* Maclaren (N. H. W.)

Calder (Mary G). Notes on the Kidston Collection of Fossil Plant Slides No. I The Anatomy of the Axis of *Lepidodendron Brownii* Unger sp, with special reference to the Relationship between the Stem and *Lepidostrobus Brownii* Unger sp., vol. lvii, No. 19, p 547.
——— Notes on the Kidston Collection of Fossil Plant Slides. No. II. The Anatomy of the Axis of *Bothrodendron mundum* Williamson sp., vol lvii, No. 26, p. 665.
Cannon (H. G.). On the Feeding Mechanism of certain Marine Ostracods, vol. lvii, No. 30, p. 739.

Davies (L. M.). The Genera *Dictyoconoides* Nuttall, *Lockhartia* nov., and *Rotalia* Lamarck: Their Type Species, Generic Differences, and Fundamental Distinction from the *Dictyoconus* Group of Forms, vol. lvii, No 13, p. 397.

Fleming (Amy M.). The Innervation of the Uterus, vol. lvii, No. 16, p. 473.

Hamilton (W. J.). The Restoration and Regeneration of the Epithelium and Endometrium of the Uterus of *Cavia* Post Partum in Non-pregnant Animals, vol. lvii, No 21, p. 593
Hora (S. L) Development and Probable Evolution of the Suctorial Disc in the Tadpoles of *Rana afghana* Günther, vol. lvii, No 15, p. 469.
Hutchison (A. G). The Metamorphism of the Deeside Limestone, Aberdeenshire, vol lvii, No 20, p. 557.

Lang (W H.). Contributions to the Study of the Old Red Sandstone Flora of Scotland. VIII On *Arthrostigma*, *Psilophyton*, and some Associated Plant-remains from the Strathmore Beds of the Caledonian Lower Old Red Sandstone, vol. lvii, No. 17, p. 491.

Maclaren (N H. W.) and Bryce (T. H.). The Early Stages in the Development of *Cavia* vol. lvii, No. 25, p. 647.
Mitchell (A. C.). The Diurnal Incidence of Disturbance in the Terrestrial Magnetic Field, vol. lvii, No. 23, p. 617.

Simpson (J. B.). The Late-Glacial Readvance Moraines of the Highland Border West of the River Tay, vol. lvii, No. 24, p. 633.

Stephen (A. C) Studies on the Scottish Marine Fauna: The Natural Faunistic Divisions of the North Sea as shown by the Quantitative Distribution of the Molluscs, vol. lvii, No. 22, p. 601.

Trapp (G) A Study of the Foliar Endodermis in the Plantaginaceæ, vol lvii, No. 18, p 523.
Tyrrell (G. W.). Stratigraphical Observations in the Stor Fjord Region of Spitsbergen With an Appendix on the Mesozoic Fossils from Spitsbergen collected by Dr G. W. Tyrrell, by J. Weir, vol. lvii, No. 27, p. 675.

Weir (J). *See* Bailey (E B).
——— *See* Tyrrell (G W.)
Williams (S). A Contribution to the Experimental Morphology of *Lycopodium Selago*, with special reference to the Development of Adventitious Shoots, vol. lvii, No. 29, p 711

PRINTED IN GREAT BRITAIN BY NEILL AND CO., LTD., EDINBURGH.

I.A.R.I. 75

INDIAN AGRICULTURAL RESEARCH
INSTITUTE LIBRARY, NEW DELHI.

Date of Issue	Date of Issue	Date of Issue
12.10.60		
2-1-63		
8 7-63		
27 12-67		
1 OCT 1977		
11-78		

GIPNLK—H-40 I.A.R.I.—29-4-55—15,000